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► **To cite this version:**

Riccardo Andreotti, Leonardo Marchetti, Luca Sanguinetti, Mérouane Debbah. Distributed power control over interference channels using ACK/NACK feedback. 2014 IEEE Global Communications Conference (Globecom), Dec 2014, Austin, United States. 10.1109/glocom.2014.7037464 . hal-01098840

HAL Id: hal-01098840

<https://hal.science/hal-01098840>

Submitted on 29 Dec 2014

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Distributed power control over interference channels using ACK/NACK feedback

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Abstract—In this work, we consider a network composed of several single-antenna transmitter-receiver pairs in which each pair aims at selfishly minimizing the power required to achieve a given signal-to-interference-plus-noise ratio. This is obtained modeling the transmitter-receiver pairs as rational agents that engage in a non-cooperative game. Capitalizing on the well-known results on the existence and structure of the generalized Nash equilibrium (GNE) point of the underlying game, a low complexity, iterative and distributed algorithm is derived to let each terminal reach the GNE using only a limited feedback in the form of link-layer acknowledgements (ACK) or negative acknowledgements (NACK). Numerical results are used to prove that the proposed solution is able to achieve convergence in a scalable and adaptive manner under different operating conditions.

I. INTRODUCTION

The power consumption of the communication technology industry is becoming a major societal and economical concern [1], which has stimulated academia and industry to an intense activity in the new research area of green cellular networks [2]–[4]. The ultimate goal is to design new innovative network architectures and technologies needed to meet the explosive growth in cellular data demand without increasing the power consumption. Along this line of research, in this work we focus on a network composed of several single-antenna transmitter-receiver pairs operating over the same frequency band (or time slot) in which each pair aims at selfishly minimizing the power required to achieve a given signal-to-interference-plus-noise ratio (SINR). The mutual interference due to the simultaneous transmissions gives rise to a sort of competition for the common resource. The natural framework to study the solution of such interactions is non-cooperative game theory [5], [6] in which the transmitter-receiver pairs are modeled as players that engage in a game using their own local information while fulfilling the given requirements. The existence and uniqueness properties of the equilibrium points of the underlying game have been widely studied in the literature and a large number of works already exist on this topic [7]–[9]. Particular attention has also been devoted to derive schemes based on best response dynamics that allow each player to achieve the equilibrium in an iterative and distributed manner. All these schemes rely on the assumption that the transmitter has perfect knowledge

of the SINR measured at the receiver. This assumption does not hold true in practical applications and the only way for the transmitter to acquire this knowledge is through a return control channel. Although possible, however, this solution is not compliant with most of the current wireless communication standards in which the receiver only sends back a link-layer acknowledgements (ACK) whenever is able to correctly decode the message and a negative ACK (NACK) otherwise. Most of the existing works dealing with resource allocation schemes using a 1-bit feedback are for centralized networks (see [10] and [11], and references therein), while only a few examples exist in decentralized scenarios. A first attempt in this direction is represented by [12] where the authors propose a distributed power control algorithm maximizing the sum rate in a secondary network, under a given outage probability at the primary user. The latter is evaluated by the secondary user by means of the 1-bit ACK/NACK feedback sent on the reverse link between the primary receiver and transmitter. In [13], a distributed power allocation scheme for outage probability minimization in MIMO interference channels is proposed. The optimization problem is modeled as a non-cooperative game with mixed strategies, where the probability of playing a certain strategy is updated with a reinforcement learning rule based on the ACK/NACK feedback.

