Optimal Control of a Vehicular Organic Rankine Cycle via Dynamic Programming with Adaptive Discretization Grid

Johan Peralez ^{1,2}, Paolino Tona ¹, Antonio Sciarretta¹, Pascal Dufour², Madiha Nadri²





¹ IFP Energies Nouvelles (France)
 ² LAGEP, University of Lyon (France)

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mechanical power

waste heat recovery.

exhaust heat recoverv



On board a diesel-electric train

- Main specificity: limited capacity of the cold sink.
- Objective: find an optimal control policy.

ORC: optimal control problem

2 Dynamic Programming

- DP principle and Level-Set algorithm
- Adaptive-grid algorithm





Contents

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Title	Context	ORC: optimal control problem ●○○	Dynamic Programming 000000	ORC: numerical solution	Conclusion
Sys	tem de	escription			



- 1 Vaporization
- 2 Expansion
- 3 Condensation
- 4 Compression



Main actuators

• evaporator by-pass: lets a fraction of exhaust gas feeding ORC.

 \dot{m}_A

- pump: circulates the working fluid.
- fan: provides a cooling air flow.

Title	Context	ORC: optimal control problem ○●○	Dynamic Programming 000000	ORC: numerical solution	Conclusion
Τw	o state	ORC model			
	Superviso	ory oriented model			
	 Simplified but physically meaningful. 				



Supervisory oriented model

- Simplified but physically meaningful.
- Validated on experimental data (captures slowest dynamics).





Objective: Maximizing energy recovered

Performance index:

$$J = \int_0^{L_f} P_{turbine}(x) - P_{pump}(x) - P_{fan}(u, t), dt$$

^aSuperheating is considered as perfectly regulated by pump (Peralez et al. [2013]).



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- Input control variables^a:

$$u = \{V_{o,e}, \overset{*}{m}_{A}\} \in [0,1] \times [0, u^{max}]$$

Security constraint:

$$p_1(x,t) < 25 \text{ bar}, \ \forall t \in [0,t_f]$$

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This problem is challenging to solve

- The model is non-linear, implicit, time-variant.
- The cost function is non-convex .
- Two control variables / two dynamic states.
- State constraint.

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Generic optimal control problem

• Discretized dynamic system:

$$\begin{array}{rcl} x_{k+1} & = & f_k\left(x_k, u_k\right) \\ x_0 & = & x_0 \\ x_k & \in & \mathcal{X}_k \subseteq \mathbb{R}^n \\ u_k & \in & \mathcal{U}_k \subseteq \mathbb{R}^m. \end{array}$$

• Cost functional to minimize $\min_{u} g(x_{N}) + \sum_{k=0}^{N} h_{k}(x_{k}, u_{k}),$

Dynamic Programming (DP)

- Based on principle of optimality introduced by Bellman.
- Main advantage: no assumption on model linearity or cost convexity.
- Main drawback: the computational load grows exponentially with the dimension of the problem.



Considering the cost functional

$$g(x_N) + \sum_{k=0}^{N} h_k(x_k, u_k),$$

DP proceeds backwards in time to evaluate the optimal cost-to-go:

| Final time computation

$$\mathcal{J}_N(x^i) = \left\{ egin{array}{c} g_N(x^i), \ ext{for} \ x^i \in \mathcal{X}_N \ \infty \ ext{else.} \end{array}
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II Intermediate computations for k = N - 1 to 0

$$\mathcal{J}_k(x^i) = \min_{u_k \in U_k} \{h_k(x^i, u_k) + \mathcal{J}_{k+1}(f_k(x^i, u_k))\}.$$





^aElbert et al. [2013]



At each time step, a second discretization of state space is introduced:



 \Rightarrow improves the estimation accuracy (but requires additional computations).

Title	Context	ORC: optimal control problem 000	Dynamic Programming ○○○○●○	ORC: numerical solution	Conclusion	
Adaptive grid efficiency $(1/2)$						
	A simple energy management example					
	 Analytic solution is known: Petit and Sciarretta [2011]. 					
	 Prol min 	blem: finding the speed imizes the consumption	l and distance traje 1 of an electric veh	ectories which icle		



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• Results in a set of external conditions for the ORC:

- exhaust gas temperature $T_{exh}(t)$
- exhaust gas mass flow $\overset{*}{m}_{exh}(t)$.

• A large variation of ambient temperature $T_A(t)$ is considered.





Adaptive-grid algorithm solution ($N_{x1} = N_{x2} = N_{u1} = N_{u2} = 21$)

Highlights

- Energy recovered was improved by 7% (compared to static optimization).
- Strong influence of ambient temperature on energy production.

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Con	clusio	า			

Summary

- We have formulated and solved an original optimal control problem for a vehicular Rankine system.
- We have developed an Adaptive grid algorithm which achieves the required accuracy within acceptable computational time.

On-going work

- Thanks to these results it becomes possible to carry out extensive parametric studies.
- These results also prove useful for the design of an online strategy.



