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The marginal social cost of travel time variability

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Abstract

This paper investigates the cost of travel time variability for car users at the peak hour. In particular, we derive the marginal social cost of travel time variability, which takes the feedback of travel time unreliability on the congestion profile into account. This is in contrast with the value of travel time variability, which treats congestion as an exogenous phenomenon. Congestion is modeled using the standard bottleneck model of road congestion, which we amend by adding a random delay. For individuals with (α,β,γ) preferences and uniformly distributed delays, the marginal social cost of travel time variability is strictly lower than the value of travel time variability. Moreover, we show that the former tends toward the latter when σ, the standard deviation of the random delay, tends toward +∞. For normally distributed delays, numerical application leads to similar conclusions. Analysis of data from the Paris area suggests that given the plausible range of σ, the marginal social cost of travel time variability is markedly lower than the value of travel time variability. When appraising the economic benefits of reliability improvements, one should prefer the marginal social cost of travel time variability for the peak period, and the value of travel time variability for the off-peak period.

Key words: Reliability; Congestion; Welfare; Bottleneck; Scheduling.

1. Introduction

Travel time reliability is an important consideration when people make travel decisions (Li et al., 2010; Carrion and Levinson, 2013). Bates et al. (2001) distinguish two main ways in which unreliable travel times impact people. First, it may disturb their activity schedule by causing early or late arrivals at destination. When excessive, the delay can even lead to the cancellation of an activity: for instance, an unpredicted traffic jam can cause one to arrive too late at the theater and miss the play. Second, people could dislike travel time variability *per se*. The unpredictability of the travel time would generate some form of anxiety, or cause some additional planning costs (Noland et al., 1998).

The valuation of travel time variability has accordingly become an increasing concern in transport economics, to the extent that the value of travel time variability (also referred to as the value of reliability) is now along with the value of time one of the most important values obtained from travel demand studies (Carrion and Levinson, 2012). The work of Fosgerau and Karlström (2010) has been central in this regard. In line with Polak (1987) and Noland and Small (1995), the authors study the choice of departure time for a traveler facing variable travel times. Their major contribution lies in deriving the value of travel time variability (VTTV) in a case relatively general, thereby providing a sound theoretical foundation to this notion. Still, one element limits the extent of their results. In line with the others before them, Fosgerau and Karlström (2010) study the behavior of one traveler only and model congestion as an exogenous phenomenon. Hence, the VTTV does not capture the impact that

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changes in travel time variability have on congestion. Likely limited for the off-peak period, this feedback effect could be sizable at rush hour when congestion becomes a primary feature of traffic. As all individuals (and not only one) adapt their departure time in response to changes in travel time variability, the congestion time profile is likely to change. To address this shortcoming of the VTTV, we propose an alternative indicator, the marginal social cost of travel time variability (MSCTTV), which differs from the VTTV in that it does capture the equilibrium mechanism just mentioned.

This paper studies the cost of travel time variability for car users when congestion is modeled as an endogenous phenomenon. We are especially interested in the spread between the value of travel time variability and the marginal social cost of travel time variability: for a given individual, it reflects the difference between the short term and long term (i.e. before and after other individuals adjust their departure time) costs of a change in the variability of travel times. We model congestion using the standard bottleneck model of peak-load congestion, which we amend by introducing a random delay. Derivation of the Nash equilibrium in departure times yields first the equilibrium expected scheduling disutility as a function of the standard deviation of the delay, then the MSCTTV by differentiation. We focus the analysis on the case of uniformly distributed delays as it leads to closed-form solutions. This allows a direct analytical comparison of the VTTV and the MSCTTV. The more realistic case of a normal law is also considered using numerical simulation, analytical derivation of the equilibrium solution being unfeasible in this case.

The primary impact of travel time variability on the Nash equilibrium is peak spreading. Departures are spread more evenly over the rush hour than under certain travel times, resulting in less congestion. As a result of this mitigating mechanism, the MSCTTV is strictly lower than the VTTV. When the scale of the random delay is small enough compared to the total length of the rush hour, the MSCTTV is even null in the case of a uniform law, and close to null in the case of a normal law. Individual adjustments regarding the departure time offset, partly or even entirely, the direct cost of an increase in travel time variability. As far as cost-benefit analysis is concerned, this means that direct uses of the VTTV, in particular in 4-step models, overestimate the benefits of policies aiming to reduce travel time variability on a road network, and likely strongly so. One should consider the effect of such policies on the departure time profile (and thus on the O-D matrix when using a 4-step model) to avoid this bias, one solution being to use the MSCTTV. Additionally, our relatively strong result that the MSCTTV is null in some cases tends to corroborate the finding of Börjesson et al. (2012), which is that (α,β,γ) preferences only capture part of the cost that travel time variability puts on travelers. Future research might want to delve into this issue.

The layout of the paper is as follows. Section 2 reviews the prior literature. The model is presented in Section 3. Section 4 elaborates on the case where the random delay follows a uniform law, while Section 5 addresses more succinctly the case of a normal law, using numerical simulation. Section 6 discusses the results, and Section 7 concludes.

2. Prior literature

This paper is directly related to two main strands of the literature on the cost of travel time variability. The first one focuses on the individual and his preferences through the notion of value of reliability. The second one considers interactions between individuals and studies the impact of travel time variability on the traffic equilibrium and the cost of travel. We shortly review both.

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2.1. *Value of travel time variability*

The value of travel time variability captures how much an individual values (in monetary terms) the cost associated with a marginal increase in travel time variability. As such, it is mainly a property of preferences. The literature on this topic is structured according to three main modeling frameworks (Li et al., 2010; Carrion and Levinson, 2012): the mean-variance model, the mean lateness model, and the scheduling model. The mean-variance and mean lateness models follow a descriptive approach. While they assume that individuals dislike travel time variability, they do not purport to explain why. These two modeling frameworks provide valuable econometric specifications to estimate the VTTV, but lack theoretical foundations.[1]

Scheduling models aim to fill this gap and provide a micro-economic foundation to the value of reliability. They represent the choice of departure time for an individual who faces time constraints (e.g. work start time). Typically, they assume that a traveler has a scheduling disutility function $U(t,T)$, which depends on the departure time t and travel time T, and study the optimal departure time and the associated minimum disutility. Building on Gaver (1968) and Vickrey (1969), Small (1982) has specified and estimated the following functional form for the scheduling disutility, which is widely used in works on the value of travel time variability:

$$U(t,T) = \alpha T + \beta(t^* - t - T)^+ + \gamma(t + T - t^*)^+ + \delta \mathbf{1}(t + T - t^*) \tag{1}$$

where $(x)^+ = x$ if x is positive, 0 otherwise, and $\mathbf{1}(x)$ is the Heaviside step function (equal to 1 if $x \geq 0$, 0 otherwise). $t^* - t - T$ is the schedule delay. It is measured relatively to a preferred arrival time t^*, which usually represents the work starting time. The cost of one minute of travel time is α; the cost of being one minute early at your destination is β and the cost of being one minute late is γ. Lastly, δ is the fixed penalty for being late. These parameters set the terms of the trade-off between travel time and schedule delay when choosing the departure time. We will refer to (1) as $(\alpha,\beta,\gamma,\delta)$ preferences, or more simply (α,β,γ) when the late dummy is not included ($\delta = 0$).

To study the value of travel time variability, most authors consider the case of a single individual with $(\alpha,\beta,\gamma,\delta)$ preferences who faces random travel times (e.g. Noland and Small, 1995; Bates et al., 2001; Batley, 2007).[2] Travel time T is the sum of a deterministic term and of a random delay, with two central assumptions: 1) the deterministic component is an exogenous function of the departure time; 2) the distribution of the delay is independent of the departure time. In the special case where the traveler has (α,β,γ) preferences and where the deterministic travel time is a constant T_0, Fosgerau and Karlström (2010) show that:

$$t_d = t^* - T_0 - \sigma\Phi^{-1}\left(\frac{\gamma}{\beta+\gamma}\right) \tag{2}$$

$$E\left[U(t_d, T_0)\right] = \alpha T_0 + (\beta+\gamma)\sigma\int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(s)\,ds \tag{3}$$

[1] As pointed out later, Fosgerau and Karlström (2010) provided a theoretical grounding to the mean-variance formulation *a posteriori*, using the scheduling model. The mean-variance model remains a descriptive model *per se* nonetheless.