In this work, we consider packet-oriented transmission links and propose a novel iterative and distributed algorithm that allows the transmitters to converge to the equilibrium point using only the limited feedback in the form of ACK or NACK. The proposed solution relies on a learning algorithm that is reminiscent of the scheme proposed in [10] and allows each transmitter to locally update an estimate of the received SINR while converging towards the equilibrium. This is achieved applying a simple updating rule completely unaware of the structure of the underlying game and requiring knowledge of local information only. Numerical results are used to assess the convergence and performance (in terms of number of iterations required) of the proposed solution in the uplink of a small-cell network.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a K -user Gaussian interference channel, in which there are K transmitter-receiver pairs sharing the same Gaussian channel, that might represent a time or frequency bin. The transmission takes place at the same time over the same frequency band and it is organized in frames. Each frame counts a certain number of packets, each one composed of M

L. Sanguinetti is funded by the People Programme (Marie Curie Actions) FP7 PIEF-GA-2012-330731 Dense4Green. This research has been supported by the FP7 NEWCOM# (Grant no. 318306), the ERC Starting MORE (Grant no. 305123), and the French pôle de compétitivité SYSTEM@TIC within the project 4G in Vitro.

data symbols of unity-energy. We call $x_k(m)$ the m th data symbol of transmitter k within a generic packet and denote $\mathbf{x}_k = [x_k(1), x_k(2), \dots, x_k(M)]^T$. Each \mathbf{x}_k is encoded at a rate $r_k \in \mathcal{R}_k$ with \mathcal{R}_k being the set of feasible rates and transmitted with an amount of power $p_k \in \mathbb{R}_+$. The channel is assumed to be constant over a frame and to change independently from one frame to another (block-fading channel). We assume that the transmitters do not have any a-priori knowledge of the channel.

Letting $h_{k,i}$ denote the channel coefficient between transmitter i and receiver k over a generic packet, the vector $\mathbf{y}_k \in \mathbb{C}^{M \times 1}$ received at the k th receiver within the generic packet can be written as

$$\mathbf{y}_k = \sum_{i=1}^K h_{k,i} \sqrt{p_i} \mathbf{x}_i + \mathbf{w}_k \quad (1)$$

where $\mathbf{w}_k \in \mathcal{CN}(0, \sigma^2 \mathbf{I}_M)$ accounts for the additive white Gaussian noise. The corresponding signal-to-interference-plus-noise ratio (SINR) is given by

$$\gamma_k = \frac{p_k |h_{k,k}|^2}{\sum_{i=1, i \neq k}^K |h_{k,i}|^2 p_i + \sigma^2}. \quad (2)$$

For later convenience, we call

$$\mu_k = \frac{|h_{k,k}|^2}{\sum_{i=1, i \neq k}^K |h_{k,i}|^2 p_i + \sigma^2} \quad (3)$$

the channel-to-interference-plus-noise ratio (CINR) and denote $\mathbf{p}_{-k} = [p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_K]^T$ the vector collecting all the transmit power except that of transmitter k . In this work, we aim at solving for any $k = 1, 2, \dots, K$ the following power minimization problem

$$\begin{aligned} \min_{p_k \in \mathbb{R}_+} \quad & p_k \\ \text{subject to} \quad & \gamma_k \geq \bar{\gamma}_k \end{aligned} \quad (4)$$

where $\bar{\gamma}_k > 0$ are given QoS requirements. The interplay among the pairs through (2) makes (4) a multidimensional optimization problem in which each transmitter-receiver pair aims at unilaterally choosing the minimum transmit power p_k so as to full fill its own requirement. In doing this, each pair affects the choice of all other pairs as well.

III. GAME FORMULATION

The natural framework to study the solution of problems in the form of (4) is non-cooperative game theory [5]. Interpreting (4) as a game \mathcal{G} leads to the definition of the tuple $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}, \{u_k\})$, where $\mathcal{K} = \{1, 2, \dots, K\}$ is the set of players, \mathcal{A}_k is the k th player's strategy set such that the constraints in (4) are satisfied, and $u_k = p_k$ is the utility function of player k . Note that player k 's action set depends on the actions of the other players, i.e., $\mathcal{A}_k = \mathcal{A}_k(\mathbf{p}_{-k})$ due to the presence of coupling constraints. In this case, the solution concept to be used is the GNE that is defined as the point collecting all the system states stable to unilateral deviations [14]. The GNE of the power allocation problem in (4) has been extensively studied in

the literature. The main results are summarized in the following theorem.

