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HELICOPTER PRESIZING
BY MULTIDISCIPLINARY - MULTI OBJECTIVE OPTIMIZATION

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Abstract. The ONERA project C.R.E.A.T.I.O.N.: Concepts of Rotorcraft Enhanced Assessment Through Integrated Optimization Network, has for main goal the development of a multidisciplinary computational workshop for the evaluation of rotorcraft concepts. The evaluation concerns both flight performances and environmental impacts (acoustics, air pollution/fuel consumption). The CREATION workshop must allow the evaluation of any rotorcraft concept whatever the level of details in its description data initially available. Therefore the tool must cope with the preliminary conception and presizing problems. In this paper, two different kinds of methodologies are presented. The first one is based on a genetic algorithm for addressing the multi objective dimension of the sizing problem and for exploring largely the design space. Then once the Pareto front has been globally captured, a determinist algorithm is used for the final local optimization. The second approach makes use of Mixture of Experts for dealing with the fact that the design parameters contain both discrete and continuous variables. It uses a determinist algorithm for finding directly a local optimum without requiring the computation of the Pareto front. Both methodologies have been applied successfully to the presizing of a large civil helicopter for the transportation of 90 passengers. The results will be presented and compared.
INTRODUCTION

The ONERA project C.R.E.A.T.I.O.N.: “Concepts of Rotorcraft Enhanced Assessment Through Integrated Optimization Network”, has for main goal the development of a multidisciplinary computational workshop for the evaluation of rotorcraft concepts. The evaluation concerns both flight performances and environmental impacts (acoustics, air pollution/fuel consumption).

The CREATION numerical workshop must be able to evaluate any rotorcraft concept whatever the level of details in its description data initially available. This includes the cases for which very few data are known, for example: when only an idea of new concept and of its potential application must be assessed or when only an expression of needs is given by future operational users. Therefore the tool must cope with the preliminary conception and presizing problems. Hence, several levels of modeling are included allowing the assessment of a rotorcraft from the appropriate modeling level depending on the first data available.

The seven disciplinary modules which are the “seven funding pillars” of the tool include two “aims modules”: “Flight performances" and “Environmental impacts”, plus five “means modules”: “Missions & Specifications”, “Architecture & Geometry”, “Weights & Structures”, “Aerodynamics”, “Power Generation (engine)”. Within each of these modules, four modeling levels have been set from Response Surface Models and statistics, more or less comprehensive analytical models until expert numerical models (see [1]).

A computational chain coupling these disciplinary modules has been applied for the helicopter presizing from a set of mission requirements given in [2]. Then two methodologies have been set for dealing with this multidisciplinary multi objective problem. The first one is based on a genetic algorithm for addressing the multi objective dimension of the sizing problem and for exploring largely the design space. Then once the Pareto front has been globally captured, a determinist algorithm is used for the final local optimization. The second approach makes use of Mixture of Experts for dealing with the fact that the design parameters contain both discrete and continuous variables. It uses a determinist algorithm for finding directly a local optimum without requiring the computation of the Pareto front.

After setting the background of this study, the main lines of the CREATION project will be presented. Then the practical case of application will be exposed in terms of helicopter presizing by giving the mission requirements. The two different methodologies will be described and their results will be presented. They will be compared and their respective advantages and drawbacks will be drawn.

BACKGROUND

Of course there are as many predesign tools as industrial rotorcraft builders. These “house-tools” are generally based on databases of existing helicopters and make use of rather empirical methods. Such a pragmatic way is well suited for a quick and efficient presizing of a helicopter by interpolation within these databases, but not for the extrapolation of rotorcraft concepts out of the scope of these databases. In the CREATION project the presizing is not a goal but rather a means, in order to be able to evaluate any rotorcraft concept even when no or very few data are available. Such a capability is built by putting the priority on the universality of the models by contrast with empirical laws and by taking into account as soon as possi-
ble in the predesign loops its multidisciplinary character and the associated complex optimization.

The scientific literature on the topic can be split into two main groups. First the abundant literature on the Multidisciplinary Design Optimization (MDO) methods presents a wide variety of possible formulations (e.g. [3]-[4]). Up until now their applications in aeronautics have mainly concerned fixed wings aircraft [5], but also launchers and satellites.