[2] A few works consider other utility functions than the standard $(\alpha,\beta,\gamma,\delta)$ preferences (e.g. Siu and Lo, 2009; Fosgerau and Engelson, 2011), including the Vickrey-Tseng-Verhoef preferences popularized by Vickrey (1973) and Tseng and Verhoef (2008). The general framework remains the same.

where t_d is the optimal departure time and Φ the cumulative distribution function (c.d.f.) of the standardized travel time. Compared to when travel times are deterministic, the traveler takes a safety margin (which may be positive or negative), which is represented by the term $\sigma\Phi^{-1}(\gamma/(\beta+\gamma))$ in (2). Travel time variability lowers the capacity of the individual to be on time nonetheless, and the cost of travel (or scheduling disutility) is increased by the integral term in (3).

Using the standard definition of the value of travel time variability as the derivative of the scheduling disutility (expressed in monetary terms) with respect to the standard deviation of travel times (Carrion and Levinson, 2012), Fosgerau and Karlström (2010) find:

$$\text{VTTV} = (\beta+\gamma)\int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(s)\,ds \tag{4}$$

The value of reliability only depends on the schedule delay cost parameters β and γ and on the c.d.f. Φ of the standardized random delay. This result has had considerable influence at various levels. First, it shows that the mean-variance model is formally equivalent to the scheduling model (given the above assumptions). This validates prior empirical studies based on the mean-variance framework.[3] Second, it provides a simple formula for the VTTV, making it easy to integrate in national guidelines for the economic appraisal of transportation projects.

Formula (4) is based on the assumption that the deterministic component of the travel time is constant. Noland and Small (1995) and Fosgerau and Karlström (2010) consider the more general case where the deterministic component may vary with the departure time to represent congestion.[4] Notwithstanding, congestion is always treated as an exogenous phenomenon, which is consistent with the view that the VTTV should only reflect individual preferences, not how the whole system reacts when travel time variability changes. Thus, and by definition, the VTTV does not consider the feedback of travel time unreliability on congestion: it measures the marginal cost of travel time variability in disequilibrium, that is, when the other individuals do not adjust their departure times.

2.2. Equilibrium models

Yet, congestion is the sum of the trip timing decisions of all concerned individuals. If one traveler optimizes his departure time when facing random travel times, there is no reason to assume that the other travelers would not do the same. The congestion profile is likely to change depending on the level of travel time variability, and subsequently the cost of travel.

To tackle this question, several works study the impact of travel time variability on the departure time profile and the resulting traffic equilibrium. Noland et al. (1998) analyze the impact of road incidents in a simulation model which combines a supply side model *à la* Henderson[5] with a discrete choice demand model that predicts scheduling choices for morning commute trips. The bottleneck model provides an alternative representation of the supply side, and was applied to study various aspects contributing to the variability of travel time: stochastic demand (de Palma et al., 1983; Arnott et al., 1991), stochastic capacity (Arnott et al., 1999, 1996; Li et al., 2008) and incidents (Fosgerau, 2010).

[3] This result is actually controversial, not on a theoretical ground but when trying to put it to practice. While Noland et al. (1998) support it, Börjesson et al. (2012) find that the valuations of travel time variability given by the mean-variance approach and the scheduling approach are inconsistent.
[4] Fosgerau and Karlström (2010) find that doing so only marginally changes the VTTV and that (4) still applies in first approximation.
[5] In his model, Henderson (1981) treats time as a discrete variable and represents congestion using the standard speed-flow curves. See Chu (1995) for a comparison of the bottleneck model with Henderson's approach.

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However, these works do not specifically analyze the relationship between the cost of travel and the standard deviation of travel times.

This paper studies the cost of travel time variability in the case of endogenous congestion. In particular, we seek to compare the marginal social cost of variability and the value of travel time variability, for the rush hour period. In line with the literature using equilibrium models, the scope of our analysis is restricted to car users, and to the impact of travel time variability on the choice of departure time. We do not consider other possible reactions such as changing mode (Chorus et al., 2006) or route (Liu et al., 2004). This work assumption seems sensible in a first approach, given that changing departure time is typically the easiest response to travel time variability (Li et al., 2010). Last, our work bears similarities with that of Siu and Lo (2009), who also study the impact of travel time variability when congestion is endogenous. However, the two works have different focuses: the dynamics of congestion in the case of Siu and Lo (2009), and the relationship between the cost of travel and travel time variability in our case.

3. The model

3.1. Model set-up

In the bottleneck model of road congestion with peak-load demand, a fixed number N of individuals commute every day from point A (home) to point B (work). Each individual travels with his own car, along the single road that joins A and B. The road starts with a bottleneck with fixed capacity s. Accordingly, whenever the departure rate $r(t)$ exceeds s, a queue develops at the bottleneck entrance (if it does not yet exist).

In the standard setting, the travel time is the sum of two elements: a fixed component, representing the time necessary to go from A (or more accurately from the bottleneck exit) to B, and a variable one, representing the time spent in the queue (if any). Both terms are deterministic. In line with Coulombel and de Palma (2013), we add a stochastic component to model travel time variability. This takes the form of a centered random delay, noted ε_t, which may vary over the course of the day.[6] Accordingly, the travel time $\tilde{T}$ takes the following form:

$$\tilde{T}(t) = T_0 + \frac{Q(t)}{s} + \varepsilon_t \tag{5}$$

where a tilde emphasizes the stochastic nature of a variable. T_0 is the free flow travel time. $Q(t)/s$ is the congestion delay, equal to the ratio between the queue length $Q(t)$ and the bottleneck capacity s. For reminder, the queue length varies as follows:

$$\begin{cases} Q'(t) = \left[r(t) - s\right]^+ & \text{if } Q(t) = 0 \\ Q'(t) = r(t) - s & \text{if } Q(t) > 0 \end{cases} \tag{6}$$

[6] Strictly speaking, ε_t encompasses all phenomena leading to travel time variability other than those related to fluctuations in demand (N) or in the bottleneck capacity (s). This includes weather conditions, variations in the driving speed after exiting the bottleneck and the parking time at destination among other things. Furthermore, the role of the provision of dynamic information regarding ε_t is not considered in this paper. This excludes dynamic strategies based on such information (like choosing to leave when travel times are lower than usual). The time dynamics of ε_t has therefore no importance here, and any behavior can be assumed as long as the basic assumptions mentioned further hold.

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In order to allow a direct comparison of our results with Fosgerau and Karsltröm (2010), we assume that individuals have (α,β,γ) preferences. The scheduling disutility is given by the random variable:

$$U(t) = \alpha T(t) + \beta\left[t^* - t - T(t)\right]^+ + \gamma\left[t + T(t) - t^*\right]^+ \tag{7}$$

As is usual in the literature, we set the following constraint: $\beta \leq \alpha$.[7]

The choice of (α,β,γ) preferences implies that we disregard anxiety costs or any additional planning cost that would not be captured by this specification. While Noland et al. (1998) find that (α,β,γ) preferences well fit the behavior of individuals in their survey, Tseng and Verhoef (2008) argue that the assumption of time-invariant shadow prices is unrealistic, a point corroborated by their empirical analysis. Moreover, Börjesson et al. (2012) find that (α,β,γ) preferences do not well capture the aversion of individuals to travel time variability. While acknowledging these points, the choice of (α,β,γ) preferences is driven by one of our main objectives, which is to compare our findings to those of Fosgerau and Karsltröm (2010). It leads to closed-form solutions in Section 4 to boot.[8]

Also, we assume that individuals have homogeneous preferences: they all wish to arrive at the same t^* and all have the same triplet (α,β,γ). True, relaxing this assumption would make the model more realistic. For instance, considering a distribution of the (α,β,γ) among the population instead of a unique value would allow to model the fact that the trip purpose may vary among travelers.[9] Still, this assumption makes the model analytically tractable, while not being central to our main findings.[10]

Each individual seeks to minimize his expected scheduling disutility by adjusting his departure time. A Nash equilibrium is reached when no one has an incentive to shift his departure anymore. The equilibrium is fully characterized by the departure rate function $r(t)$, with the condition that the expected trip cost $\mathbf{E}\left[U(t)\right]$ must be minimum and constant on the set $\{t \,/\, r(t) > 0\}$.