Theorem 1: If the problem (4) is feasible, then there exists a unique power allocation vector $\mathbf{p}^* = [p_1^*, p_2^*, \dots, p_K^*]^T$ that is the GNE of the game \mathcal{G} . The elements of \mathbf{p}^* are the solutions to the following fixed-point system of equations:

$$p_k^* = \text{BR}(\mathbf{p}_{-k}^*) = \frac{\bar{\gamma}_k}{\mu_k(\mathbf{p}_{-k}^*)} \quad \forall k \in \mathcal{K} \quad (5)$$

where the operator BR stands for the best-response of user k to given other users' strategy \mathbf{p}_{-k}^* and μ_k is defined in (3).

As shown in [15], a necessary and sufficient condition for which problem (4) is feasible is that $\rho_{\mathbf{G}} < 1$ where $\rho_{\mathbf{G}}$ is the spectral radius of matrix $\mathbf{G} \in \mathbb{R}^{K \times K}$, whose (k, i) th element is given by

$$[\mathbf{G}]_{k,i} = \begin{cases} 0 & k = i \\ \frac{\bar{\gamma}_k |h_{k,i}|^2}{|h_{k,k}|^2} & k \neq i. \end{cases} \quad (6)$$

The existence and the uniqueness follows observing that the best-response is a standard function [16], [17].

In addition to this, using the results of [16] it follows that the optimal point \mathbf{p}^* can be reached via a distributed iterative power control policy based on best response dynamics according to which every player k updates its power (strategy) $p_k^{(n+1)}$ at time $n+1$ as

$$p_k^{(n+1)} = \frac{\bar{\gamma}_k}{\mu_k(\mathbf{p}_{-k}^{(n)})}, \quad (7)$$

with $\mu_k(\mathbf{p}_{-k}^{(n)})$ being the CINR within the transmission time n .

IV. DISTRIBUTED ALGORITHM

Using (2), we may rewrite (7) as

$$p_k^{(n+1)} = p_k^{(n)} \frac{\bar{\gamma}_k}{\gamma_k(\mathbf{p}^{(n)})} \quad (8)$$

from which it follows that the computation of $p_k^{(n+1)}$ for a given $\mathbf{p}_{-k}^{(n)}$ requires knowledge of $\gamma_k(\mathbf{p}^{(n)})$. Most of the existing works rely on the assumption that each transmitter has perfect knowledge of it. Unfortunately, this assumption does not hold true in practical applications and the only way for the transmitter to acquire this knowledge is through a return control channel. Although possible, however, this solution is not compliant with current cellular standards in which the receiver only sends back an ACK ($f_k = 0$) whenever is able to correctly decode the packet and a NACK ($f_k = 1$) otherwise. Assume that a maximum likelihood (ML) decoder is used at the receiver and denote by $\hat{\mathbf{x}}_k \in \mathbb{C}^{M \times 1}$ the ML estimate of \mathbf{x}_k obtained from \mathbf{y}_k . Therefore, an ACK or NACK is sent to transmitter k with probability

$$\Pr \{f_k = \bar{f}\} = \begin{cases} \varepsilon_k(\mu_k, r_k, p_k) & \bar{f} = 1 \\ 1 - \varepsilon_k(\mu_k, r_k, p_k) & \bar{f} = 0 \end{cases} \quad (9)$$

where $\varepsilon_k(\mu_k, r_k, p_k)$ stands for the ML decoding error probability, which is clearly a function of the CINR μ_k , the transmit

Algorithm 1 Distributed resource allocation algorithm

1) At $n = 1$ for any $k \in \mathcal{K}$, choose a feasible $r_k^{(1)} \in \mathcal{R}_k$ and an arbitrary estimate $\hat{\mu}_k^{(1)}$. Then, set $\hat{p}_k^{(1)} = \bar{\gamma}_k / \hat{\mu}_k^{(1)}$;