A second group of references deals with rotorcraft. They are themselves of two kinds:
- either rather “upstream” by comparison with our objectives in the meaning that they study mainly the methods for preliminary design (MDO formulations [6] or probabilistic approach [7]) but with models which seem to be below the level of expertise intended in CREATION;
- or rather “downstream” by being focused on the study of a few concepts in the context of a precise flight performance objective (e.g. “joint heavy lift rotorcraft systems” [8] and “High speed rotorcraft” [9] studies).

The seldom works the closest to the CREATION project are indicated hereafter.

In Europe:
A collaboration has been settled between ONERA and DLR on this topic from the beginning of the proposal. In the RIDE project (« Rotorcraft Integrated Design and Evaluation », e.g. [10]), the DLR develops a rotorcraft evaluation platform following a similar multidisciplinary approach as already applied for fixed wings aircraft in their TIVA project (« Technology Integration for the Virtual Aircraft »). Two independent tools dedicated to rotorcraft are developed with exchanges on some aspects (e.g. rotorcraft databases) between ONERA and DLR.

In the USA:
Besides the already mentioned works of the « Georgia Institute of Technology » on the methods for the preliminary design of rotorcraft ([6]-[7]), a software tool was also developed to put them into practice: « CIRADS: Concept Independent Rotorcraft Analysis and Design Software » [11].

Recent papers have shown that NASA has developed a tool called NDARC: « NASA Design and Analysis of Rotorcraft » presented by Wayne Johnson in January 2010 (e.g. [12]-[13]).

The role of this tool is both to sustain the research and the expertise in particular for the «Department of Defence» (DoD). The research activities find in that kind of preliminary design tool a means for evaluation of the impact of a new concept or a new technology by integrating it in a complete adapted rotorcraft system. The support to the expertise during acquisition studies of new rotorcraft by the DoD concerns the exploration of concepts, their selection and improvement. During these different phases of acquisition, it is required to be able to evaluate independently a large variety of rotorcraft concepts and designs.

These two purposes are also those of the CREATION project: support the innovation and the expertise on rotorcraft at ONERA.
In the rest of the world:

In particular the work of the Technion Israël Institute [14]-[15] is noticeable. A computational platform for the analysis of rotorcraft concepts has been built called: «RAPID/RaTE» for «Rotorcraft Analysis for Preliminary Design / Rand Technologies & Engineering». It makes use of different levels of modelling from simple analytical models based on statistical analysis on existing rotorcraft databases (helicopters in [14] and extended with other rotorcraft concepts mainly of “Uninhabited Aircraft Vehicles” type in [15]), until more sophisticated models including for example aero-elastic blade modelling.

At ONERA, numerous works on rotor optimization both on aero-acoustics and aero-elasticity have been performed (e.g. [16]-[17]-[18]). The preliminary conception of a whole aircraft by using MDO techniques has been addressed on fixed wings (airplanes and missiles [5]). In the field of rotorcraft, some first studies were carried out for the European project CAPECON on civil applications of Uninhabited Aircraft Systems (UAS) (see [19]-[20]), as well as for some specific concepts for the industry and the French Ministry of Defence.

As reported in [21], these rotorcraft conceptual studies have emphasized the need for a multi-department collaboration in order to better address the multidisciplinary character of rotorcraft evaluation studies. That is why the federative project CREATION has been launched involving six research units spread in five ONERA departments with expertise in aerodynamics, flight dynamics, structures, acoustics, system conception and optimization.

3 THE CREATION PROJECT

As mentioned in the introduction, the goal of CREATION is to build a computational workshop for evaluating existing or new rotorcraft concepts mainly from the flight performances and environmental impact points of view. Hence it provides a means for the expertise and innovation, for example by allowing the exploration of the design space in terms of rotorcraft architectures for finding what is the best configuration the most suited to a certain set of mission needs.

For building this tool, three main milestones have been defined for developing incrementally its capabilities.

- **Milestone 1 - evaluation capability:** the flight performances and environmental impact (external noise and air pollution) of a known helicopter can be assessed whatever its degree of description (from a minimal set of about ten data);
- **Milestone 2 - predesign capability:** the presizing of a helicopter suited for a set of mission requirements can be addressed starting from a white page;
- **Milestone 3 - innovation capability:** the computational workshop provides a means for the investigation of alternate concepts (with respect to the conventional helicopter).