3.2. Equilibrium solution with deterministic travel times

We start by reviewing the main findings when travel times are deterministic, *i.e.* when $\varepsilon_t = 0 \;\; \forall t \in \Re$, which is the standard bottleneck model of road congestion with peak-load demand.[11]

First, there exists a unique pure Nash equilibrium. Its main characteristics are as follows (subscript d denoting the deterministic case as opposed to the stochastic case that will follow): (1) individuals leave home in a continuous (but not homogenous) way between t_d^0 and t_d^1; (2) the queue length is strictly positive on $\left]t_d^0, t_d^1\right[$, and the bottleneck always operates at full capacity on this interval. Consequently, the length of the rush hour is: $t_d^1 - t_d^0 = N/s$; (3) the departures rate varies so as to equalize the scheduling disutility throughout the rush hour. This entails the following condition:

[7] The case $\beta > \alpha$ leads to counterintuitive behaviors, such as preferring to waste time in the vehicle (by making circles around the destination for instance) rather than to get early at one's destination.
[8] We could easily apply the same framework to another utility function. Closed-form solutions would not be guaranteed in the uniform case, however, meaning that we would have to resort to numerical analysis.
[9] Heterogeneity in the bottleneck model with deterministic travel times is discussed in depth in Lindsey (2004).
[10] As we will see in Section 4, the primary impact of travel time variability is that departures are spread more evenly over the rush hour than when travel times are certain. This phenomenon would still occur even if the parameters t^* and (α,β,γ) were to be distributed, which is why we believe our main findings to be robust to the assumption of constant t^* and (α,β,γ).
[11] This case is extensively discussed in the literature (see Arnott et al., 1990, 1993, among others).

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$$\begin{cases} T'(t) = \dfrac{\beta}{\alpha - \beta} & \text{for } t \in \left[t_d^0, t_d^p\right] \\ T'(t) = -\dfrac{\gamma}{\alpha + \gamma} & \text{for } t \in \left[t_d^p, t_d^1\right] \end{cases} \tag{8}$$

Outside $\left[t_d^0, t_d^1\right]$, there is no congestion but the cost of travel is greater than inside $\left[t_d^0, t_d^1\right]$ because of schedule delay costs. During the first part of the rush hour $\left[t_d^0, t_d^p\right]$, the departure rate is greater than *s*. Queue length increases and reaches its maximum at the peak t_d^p. Waiting time is also greatest at t_d^p, conversely the schedule delay is minimal and null. People leaving before the peak are thereby early, while those leaving after the peak are late.

These various results are illustrated in Figure 1. As we can see, cumulative departures rise quickly during the first part of the rush hour $\left[t_d^0, t_d^p\right]$, then grow at a slower rate, until everybody has left at t_d^1. Because it has a limited capacity, the bottleneck acts as a filter and people arrive at their destination at a constant rate *s*, which one can observe on the curve of cumulative arrivals. The horizontal distance between the two curves gives the total travel time, which is the sum of the free flow travel time T_0 and the queuing time. Again, we see that the queuing time is null for people leaving at the beginning or at the end of the rush hour, while it is maximum for people leaving at t_d^p.

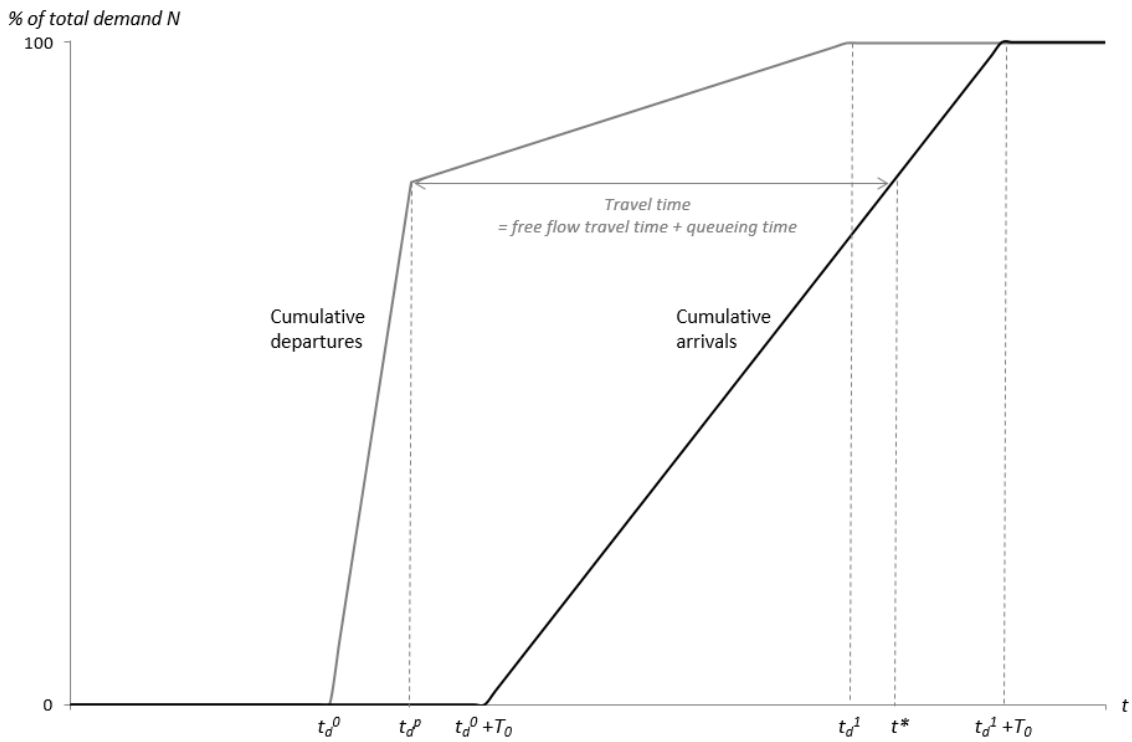

Figure 1: The standard bottleneck model

Finally, solving (8) yields the equilibrium solution:

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$$\begin{cases} t_d^0 = t^* - T_0 - \dfrac{\gamma}{\beta+\gamma}\dfrac{N}{s} \\ t_d^p = t_d^0 + \dfrac{\alpha-\beta}{\alpha}\dfrac{\gamma}{\beta+\gamma}\dfrac{N}{s} \\ t_d^1 = t^* - T_0 + \dfrac{\beta}{\beta+\gamma}\dfrac{N}{s} \end{cases} \quad \text{and} \quad \begin{cases} T\left(t_d^p\right) = T_{\max} = T_0 + \dfrac{1}{\alpha}\dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} \\ U_d^* = \alpha T_{\max} = \alpha T_0 + \dfrac{\beta\gamma}{\beta+\gamma}\dfrac{N}{s} \end{cases} \tag{9}$$

where U_d^* is the equilibrium disutility in the deterministic case.

3.3. Equilibrium solution with stochastic travel times

As previously pointed out by Bates et al. (2001), the most general case where the set of distributions of $(\varepsilon_t)_{t\in\Re}$ has no specific properties does not yield any significant result. To carry the analysis further, we assume that the $(\varepsilon_t)_{t\in\Re}$ follow the same law, are centered, and have the same standard deviation σ.

The assumption of a constant standard deviation is likely the most questionable. On theoretical grounds, it implies that there is no direct relationship between the bottleneck and the random delay: the random delay takes place after the bottleneck, and is not influenced by the current queue.[12] On empirical grounds, while a few studies corroborate it (*e.g.* Hurdle et al., 1983), more recent studies typically find a positive correlation between the congestion level and the standard deviation of travel times (Fosgerau, 2010; OECD, 2010). Still, this assumption holds several merits. First and foremost, it is consistent with our goal to compare the MSCTTV and the VTTV, seeing that the VTTV is based on the same assumption of a constant standard deviation of travel time, whatever the departure time. Moreover, assuming a constant standard deviation makes computation much easier to follow, with no evidence at this point that it strongly determines our main results.[13] The probability distribution function (p.d.f.) and the c.d.f. of the normalized random delay ε_t/σ are denoted φ and Φ, respectively.

Existence and uniqueness of the equilibrium solution is demonstrated in the general case (*i.e.* for any distribution of the random delay) in Coulombel and de Palma (2013). In particular, they show that the equilibrium solution is characterized by the following conditions (where subscript s denotes the stochastic case):

$$\begin{cases} \mathbf{E}\left[U\left(t_s^0\right)\right] = \mathbf{E}\left[U\left(t_s^1\right)\right] = U_s^* \\ \bar{T}'(t) = \dfrac{\beta\Phi\left(m(t)\right) - \gamma\left(1-\Phi\left(m(t)\right)\right)}{\alpha - \left(\beta\Phi\left(m(t)\right) - \gamma\left(1-\Phi\left(m(t)\right)\right)\right)} \quad \forall t \in]t_s^0, t_s^1[\end{cases} \tag{10}$$

$$\text{with } t_s^1 = t_s^0 + N/s \text{ and } m(t) = \frac{t^* - t - \bar{T}(t)}{\sigma}$$

where $\bar{T}(t)$ is the expected travel time and $m(t)$ the normalized safety margin. t_s^0 and t_s^1 mark the beginning and the end of the rush hour in the stochastic case; they are determined using (10). Last, U_s^* is the equilibrium disutility in the stochastic case.

[12] However, the existence of the random delay does lead to changes in the departure times of the various individuals and thus to changes in the queue dynamics.