2) At $n = 2, 3, \dots$ for any $k \in \mathcal{K}$

a) compute

$$\hat{\mu}_k^{(n)} = \hat{\mu}_k^{(n-1)} + \frac{f_k^{(n-1)} - \varepsilon(\hat{\mu}_k^{(n-1)}, p_k^{(n-1)}, r_k^{(n-1)})}{(n-1)^\beta \varepsilon'(\hat{\mu}_k^{(n-1)}, p_k^{(n-1)}, r_k^{(n-1)})}$$

and set

$$r_k^{(n)} = \arg \max_{r \in \mathcal{R}_k} \Phi(\hat{\mu}_k^{(n)}, p_k^{(n-1)}, r)$$

b) update

$$p_k^{(n)} = \frac{\bar{\gamma}_k}{\hat{\mu}_k^{(n)}}$$

power p_k and the encoding rate r_k . In particular, assuming Gaussian random codes a generic $\varepsilon_k(\mu_k, r_k, p_k)$ can be approximated as follows [18]

$$\varepsilon_k(\mu_k, r_k, p_k) \approx \exp \left(M \rho \left[r_k \log 2 - \frac{1}{2} \log \left(1 + \frac{\gamma_k}{1 + \rho} \right) \right] \right),$$

where $\rho \in [0, 1]$ is the union bound parameter and M is the number of data symbols per packet encoded at a rate of r_k bits/symbol.

Based on the above considerations, we propose an iterative and distributed two-step algorithm that allows each transmitter-receiver pair to reach the GNE of the game only exploiting the knowledge of $\{p_k^{(n-1)}, r_k^{(n-1)}, f_k^{(n-1)}\}$. The first step is reminiscent of the iterative solution proposed in [10] and aims at locally computing a reliable estimate $\hat{\mu}_k^{(n)}$ of $\mu_k^{(n)}$. Mathematically, $\hat{\mu}_k^{(n)}$ is obtained as follows

$$\hat{\mu}_k^{(n)} = \hat{\mu}_k^{(n-1)} + \frac{f_k^{(n-1)} - \varepsilon(\hat{\mu}_k^{(n-1)}, p_k^{(n-1)}, r_k^{(n-1)})}{(n-1)^\beta \varepsilon'(\hat{\mu}_k^{(n-1)}, p_k^{(n-1)}, r_k^{(n-1)})} \quad (10)$$

where β is a design parameter that regulates the convergence speed of the iterative procedure. The larger β , the smaller the convergence time. In addition, ε' denotes the derivative of ε with respect to μ and is given by

$$\varepsilon' = \frac{-M \rho p_k}{2(1 + \rho + \mu_k p_k)} \varepsilon_k$$

where we have dropped the functional dependence from (μ_k, r_k, p_k) for notational simplicity. The value of $\hat{\mu}_k^{(n)}$ is then used to update $r_k^{(n)}$ according to:

$$r_k^{(n)} = \arg \max_{r \in \mathcal{R}_k} \Phi(\hat{\mu}_k^{(n)}, p_k^{(n-1)}, r) \quad (11)$$

where

$$\Phi(\hat{\mu}_k^{(n)}, p_k^{(n-1)}, r) = \frac{[\varepsilon'(\hat{\mu}_k^{(n)}, p_k^{(n-1)}, r)]^2}{\varepsilon(\hat{\mu}_k^{(n)}, p_k^{(n-1)}, r)[1 - \varepsilon(\hat{\mu}_k^{(n)}, p_k^{(n-1)}, r)]}$$

is the Fisher information associated to the random variable f_k . Following the same arguments of [10], it can be proven that for any unbiased estimator based on n ACK/NACK, the estimation error variance of $\mu_k^{(n)}$ is lower bounded by the reciprocal of the cumulative Fisher information given by $\sum_{i=1}^n \Phi(\mu_k^{(i)}, p_k^{(i-1)}, r_k^{(i)})$.