These capabilities allows evaluations for quantifying objectively (as unbiased as possible) performances and environmental metrics. These evaluations rely on models which are organized in multidisciplinary modules and in multi modeling levels within each disciplinary module. A 3 Dimensional view of this framework representing the CREATION workshop is shown on Figure 1.
From the disciplinary prism decomposition, which corresponds to the “horizontal organization” of the tool on Figure 1, the two central modules are “Flight performances” and “Environmental impacts”. Around them, five “means modules” provide the required data:

- Missions & Specifications
- Architecture & Geometry,
- Weights & Structures (including aeroelasticity),
- Aerodynamics,
- Power Generation (engine).

Except for the “Mission & Specification” module which provides the information on the mission profile and specifications, within each of the other modules several modeling levels have been implemented. This “vertical structuration” in modeling levels is needed for dealing with the available data describing the rotorcraft. If there are enough data for applying the highest level of model, then expert numerical models can be used. Otherwise, the starting level of modeling must be adapted to the available data and then the models provide an enrichment of the describing data allowing rising up the modeling level until the one required for the intended evaluation. In the case no data are available but the mission requirements, the presizing is performed from the lowest level of modeling (the least demanding in terms of input data) and then the data describing the rotorcraft are enriched step by step through the modeling levels.

Four main levels of modeling have been implemented in most of the modules:

- Level 0: Response Surface Models (RSM) based on databases or simulations,
- Level 1: simple analytical models based on physics,
- Level 2: more comprehensive analytical models,
- Level 3: numerical models.
For example in the case of the “Flight performances” module:
1. at level 0, statistical models: give design trends between the main rotorcraft dimensions and the main performances, they are only use for a first rough initialization if needed;
2. at level 1, the Power Balance method (PB) also called energy method: allows the calculation by analytical expressions of the required power (Preq) to make the rotorcraft fly at a certain flight point, this power demand is the sum of the induced power, the airfoil profile drag power, the power for overcoming the drag of all the other elements except the blade airfoils (i.e. fuselage, empennage, etc.), the anti-torque power (in the case of rotorcraft concepts with anti-torque device);
3. at level 2, Analytical Flight Mechanics (AFM): the complete trim of the forces and moments resulting at the rotorcraft center of gravity is computed iteratively with analytical expressions (for example for the rotors, a rotor disk model with linear blade airfoil aerodynamics);
4. at level 3, Numerical Flight Mechanics (NFM): more precise models are used for each component of the rotorcraft. In particular, the rotor model is based on a blade element discretization allowing a more detailed modeling of the blade (airfoils, chords, twist laws, etc.).

4 PRACTICAL TEST CASE FOR HELICOPTER PRESIZING

The present paper is focused on the milestone 2, i.e. the presizing “ab initio” of a helicopter answering to a set of mission requirements. In order to be able at the end of the presizing to compare and to validate it with respect to a reference, it has been chosen to start with the same mission specifications dealt with in [2]. NASA investigates the potential of rotorcraft for contributing to the decongestion of the air traffic through the airports with a time horizon of application of about 30 years from now.

The helicopter must transport 90 passengers over about 500 nm with a cruise speed of at least 150 kt. As in [2], the cruise altitude is fixed at 12000 ft. The specifications are given more in details hereafter in table 1 in the international standard units system. An extra demanding condition is a failure case with One Engine Inoperative (OEI) in which the helicopter must be able to hover at 1500 m ISA+20 (the temperature is 20°C above the International Standard Atmosphere condition).

<table>
<thead>
<tr>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload weight</td>
</tr>
<tr>
<td>Range</td>
</tr>
<tr>
<td>Cruise speed</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flight points of the mission profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight point</td>
</tr>
<tr>
<td>Difference (in °C) wrt. ISA</td>
</tr>
<tr>
<td>Step duration</td>
</tr>
<tr>
<td>$V_h$</td>
</tr>
</tbody>
</table>
 Specifications

<table>
<thead>
<tr>
<th>Vz</th>
<th>Vroc</th>
<th>Vroc</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Flight type:
- Taxi
- hover
- climb
- cruise
- hover

Other steps

Fuel reserve (Alternate airport): 185 km at 12000 ft and Vcr
Reserve (emergency): 30 min at 5000 ft and Vcr (i.e. 140 km)

Table 1: summary of mission requirements

The corresponding mission profile is presented on Figure 2.

Figure 2: mission profile.