[13] Preliminary analyses corroborating that the assumption of a constant standard deviation does not play a central role in our main results regarding the MSCTTV are available upon request from the authors.

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Considering that there exists an equilibrium and that it is unique, we can define the marginal social cost of travel time variability as the derivative of the equilibrium expected disutility with respect to the standard deviation of travel times:

$$\text{MSCTTV} = \frac{\partial U_s^*}{\partial \sigma} \tag{11}$$

The definition of the MSCTTV is almost identical to that of the VTTV. The only difference lies in how the expected disutility is computed: for the MSCTTV it is computed at the Nash equilibrium, while for the VTTV it is computed at the individual optimum, assuming that the other individuals do not adjust their departure time.

We will now use (10) to analytically derive the Nash equilibrium and the marginal social cost of travel time variability in the case of uniformly distributed delays.

4. Uniformly distributed delays

This section examines the case where the random delay is distributed according to a uniform law. By doing so we do not aim at realism: most empirical studies find that travel times follow an either normal or log-normal law (Richardson and Taylor, 1978; Giuliano, 1989). The choice of a uniform law leads to closed-form solutions, however, which is not the case for most other distributions, including the normal and log-normal cases (to the best of our knowledge). The uniform case is thus of utmost interest as we can derive the MSCTTV analytically and directly compare it with the VTTV, using the formula provided by Fosgerau and Karlström (2010). The more realistic situation where delays follow a normal law is discussed in the next section, using numerical simulation given what was just said.

For reminder, in the case of a uniform distribution, φ and Φ are given by:

$$\begin{cases} \varphi(x) = \dfrac{1}{2\sqrt{3}} \\ \Phi(x) = \dfrac{1}{2} + \dfrac{x}{2\sqrt{3}} \end{cases} \quad \text{for } x \in \left[-\sqrt{3}, \sqrt{3}\right]. \tag{12}$$

4.1. Expected scheduling disutility

Using (7), the expected scheduling disutility when leaving at t is:

$$\mathbf{E}\left[U(t)\right] = \alpha\bar{T}(t) + \beta\int_{-\infty}^{\frac{t^*-t-\bar{T}(t)}{\sigma}} \left[t^*-t-\bar{T}(t)-\sigma u\right]\varphi(u)\,du + \gamma\int_{\frac{t^*-t-\bar{T}(t)}{\sigma}}^{+\infty} \left[t+\bar{T}(t)+\sigma u-t^*\right]\varphi(u)\,du \tag{13}$$

where $\bar{T}(t) = T_0 + Q(t)/s$ is the expected travel time. Using the notation $m(t) = \left[t^*-t-\bar{T}(t)\right]/\sigma$ for the normalized safety margin, we can rewrite (13) as follows:

$$\mathbf{E}\left[U(t)\right] = \alpha\bar{T}(t) + (\beta+\gamma)\sigma\left[m(t)\Phi(m(t)) + \Gamma(m(t))\right] - \gamma\sigma m(t) \tag{14}$$

where $\Gamma(x) = \int_x^{+\infty} u\varphi(u)\,du$ is a positive function. Calculation yields $\Gamma(x) = \frac{3-x^2}{4\sqrt{3}}$ for $x \in \left[-\sqrt{3}, \sqrt{3}\right]$, 0 otherwise. It ensues:

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$$\begin{cases} \mathbf{E}\left[U(t)\right] = \alpha\bar{T}(t) + \beta\sigma m(t) & \text{if } m(t) \geq \sqrt{3} \\ \mathbf{E}\left[U(t)\right] = \alpha\bar{T}(t) + \frac{\beta-\gamma}{2}\sigma m(t) + \frac{\beta+\gamma}{4}\sigma\frac{3+m(t)^2}{\sqrt{3}} & \text{if } m(t) \in \left[-\sqrt{3}, \sqrt{3}\right] \\ \mathbf{E}\left[U(t)\right] = \alpha\bar{T}(t) + \gamma\sigma m(t) & \text{if } m(t) \leq -\sqrt{3} \end{cases} \tag{15}$$

Because the random delay is bounded, people who leave with a large enough head start ($|m(t)| \geq \sqrt{3}$) are sure to be always early ($m(t) > 0$) or always late ($m(t) < 0$) at their destination, whatever the delay. For those, the expected scheduling disutility in (15) has the same formula as in the deterministic case (remember that $\sigma m(t) = t^* - t - \bar{T}(t)$). When choosing a smaller safety margin (case $|m(t)| \leq \sqrt{3}$), one may arrive before or after t^* depending on the delay one experiences, hence a different formula.

4.2. Travel time dynamics

Based on (10), the expected travel time satisfies:

$$\forall t \in \left[t_s^0, t_s^1\right] \quad \begin{cases} \bar{T}'(t) = \frac{\beta}{\alpha-\beta} & \text{if } m(t) \geq \sqrt{3} \\ \bar{T}'(t) = \dfrac{\frac{\beta+\gamma}{2}\frac{m(t)}{\sqrt{3}} + \frac{\beta-\gamma}{2}}{\alpha - \left\{\frac{\beta+\gamma}{2}\frac{m(t)}{\sqrt{3}} + \frac{\beta-\gamma}{2}\right\}} & \text{if } m(t) \in \left[-\sqrt{3}, \sqrt{3}\right] \\ \bar{T}'(t) = -\frac{\gamma}{\alpha+\gamma} & \text{if } m(t) \leq -\sqrt{3} \end{cases} \tag{16}$$

Again, when the safety margin is large enough (in absolute value), the expected travel time varies like in the deterministic case. In the case of a smaller margin, the travel time derivative is smoothed compared to when travel times are perfectly reliable. Indeed, when $m(t)$ varies from $-\sqrt{3}$ to $\sqrt{3}$, $\bar{T}'(t)$ shifts continuously from $\beta/(\alpha-\beta)$ to $-\gamma/(\alpha+\gamma)$, and not instantly and discontinuously as in the deterministic case.

The differential equation corresponding to $|m(t)| \leq \sqrt{3}$ can be integrated, which yields:

$$\bar{T}(t) = t^* - t - \frac{2\sqrt{3}\sigma}{\beta+\gamma}\left[\alpha + \frac{\gamma-\beta}{2} - \sqrt{\left(\alpha + \frac{\gamma-\beta}{2}\right)^2 - \frac{\beta+\gamma}{\sqrt{3}}\left(A - \frac{\alpha}{\sigma}t\right)}\right] \quad \text{if } m(t) \in \left[-\sqrt{3}, \sqrt{3}\right] \tag{17}$$

where A is a constant to be determined using boundary conditions.

4.3. Equilibrium solutions

Given the above elements, there are four classes of solutions based on the value of σ and the relative magnitude of β and γ.

The general intuition is as follows: the greater the travel time variability, *i.e.* the greater σ, the greater the number of people who adjust their departure time. At first, only a small fraction of the N drivers change their departure time, those who leave around the peak (case 1). Adjustments remain minor and perfectly offset the cost of travel time variability. At some point, either the first (case 2) or the last (case 3) driver to leave is impacted, and the rush hour period starts to shift (later in case 2, earlier

in case 3). Adjustments become more important and the equilibrium disutility increases. Eventually, σ is so important that all drivers may be early or late at their destination depending on the delay they experience (case 4). Individual adjustments become less and less efficient, and the cost of travel time variability markedly increases.

Case 1: $\sigma \le \frac{1}{\sqrt{3}} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s}$

Case 1 corresponds to the set of equilibrium solutions verifying $\left|m\left(t_s^0\right)\right| \ge \sqrt{3}$ as well as $\left|m\left(t_s^1\right)\right| \ge \sqrt{3}$. The maximum possible delay (equal to $\sqrt{3}\sigma$ in absolute value) is small enough compared to the length of the rush hour N/s. The first commuter to leave is sure to be early at his destination whatever the delay he experiences (condition $\left|m\left(t_s^0\right)\right| \ge \sqrt{3}$), similarly the last one is sure to be late (condition $\left|m\left(t_s^1\right)\right| \ge \sqrt{3}$). People leaving near the peak can arrive before or after t^* depending on the delay they experience.

The equilibrium solution verifies:

$$\begin{cases} t_s^0 = t^* - T_0 - \frac{\gamma}{\beta+\gamma}\frac{N}{s} \\ U_s^* = \alpha T_0 + \frac{\beta\gamma}{\beta+\gamma}\frac{N}{s} \end{cases} \tag{18}$$

The timing of the peak period and the equilibrium disutility are the same as in the deterministic case. On the other hand, and as we will observe in the numerical application in subsection 4.5, departure rates are different, leading to changes in the congestion profile.