The estimate $\hat{\mu}_k^{(n)}$ is eventually used in the last step for updating the transmit power as specified in (7). The main steps of the proposed solution are summarized in **Algorithm 1**.

Remark 1. Observe that similarly to a reinforcement learning approach in which at each step the probability function is updated according to a certain rule and then a strategy is randomly played according to this probability, in the proposed solution the estimate of μ_k is updated through (10), but then, the strategy is deterministically played, exploiting the knowledge of the optimal solution of the game with complete information.

Remark 2. The analytical study of the convergence of the proposed algorithm is still much open and left for future work. In the next section, we limit to assess the convergence of **Algorithm 1** by means of Monte Carlo simulations. Interestingly, it turns out that the proposed solution converges (within the required accuracy) whenever the game with complete information is feasible and thus the existence of the unique GNE point is guaranteed. Moreover, the convergence point is the same meaning that the same performance can be achieved despite the amount of required information is much lower. The only price to pay is a greater convergence time.

V. SIMULATION RESULTS

The performance of the distributed algorithm is now assessed by means of an extensive simulation campaign. To this end, we consider the uplink of a small-cell network [20] consisting of up to $K = 6$ single-antenna small cells, each serving a single UE. We set $\bar{\gamma} = [0.5, 1, 1.5, 2, 2.5, 3]^T$ dB and assume that the coverage area of each small cell is circular with radius $R = 50$ m and minimum distance $R_{\min} = 5$ m. The small cells are randomly distributed over a $200 \times 50K$ area. Moreover, we consider a system in which the large-scale fading is dominated by the path-loss. This amounts to saying that the channel coefficients $h_{k,i}$ can be modeled as

$$h_{k,i} = \frac{\bar{d}}{d^\alpha} \bar{h}_{k,i} \quad \text{for } d \geq R_{\min} \quad (12)$$

where $\bar{h}_{k,i} \sim \mathcal{CN}(0, 1)$ accounts for the small-scale fading, d is the distance between transmitter k and receiver i , $\alpha \geq 2$ is the path-loss exponent and $\bar{d} > 0$ is a constant that regulates the channel attenuation at distance R_{\min} [19]. We set $\bar{d} = 10^{-3.53}$ and $\alpha = 3.76$. We assume that the channel coefficients maintain constant in time. Moreover, the noise power level is set to $\sigma^2 = -100$ dBm and each packet is assumed to contain $M = 500$ symbols. The proposed algorithm is initialized for any $k \in \mathcal{K}$ as follows: $r_k^{(1)} = 1$ bit/s/Hz, $\hat{\mu}_k^{(1)} = |h_{k,k}|^2 / \sigma^2$ and $p_k^{(1)} = \bar{\gamma}_k / \hat{\mu}_k^{(1)}$.

Fig. 1 illustrates the values of $\gamma_k^{(n)}$ (dashed lines) measured at the BS as a function of the number n of transmitted packets in a scenario of $K = 4$ small cells when $\beta = 0.9$. The target SINRs

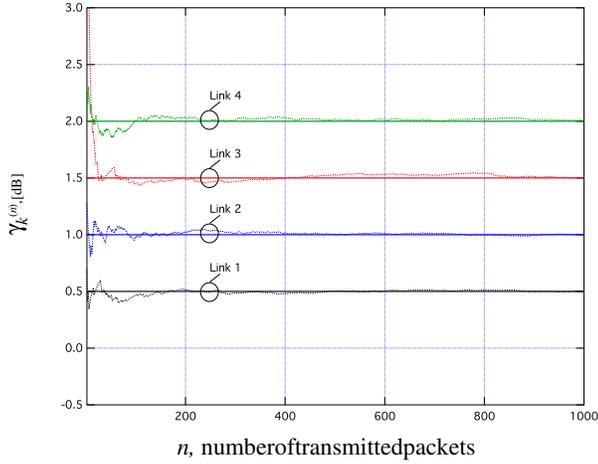


Fig. 1: SINR vs. number of packets when $K = 4$ and $\beta = 0.9$.