5 HYBRID METHOD USING BOTH GENETIC AND DETERMINIST ALGORITHMS

The proposed method consists in twelve steps which can be gathered in four main phases:
- the preparation of the optimization,
- the global optimization by exploring the design space,
- the local optimization,
- the final evaluation of the predesign solution.

Preparation of the optimization

This preparation phase contains five steps:
- Step 1: Choices of the objectives, design variables and constraints
- Step 2: Chaining of the models needed for the presizing and evaluation (MultiDisciplinary Analysis, MDA)
- Step 3: Setting a Design of Experiment (DoE)
- Step 4: Computation of the DoE with the MDA
- Step 5: Generation of the corresponding Response Surface Models (RSM)
Step 1: Objectives, design variables and constraints

The helicopter presizing consists in finding the best set of design variables, submitted to certain constraints, for reaching the best compromise with respect to several objectives. In the present practical case the selected objectives are:

- minimizing the fuel consumption: $W_{fuel}$, weight of required fuel,
- minimizing the empty weight: $W_{empty}$ (all the helicopter weights except the payload weight corresponding to the 90 pax and the fuel weight),
- minimizing the external noise: $F_{acou}$ is an acoustic metrics representing the average noise level resulting on the ground footprint during the approach.

Other objectives can of course be used. The method is robust even if additional objectives are considered. Here the choices in terms of objectives, design variables, constraints, modeling level, etc. are done for illustrating in practice the method which is more general than these choices.

The design variables considered here are the main parameters describing the main rotor:

- $R(m)$ the main rotor radius,
- $C(m)$ the blade mean chord,
- $U(m/s)$ the blade tip speed due to rotation ($U=\Omega R$),
- $b$ the number of blades.

The first three parameters are continuous variables whereas the last one is an integer taking only discrete values.

The constraints are here the boundaries of the design space, i.e. the bounds of the design variables and some limits imposed by experience for example on the blade aspect ratio ($R/C$) and on the maximum gross weight.

The bounds of the design variables have been set by considering the first rough values estimated for these design parameters by the level 0 models based on design trends given by statistics on a helicopter database (about 200 helicopters):

- $R(m) \in [10;20]$, $C(m) \in [0.5;1.5]$, $U(m/s) \in [200;230]$, $b \in \{6;7;8\}$

The blade aspect ratio is limited for respecting a structural constraint which cannot be captured by the modeling levels under the level 3 (when using elastic blade models). The helicopter database allows anticipating on that kind of constraints because it provides the design results of existing helicopters:

- $R/C \in [10;20]$

The maximum gross weight is limited to 50 tons. The design gross weight ($W_{dgw}$) is the sum of the empty weight, the payload weight and the fuel weight for this nominal mission.

Therefore, the optimization problem is mathematically summarized as follows:
Step 2: Chain of models required for the MultiDisciplinary Analysis, MDA

The first goal of this modeling chain is to compute the objectives (cost functions) for a certain set of values of the design variables. That is done through two subsets of modeling chains: a presizing chain and an evaluation chain.

The presizing chain calculates from the set of design variables: the other helicopter dimensions, the most demanding power demand, the required engine, the weight breakdown and the resulting empty weight $W_{\text{empty}}$. That includes a convergence loop on the helicopter gross weight.

The evaluation chain calculates the two other objectives: a more accurate assessment of required fuel weight $W_{\text{fuel}}$ by calculating the consumed fuel over the complete mission profile as well as the fuel reserves, the acoustic metrics $F_{\text{acou}}$ on the landing approach. (Notice that in the last version, the calculation of $W_{\text{fuel}}$ on the entire mission profile including the reserves is done once in the presizing loop whereas the evaluation chain is dedicated to the calculation of the environmental metrics: noise and air pollution.)

For the sake of clarity and brevity, the two modeling chains “Presizing” and “Evaluation” are presented schematically on Figure 4 and Figure 5.
In terms of Multidisciplinary Design Optimization (MDO), this process is a formulation of the type “Multiple Discipline Feasible”. Indeed each predesign resulting from the presizing chain is weight consistent, thanks to the convergence loop on the gross weight ($W_{dgw}$, see Figure 4). The drawback is that even with the analytical models of the level 1, the computation “Presizing + Evaluation” (MDA) for one point of the DoE (one set of values for the four design variables) is rather long, about 1 minute and 40 seconds.