Case 2: $\beta > \gamma$ *and* $\frac{1}{\sqrt{3}} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s} \le \sigma \le \frac{1}{4\sqrt{3}} \frac{\beta+\gamma}{\min(\beta,\gamma)} \frac{N}{s}$

This case regroups solutions verifying $\left|m\left(t_s^0\right)\right| \le \sqrt{3}$ and $\left|m\left(t_s^1\right)\right| \ge \sqrt{3}$ (which necessarily implies $\beta > \gamma$). Compared to case 1, travel time variability is such that the first commuter to leave arrives after t^* if the random delay is too high. The last commuter remains always late.

The equilibrium solution is now characterized by:

$$\begin{cases} t_s^0 = t^* - T_0 + \sqrt{3}\sigma - 2\sqrt{\frac{\gamma}{\beta+\gamma}\sqrt{3}\sigma\frac{N}{s}} \\ U_s^* = \alpha T_0 + \gamma\left(\frac{N}{s} + \sqrt{3}\sigma - 2\sqrt{\frac{\gamma}{\beta+\gamma}\sqrt{3}\sigma\frac{N}{s}}\right) \end{cases} \tag{19}$$

Case 3: $\gamma > \beta$ *and* $\frac{1}{\sqrt{3}} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s} \le \sigma \le \frac{1}{4\sqrt{3}} \frac{\beta+\gamma}{\min(\beta,\gamma)} \frac{N}{s}$

Case 3 is analogous to case 2 with the difference that $\gamma > \beta$. Accordingly, it is the last commuter who may be early or late while the first one is always early. The equilibrium solution satisfies:

$$\begin{cases} t_s^0 = t^* - T_0 - \sqrt{3}\sigma + 2\sqrt{\frac{\beta}{\beta+\gamma}\sqrt{3}\sigma\frac{N}{s}} - \frac{N}{s} \\ U_s^* = \alpha T_0 + \beta\left(\frac{N}{s} + \sqrt{3}\sigma - 2\sqrt{\frac{\beta}{\beta+\gamma}\sqrt{3}\sigma\frac{N}{s}}\right) \end{cases} \tag{20}$$

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Case 4: $\sigma \geq \frac{1}{4\sqrt{3}} \frac{\beta+\gamma}{\min(\beta,\gamma)} \frac{N}{s}$

The last case regroups solutions satisfying both $\left|m\left(t_s^0\right)\right| \leq \sqrt{3}$ and $\left|m\left(t_s^1\right)\right| \leq \sqrt{3}$. In this set of solutions, all individuals may be early or late depending on the delay they are subjected to. Those leaving first are naturally more likely to be early, and conversely. Last, we have:

$$\begin{cases} t_s^0 = t^* - T_0 - \frac{1}{2}\frac{N}{s} + \sqrt{3}\sigma \frac{\beta-\gamma}{\beta+\gamma} \\ U_s^* = \alpha T_0 + \sqrt{3}\sigma \frac{\beta\gamma}{\beta+\gamma} + \frac{\beta+\gamma}{16\sqrt{3}\sigma} \frac{N^2}{s^2} \end{cases} \tag{21}$$

4.4. Marginal social cost of travel time variability

Using the formulae above and then regrouping cases 2 and 3 in a single expression, we can derive the marginal social cost of travel time variability, which for reminder is defined by (11):

$$\begin{cases} \text{MSCTTV} = 0 & \text{if } \sigma \leq \frac{1}{\sqrt{3}} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s} \\ \text{MSCTTV} = \min(\beta,\gamma)\sqrt{3}\left(1 - \sqrt{\frac{1}{\sqrt{3}\sigma} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s}}\right) & \text{if } \frac{1}{\sqrt{3}} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s} \leq \sigma \leq \frac{1}{4\sqrt{3}} \frac{\beta+\gamma}{\min(\beta,\gamma)} \frac{N}{s} \\ \text{MSCTTV} = \sqrt{3}\frac{\beta\gamma}{\beta+\gamma} - \frac{\beta+\gamma}{16\sqrt{3}\sigma^2} \frac{N^2}{s^2} & \text{if } \sigma \geq \frac{1}{4\sqrt{3}} \frac{\beta+\gamma}{\min(\beta,\gamma)} \frac{N}{s} \end{cases} \tag{22}$$

MSCTTV is null when the standard deviation of travel times is below the threshold corresponding to case 1, which depends on the length of the rush hour N/s and on the preference parameters β and γ. Two elements account together for this counterintuitive result: 1) the choice of (α,β,γ) preferences; 2) the boundedness of the random delay. The scheduling disutility function $U(t,T)$ corresponding to (α,β,γ) preferences is linear in T on $]-\infty,t^*-t]$ as well as on $[t^*-t, +\infty[$, with a kink in $T = t^*-t$. Individuals leaving with a margin large enough so that they are sure to be always early (positive margin) or always late (negative margin) stay on one linear part of the disutility function; they are *de facto* risk neutral. In the case 1 of our bottleneck model, the first driver always arrives before t^*, whatever the delay, similarly the last driver always arrives after t^*. Considering what was just said, these two drivers are risk neutral.[14] As long as they remain so, they act as "anchor points" and the equilibrium disutility does not change. To conclude, the specific properties of (α,β,γ) preferences regarding risk aversion explain why for low values of σ, the cost associated with travel time variability and subsequently the MSCTTV are null.

It is straightforward to show that the MSCTTV increases with σ using (22), strictly so when $\sigma \geq \frac{1}{\sqrt{3}} \frac{\min(\beta,\gamma)}{\beta+\gamma} \frac{N}{s}$, and that it converges toward $\sqrt{3}\frac{\beta\gamma}{\beta+\gamma}$ when σ tends toward $+\infty$. Using (4), one finds that the VTTV is also equal to $\sqrt{3}\frac{\beta\gamma}{\beta+\gamma}$ when the random delay is uniformly distributed. This means that the MSCTTV is strictly lower than the VTTV, and that the former converges toward the latter when σ tends toward $+\infty$.

The fact that the MSCTTV is strictly lower than the VTTV results from the sum of all individual adjustments. As will be clear in the applied case, when σ increases, departures are more and more

[14] In case 1, not all individuals are risk neutral: only those who are sure to be early or sure to be late are, i.e. the first and the last individuals to leave. Drivers leaving in the middle of the rush hour are risk averse.

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uniformly spread over the rush hour. This reduces congestion (compared to the deterministic case) and mitigates the cost of unreliability. However, as σ increases, individual adjustments become less efficient, and the MSCTTV strictly increases. Ultimately, when σ tends toward $+\infty$, the MSCTTV is equal to the VTTV. The macroscopic equilibrium mechanism, *i.e.* the decrease in congestion, becomes ineffective, and everything happens as if congestion were exogenous and individuals no longer interacted with one another.

4.5. Applied case

We develop an applied case to illustrate and discuss at greater length the main results of our model. The values of the various parameters are summarized in Table 1.

Table 1: Parameter values for the applied case

Type of parameter	**Value**
Schedule preferences	$\alpha = 1.2$, $\beta = 1$, $\gamma = 3$
Preferred arrival time	$t^* = 9$
Expected free flow travel time	$T_0 = 0.5$
Travel demand	$N = 2\ 000$
Transportation supply	$s = 1\ 000$

This set of parameters intends to represent a somewhat plausible situation. The preferred arrival time is set to 9 am, with a free flow travel time of half an hour. Travel demand is twice the transportation supply, resulting in a rush hour that is two hours long.[15] Finally, the preference parameters α, β and γ are close to the ratio 1.0:0.8:3.0 indicated by Bates et al. (2001) as typical from the literature.[16]

Keeping these values fixed, we study how the equilibrium solution changes when travel time variability, which is measured by σ, increases. While the parameter values influence the magnitude of the various phenomena that we will observe, all results can be considered as general to the case $\gamma > \beta$. The case $\gamma < \beta$ resembles the case $\gamma > \beta$, with similar patterns for the MSCTTV. The only noteworthy difference is that the rush hour shifts in the opposite direction (it starts later and later as σ increases). Therefore, the case $\gamma < \beta$ is not considered.[17]

Travel time variability has three main effects on the congestion time profile: peak reduction (smaller top travel time), peak spreading and peak shifting. Peak shifting only occurs when travel time unreliability exceeds a certain threshold, though. These points are illustrated in Figure 2. As long as $\sigma \leq 1/2\sqrt{3}$ (condition associated with case 1), the timing of the rush hour period stays unchanged, but the congestion peak is flattened when σ increases. Beyond this threshold, the rush hour period starts earlier and earlier as variability increases, and congestion keeps decreasing.