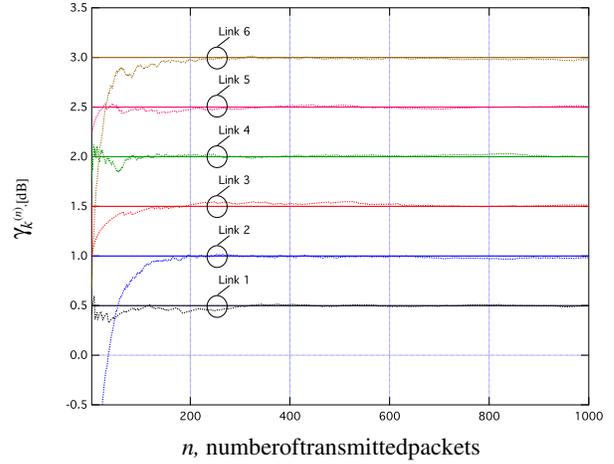


Fig. 3: SINR vs. number of packets when $K = 6$ and $\beta = 0.9$.

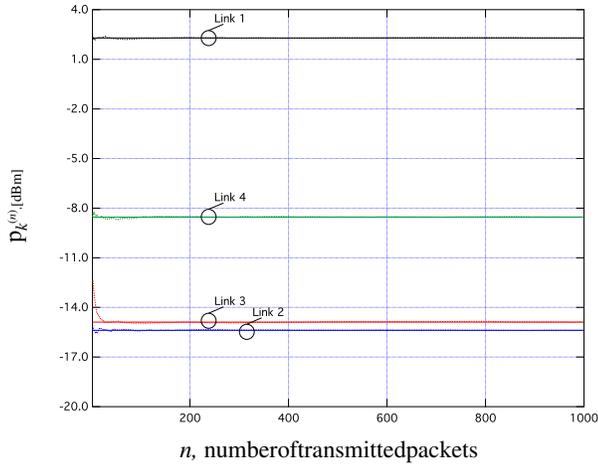


Fig. 2: Transmit power vs. number of packets when $K = 4$ and $\beta = 0.9$.

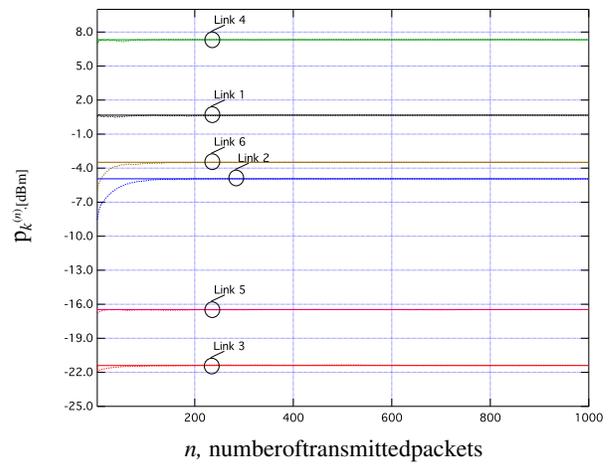


Fig. 4: Transmit power vs. number of packets when $K = 6$ and $\beta = 0.9$.

$\bar{\gamma}_k$ for $k = 1, 2, \dots, K$ (continuous lines) are also reported for comparison. Fig. 2 reports also the variations of $p_k^{(n)}$ (dashed lines) as n increases together with the power (continuous lines) required at the GNE point. The results of Figs. 3 and 4 are obtained in the same operating conditions of Figs. 1 and 2 except that now $K = 6$. As seen, in both cases, $\gamma_k^{(n)}$ converges to the target SINR $\bar{\gamma}_k$ within 200 packets. Interestingly, the attained power level is exactly the same achieved at the GNE point of the game with complete information.