[15] Note that only the ratio N/s matters, the nominal values of N and s having no importance.
[16] β and γ are not exactly in the ratio 0.8:3.0 in order to get round values for the fractions $\beta/(\beta+\gamma)$ and $\gamma/(\beta+\gamma)$.
[17] There is also the special case $\beta = \gamma$. In sum, when $\beta = \gamma$, travel time variability leads to peak spreading and peak reduction, but no peak shifting occurs. The MSCTTV behaves as when $\beta < \gamma$ or $\beta > \gamma$.

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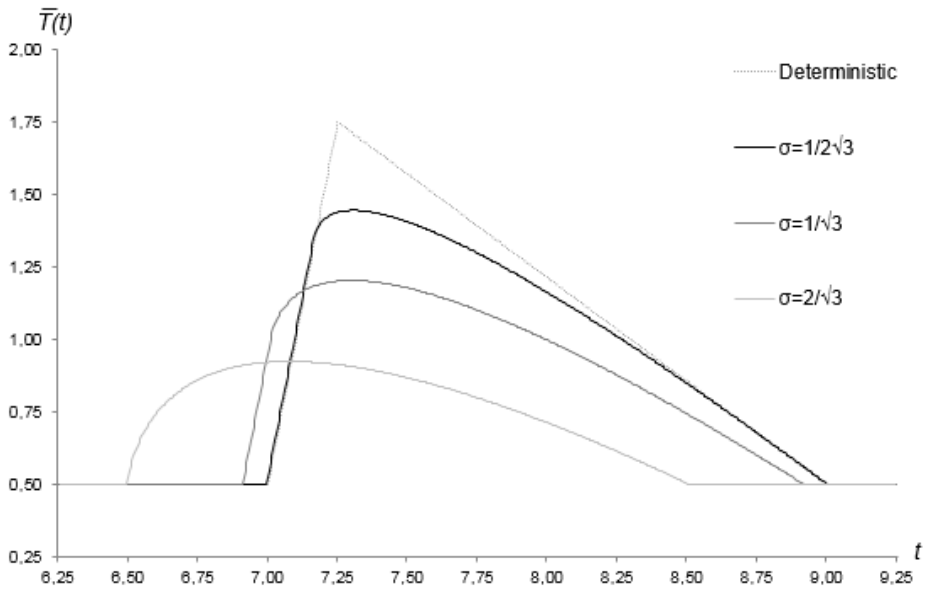

Figure 2: Impact of travel time variability on the expected travel time (uniform law)

The analysis of cumulative departures sheds light on these results. Starting from $\sigma = 0$ (deterministic travel times), an increase in σ induces adjustments before and after the peak (Figure 3). Indeed, before adjustments occur, travel time variability is most costly for people leaving at the peak, for whom schedule disutility was null under deterministic travel times. For the others, the choice not to be on time to avoid congestion insures them against the random delay to a greater or lesser extent. For instance, people leaving before the peak and arriving before t^* under deterministic travel times will suffer less from a positive delay (it will increase their travel time but reduce their schedule delay). As a result, the cumulative departure curve is only changed around the peak at first. It is drawn to the right, meaning that a certain share of departures is delayed. It is interesting to note that this prediction is the opposite of what the model with exogenous congestion predicts. As a matter of fact, applying (2) to the uniform distribution and $\gamma > \beta$, the latter model predicts that travel time unreliability causes the traveler to leave earlier than under deterministic travel times, and not later as happens in our bottleneck model.

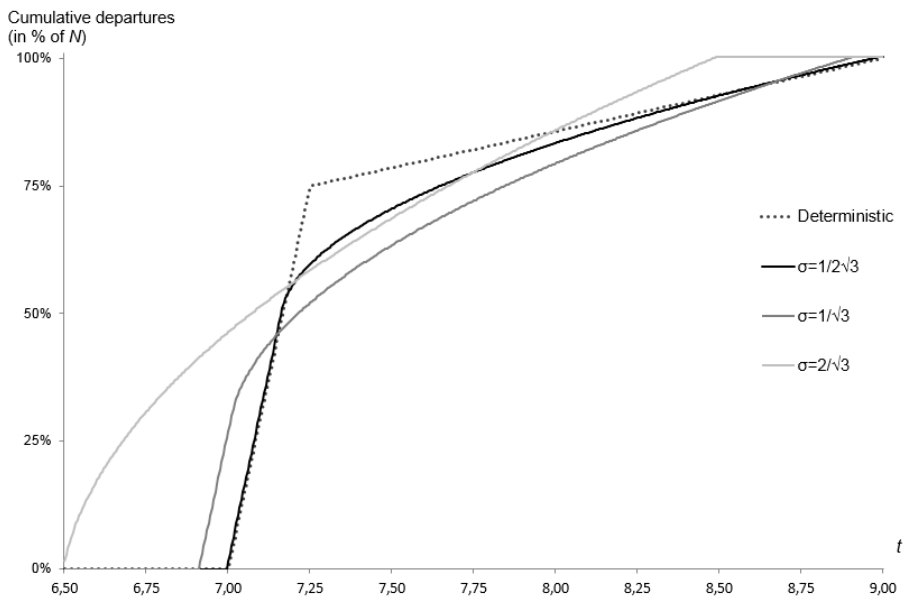

Figure 3: Influence of travel time variability on the cumulative departure curve (uniform law)

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When σ increases further, the equilibrium switches to the regime corresponding to case 3. Changes become more significant. People disliking being late more than being early ($\gamma > \beta$), they start leaving earlier, and the whole curve shifts to the left. The curve is flattened at the same time toward a single straight line, meaning that the departure rate becomes more and more homogenous. All in all, the first commuters leave earlier than in the deterministic case (this time in accordance with the model with exogenous congestion), but past a certain time they leave later. Eventually, when $\sigma \geq 2/\sqrt{3}$, the equilibrium solution switches to case 4. Unreliability is so important that the whole curve is flattened. The peak period keeps moving earlier and earlier.

We now turn our attention to the crux of this paper: the influence of σ on the marginal social cost of travel time variability. As discussed in 4.4, for low values of σ (in the interval corresponding to case 1), the MSCTTV is null (Figure 4). Then starting from $\sigma \geq 1/2\sqrt{3}$, the MSCTTV strictly increases, with a small kink at $\sigma \geq 1/\sqrt{3}$ (marking the transition between cases 2 and 4). It asymptotically converges towards the VTTV, which is for reminder given by (4).

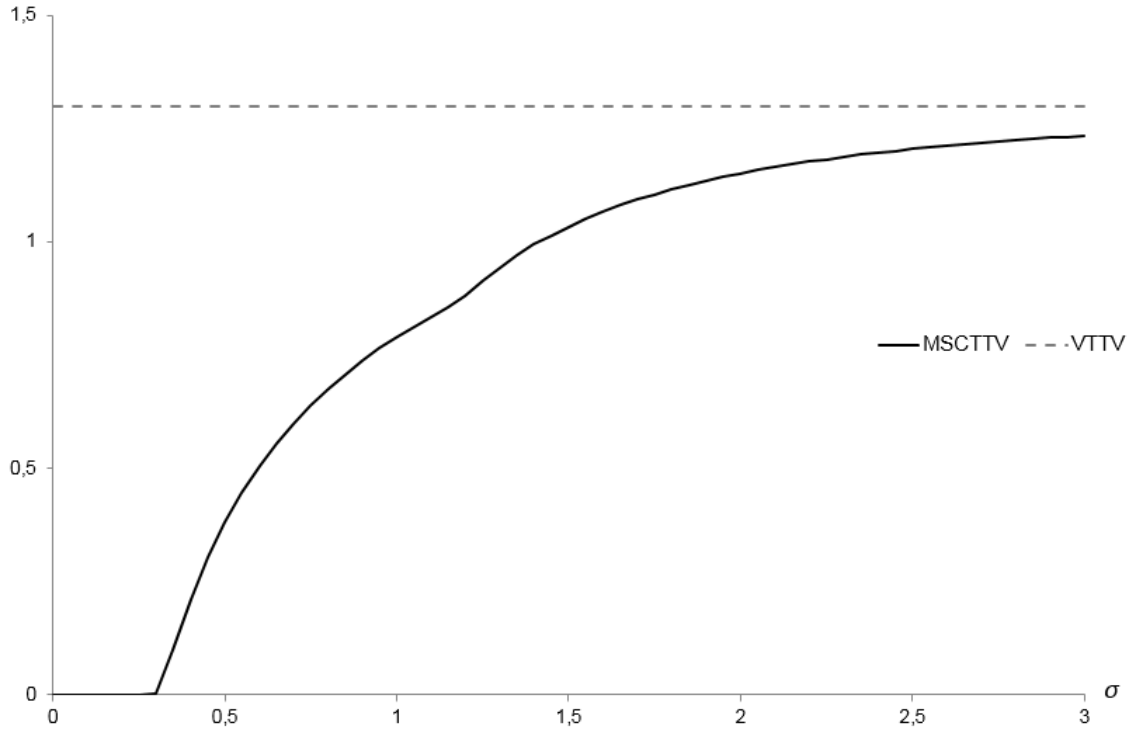

Figure 4: Comparison of the MSCTTV and the VTTV (uniform law)

Obviously, the central question is where we are on the curve. If in practice σ is in the interval corresponding to case 4 (right part of the curve in Figure 4), VTTV is a good approximation of MSCTTV. If σ is in the interval corresponding to case 1, MSCTTV is null and using VTTV as a substitute for MSCTTV is completely unfit (given the assumption of uniformly distributed delays). Before attempting an answer, we turn to the case where the delay follows a normal law to check the robustness of our results.