The results of Fig. 5 illustrates the behaviour of $\gamma_k^{(n)}$ when $K = 4$ and β is set to 0.5. As expected, reducing β allows terminals to achieve convergence in a smaller number of packets. However, this is achieved at the price of larger variations around the target values $\{\bar{\gamma}_k\}$.

Fig. 6 shows $\gamma_k^{(n)}$ when K changes during the execution of the algorithm. We assume that $K = 4$ users are active at $n = 1$. Two new users enter the network at $n = 300$ and leave it successively at $n = 600$ and 800 . As seen, the algorithm is very robust to

network perturbations and guarantees fast convergence for users entering the network at different times.

VI. CONCLUSIONS

In this work, we have focused on the problem of selfishly minimizing the power consumption while satisfying target SINR constraints in interference channels characterized by single-antenna transmitter-receiver pairs operating over the same frequency band or time slot. In particular, we have first modelled the problem as a non-cooperative game with perfect CSI and then we have solved it assuming that each transmitter has no knowledge about the propagation channel but could only exploit the ACK or NACK feedbacks generated at the link layer from the receiver. This choice has been motivated by the fact that it is compliant with many wireless communication standards and avoids the need of introducing a suitably designed return control channel. Accordingly, we have proposed an iterative and distributed algorithm inspired by best response dynamics

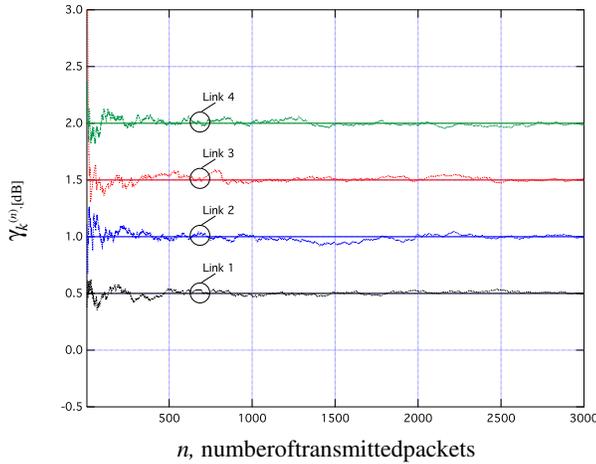


Fig. 5: SINR vs. number of packets when $K = 4$ and $\beta = 0.5$.

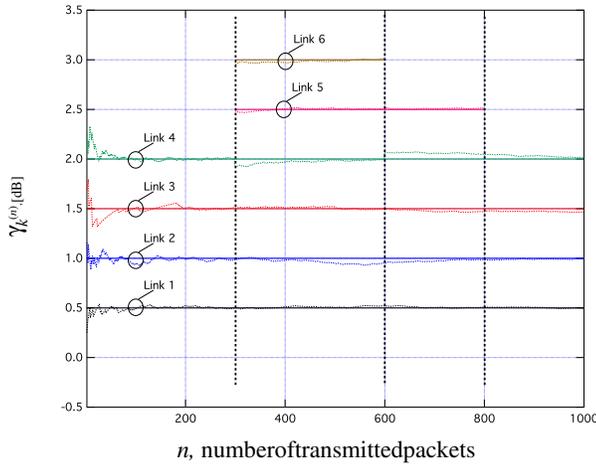


Fig. 6: SINR vs. number of packets with $\beta = 0.9$.

in which (at each step) every transmitter updates its power exploiting a local estimate of its current SINR at the receiver. The latter is learned step by step via an updating rule based on the 1-bit feedback information given by ACK or NACK. The performance of the proposed solution has been evaluated by means of numerical results in the uplink of a small cell network. It turns out that the algorithm converges reasonably fast to the GNE point of the underlying game with perfect CSI. Further research is needed to provide an analytical proof about the convergence of the iterative procedure.