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5. Normally distributed delays

This section considers the more realistic case where the random delay follows a normal distribution. In this case we have:

$$\begin{cases} \varphi(x) = \dfrac{e^{-x^2/2}}{\sqrt{2\pi}} \\ \Phi(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{x} e^{-u^2/2} du \end{cases} \tag{23}$$

The differential equation in (10) no longer has a closed form solution (to the best of our knowledge). We resort to numerical simulation to address this case, and consider the same applied case as in Section 4.

The time profiles of congestion (Figure 5) and of cumulative departures (Figure 6) vary with σ in a way extremely similar to the case of the uniform distribution, hinting that the main results weakly depend on the distribution. The same three mechanisms are observed (peak reduction, peak spreading and peak shifting), and the peak reduction is in the same order of magnitude. There are two minor differences: peak shifting occurs even for low values of σ, but is at the same time less pronounced. The fact that peak shifting occurs as soon as $\sigma > 0$ is linked to the fact that the random delay is unbounded. Because extreme delays (positive and negative) may occur, whatever his safety margin, one is never sure to be early or late at his destination. Attempting a comparison with the uniform case, we would always be in case 4 (all individuals may be early or late). This being said, the peak shifting phenomenon is barely perceptible for low values of σ.

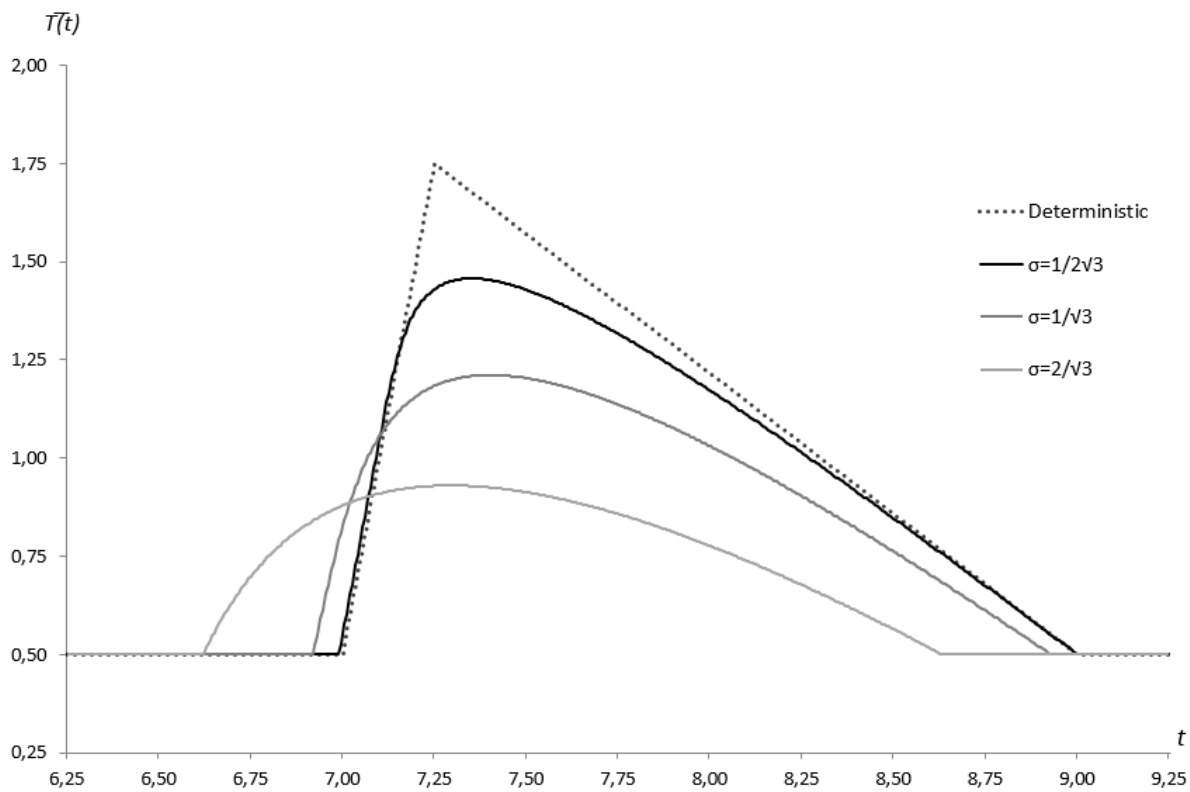

Figure 5: Impact of travel time variability on the expected travel time (normal law)

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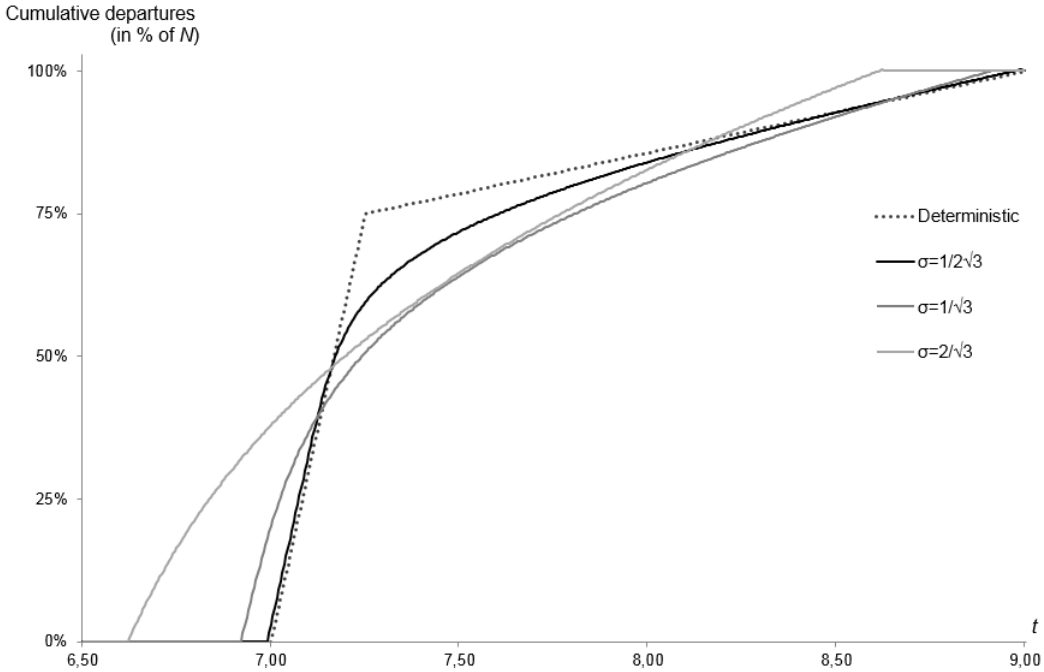

Figure 6: Influence of travel time variability on the cumulative departure curve (normal law)

The MSCTTV curve is similar to the uniform case (Figure 7) Likewise, it increases with σ and converges asymptotically toward VTTV.[18] The MSCTTV strictly increases as soon as $\sigma > 0$, however, even if only slightly, which is again linked to the fact that the normal distribution is not bounded. Convergence occurs at the same pace as for the uniform distribution.[19]