REFERENCES

- [1] G. Auer, O. Blume, V. Giannini, I. Godor, M. Imran, Y. Jading, E. Kataranaras, M. Olsson, D. Sabella, P. Skillermarck, and W. Wajda, *D2.3: Energy efficiency analysis of the reference systems, areas of improvements and target breakdown*. INFISO-ICT-247733 EARTH, ver. 2.0, 2012. [Online]. Available: <http://www.ict-earth.eu/>
- [2] Y. Chen, S. Zhang, S. Xu, and G. Li, "Fundamental trade-offs on green wireless networks," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 30–37, 2011.
- [3] "Smart 2020: Enabling the low carbon economy in the information age," The Climate Group and Global e-Sustainability Initiative (GeSI), Tech. Rep., 2008.

- [4] Green Touch Consortium, Tech. Rep. [Online]. Available: <http://www.greentouch.org>
- [5] D. Fudenberg, and J. Tirole, *Game Theory*. MIT Press: Cambridge, MA, 1991.
- [6] S. Lasaulce and H. Tembine, *Game Theory and Learning for Wireless Networks: Fundamentals and Applications*. Academic Press, Elsevier, 2011.
- [7] G. Miao, N. Himayat, and G. Li, "Energy-efficient link adaptation in frequency-selective channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 545–554, Feb. 2010.
- [8] G. Miao, N. Himayat, G. Li, and S. Talwar, "Distributed interference-aware energy-efficient power optimization," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1323–1333, Apr. 2011.
- [9] G. Bacci, E. V. Belmega, and L. Sanguinetti, "Distributed energy-efficient power optimization in cellular relay networks with minimum rate constraints," in *Proc. IEEE Intl. Conf. Acoustics, Speech and Signal Process. (ICASSP)*, Florence, Italy, May 2014.
- [10] C. E. Koksak, and P. Schniter, "Robust rate-adaptive wireless communication using ACK/NACK-feedback," *IEEE Trans. on Signal Proc.*, vol. 60, no. 4, pp. 1752 – 1765, Apr. 2012.
- [11] R. Aggarwal, C. E. Koksak, and P. Schniter, "Joint Scheduling and Resource Allocation in OFDMA Downlink Systems Via ACK/NAK Feedback," *IEEE Trans. on Signal Proc.*, vol. 60, no. 6, pp. 3217 – 3227, June 2012.
- [12] S. Huang, X. Liu, and Z. Ding, "Distributed Power Control for Cognitive User Access based on Primary Link Control Feedback," in *Proc. of IEEE INFOCOM 2010*, pp. 1 – 9, March 2010.
- [13] E. V. Belmega, H. Tembine, S. Lasaulce, "Learning to precode in outage minimization games over MIMO interference channels," in *Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems and Computers (ASILOMAR) 2010*, pp. 261 – 266, Nov. 2010.
- [14] F. Facchinei, and C. Kanzow, "Generalized Nash equilibrium problems," *Quarterly J. Operations Research*, vol. 5, no. 3, pp. 173 -- 210, Sep. 2007.
- [15] S. U. Pillai, T. Suel, and S. Cha, "The Perron-Frobenius theorem: some of its applications," *IEEE Signal Processing Magazine*, vol. 22, no. 2, pp. 62 – 75, March 2005.
- [16] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 9, pp. 1341 – 1347, Sep. 1995.
- [17] C. U. Saraydar, N.B. Mandayam, and D. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. on Commun.*, vol. 50, no. 2, pp. 291 – 303, Feb. 2002.
- [18] R. G. Gallager, *Information theory and reliable communication*. New York: Wiley 1968.
- [19] *Further advancements for E-UTRA physical layer aspects (Release 9)*. 3GPP TS 36.814, Mar. 2010.
- [20] J. Hoydis, M. Kobayashi, M. Debbah, "Green Small-Cell Networks," *IEEE Vehicular Technology Magazine*, vol.6, no.1, pp.37,43, March 2011.