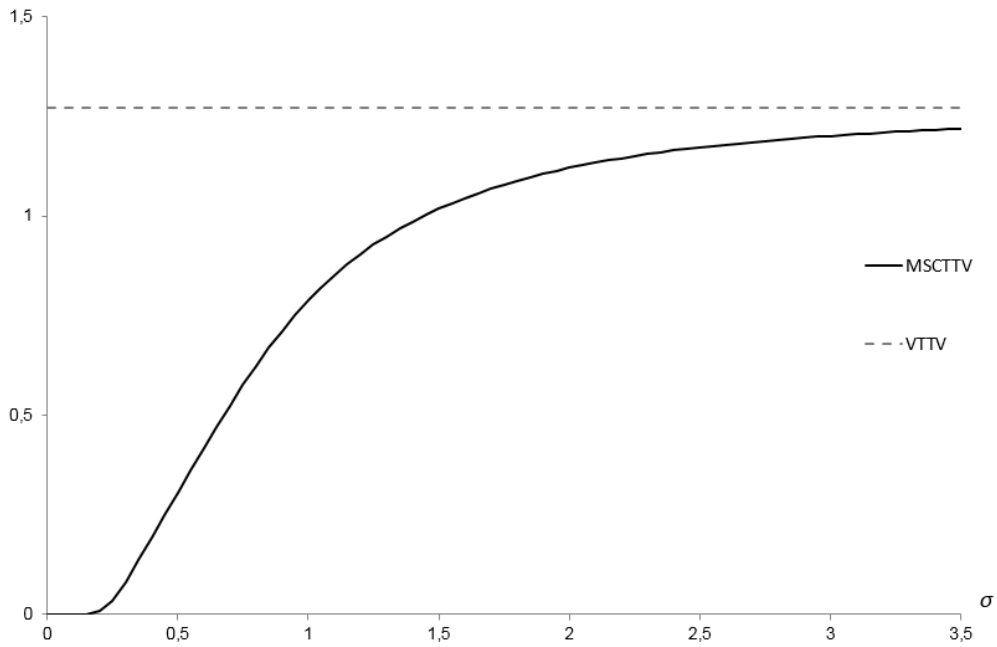

Figure 7: Comparison of the MSCTTV and the VTTV (normal law)

[18] Let us note that the VTTV is not the same for the uniform and normal distributions.

[19] A comparison of the MSCTTV for the uniform and normal distributions, not presented here for the sake of conciseness, shows that they are indeed very close whatever the value of σ.

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6. Discussion

Given that the MSCTTV curves are relatively close for the uniform and normal cases, the remaining question is where we are approximately on these curves. We can attempt an answer in the case of the Paris metropolitan area by considering the time profile of morning departures for car commuters (Figure 8). By analogy to the road bottleneck model, the peak period starts when departure rates have higher values than usual, and ends when they become close to null. For the Paris area, a conservative estimate of the peak period would be from 7h30 to 9h30, hence being approximately two hours long. In our applied case, the length of the rush hour is also 2, meaning that we can directly read σ in hours. While in practice travel time variability may substantially vary from one route to another, we can realistically assume the standard deviation of travel times to be below 0.5 (*i.e.* half an hour), or even just below 1 (*i.e.* one hour) to be very conservative. Based on Figure 4 and Figure 7, this implies that the MSCTTV is well below the VTTV.

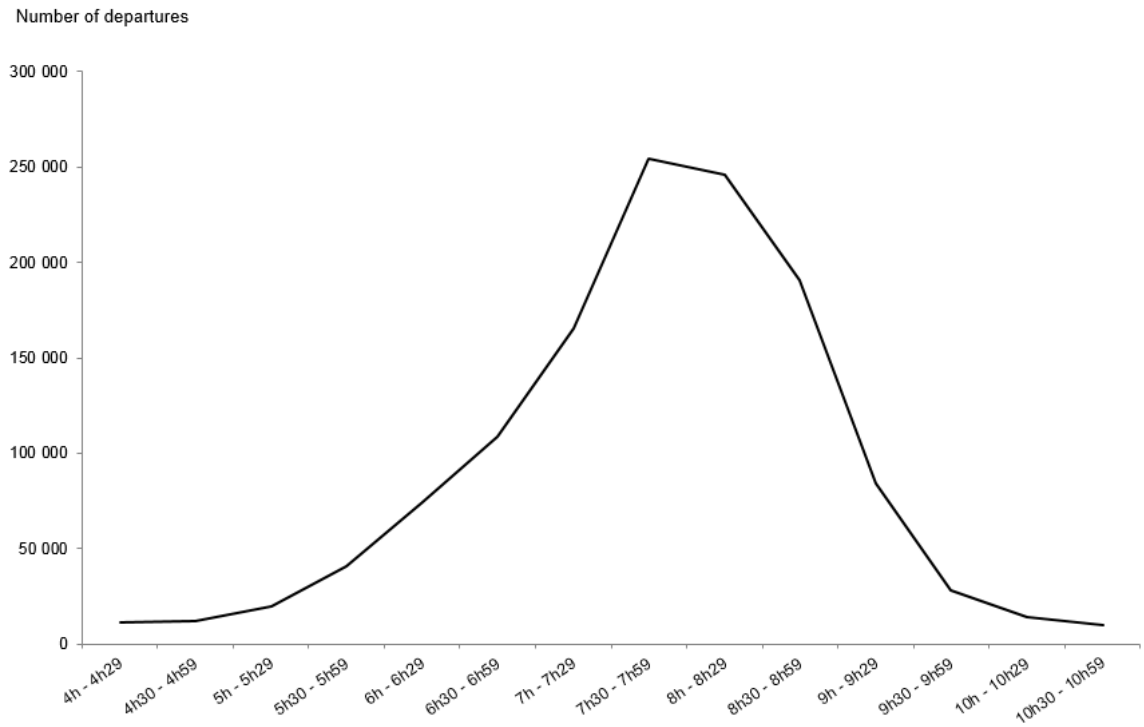

Figure 8: Home-work trips made by car (counted in number of cars), 2001 (Source: EGT, DRIEA)

We have just shown that the assumption of exogenous congestion can lead to greatly overestimate the cost of an increase in travel time variability. At rush hour, the fact that individuals adjust their departure time in response to travel time variability reduces congestion (compared to the situation where travel times are certain), which mitigates the cost caused by unreliability. However, one should not infer that travel time variability is a good instrument to struggle against congestion. The adjustments predicted by our model are unlikely to be as efficient in real life, be it only because of the difficulty for people to access good data on travel times, and in particular on their variability. Even more importantly, our model, like that of Fosgerau and Karlström, is based on the assumption that (α,β,γ) preferences correctly represent the behavior of travelers. As mentioned in subsection 3.1, this point is controversial. And, taking some perspective on our results, our finding that the MSCTTV is null for low values of σ (provided that the random delay is bounded) does seem relatively strong.

Therefore, our work tends to corroborate the idea supported by Börjesson et al. (2012) that (α,β,γ) preferences capture only part of the cost of travel time variability.

There are several possible interpretations to the "missing cost". As mentioned in the introduction, it could be related to the anxiety caused by uncertainty or to additional planning costs. Another rationale is that there exist schedule constraints at the origin (*e.g.* having to drop one's kids at school), which are not modeled by (α,β,γ) preferences and which limit the capacity of individuals to adjust their departure time. In all cases, taking this "missing cost" into account in the utility function would likely reduce the gap between the MSCTTV and the VTTV. Notwithstanding, the basic mechanism brought to light by our model (departures are more spread, which decreases congestion and thereby the cost of unreliability) would still exist, and the MSCTTV would still remain strictly lower than the VTTV.

7. Conclusions

We have extended the theory on the valuation of travel time variability to the case of endogenous congestion, thereby taking into account interactions between individuals and equilibrium mechanisms. On a theoretical note, we have established that the marginal social cost of travel time variability asymptotically converges toward the value of travel time variability for uniformly distributed delays. Numerical simulation leads to similar conclusions for a normal distribution, but with no formal proof. On a more empirical note, analysis of travel data for the Paris area strongly suggests that the marginal social cost of travel time variability is substantially lower than the value of travel time variability. For all practical purposes, when appraising economic benefits linked to reliability improvements, one should use the relevant indicator depending on the period considered (MSCTTV for the peak period, VTTV for the off-peak period).

Our analysis is subject to a number of caveats. First, we considered homogeneous (α,β,γ) preferences. Heterogeneity regarding the (α,β,γ) parameters and, in another direction, alternative scheduling preferences (*e.g.* the Vickrey-Tseng-Verhoef preferences) should be considered to test the robustness of our findings. Second, and as mentioned before in subsection 3.3, the assumption of a constant standard deviation is unrealistic. Again, the robustness of our results could be put to a test by linking the standard deviation to the congestion level. Third, the convergence of the marginal social cost of travel time variability toward the value of travel time variability was only proved for the uniform distribution. While numerical simulations incline us to reckon this result as general, a formal demonstration of this point has yet to be achieved. Finally, we only consider the impact of travel time variability on the departure time choice. Extending the model to include the mode choice as well as the route choice seems a promising way toward a better understanding of how travel time variability influences the whole trip making process.

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