**Step 3: Setting a Design of Experiment (DoE)**

For solving a multi objectives problem, different methods exist which can be classified in two main categories. The “a priori” methods allow the direct convergence on one particular
point of the Pareto front by transforming the multi objectives into a global criterion or by a hierarchical optimization ... The “a posteriori” methods begin by the calculation of the whole Pareto front and then the selection of one best compromise solution is done by different techniques.

Here in this part 5 of the paper one “a posteriori” method is illustrated. A genetic algorithm (NSGA II) is used for computing the Pareto front of this multi objectives problem. The number of computed points by the genetic algorithm is high (about 20 000 or more). Therefore using the MDA chain presented in step 2 would lead to more than 500 hundred hours of computation time.

The use of a much quicker surrogate model is required. Thus a Response Surface Model (RSM) is calculated for representing the “Presizing and Evaluation” MDA chain. That supposes first to define a Design of Experiment (DoE) containing a representative set of values which can be taken by the design variables (R, C, U, b). For defining this table of values, a mixed of Latin Hypercube Sample (LHS) and orthogonal plan techniques is applied for the continuous variables (R, C, U) which are then duplicated for three values of the blade number (6, 7, 8). After elimination of the points which are not respecting the constraints, a table of 591 points is obtained in the present example case.

- **Step 4:** Computation of the DoE with the MDA
  
  From the input values given by the DoE in terms of design variables (R, C, U, b), the objectives are calculated as outputs of the MDA chain (“Presizing and Evaluation”). This step takes a long time (about 15 hours), but is done once as long as the models are not changed. Then the different optimization approaches presented here in the parts 5 and 6 use these results as simulation database (learning points).

- **Step 5:** Generation of the corresponding Response Surface Models (RSM)
  
  From this simulation database a RSM is generated by a Kriging technique [23] which is a stochastic technique of spatial interpolation. Polynomial interpolation has been also tested, but does not give better fit goodness than the Kriging.

  From a set of values of the design variables (R, C, U, b), a RSM gives one output value corresponding to one objective. Therefore there are as many RSM as objectives. Hence in the present example case, three RSM have been generated for providing the relationships between the four design variables and each of the three objectives ($W_{fuel}$, $W_{empty}$, $F_{acou}$).

**Global optimization by exploring the design space**

The genetic algorithm can then be used with the Response Surface Models for exploring the design space and assessing the Pareto front. That is done in three steps.

- **Step 6:** Implementation of the RSM into a MDO process
  
  The RSM are implemented into the “Multidisciplinary Design Feasible” process presented previously in step 2. In practice this implementation is done here into ModelCenter©.

- **Step 7:** Optimization with a genetic algorithm
  
  Genetic algorithms are stochastic techniques allowing the optimization of “multi objectives – multi variables – under constraints” problems. Here the NSGA II algorithm has been used
as it offers a good compromise between a wide exploration of the design space and a precise convergence towards the Pareto front.

➢ **Step 8: Generation of the Pareto front**

The points of the Pareto front form a frontier of non-dominated solutions in the objectives space. In the present case the objectives space is a 3D space and the Pareto front is a surface, but with a complex shape as shown for example on Figure 6. The yellow points are for the 7 blades case, the blue ones for the 8 blades case, the grey points are projections on the three different plans corresponding to two objectives.

![Figure 6: illustration of Pareto front](image)

Finding a final best compromise solution by local optimization

From the mathematical point of view, all the points on the Pareto front are equivalent. They all correspond to non-dominated solutions. A difficulty for the engineer is to select one final solution. A best compromise can be obtained by pragmatical consideration consisting for instance in defining a certain order of priority between the objectives.

Here an approach is proposed for dealing with the case where all objectives have the same importance.

➢ **Step 9: Calculation of the normalized values**

For comparing the solutions on the Pareto front, a normalization is needed as the objectives are of different natures (here weight in kilograms and Facou is an acoustic metric in dB or in Watt). Knowing the maximum and minimum values taken by each objective on the Pareto front, the normalized values are calculated as the difference to the goal (minimal or maximal values for an objective which must be minimized or maximized) divided by the distance between the maximal and minimal values. For the case of objectives to be minimized (case of the three objectives here):

\[ \bar{F}_i(X) = \frac{F_i(X) - F_i^{min}}{F_i^{M} - F_i^{m}} \]

\( X \) being a solution on the Pareto front \( P \), the minimal and maximal values of the “objective i” are noted:

\[ F_i^{m} = \max_{x \in P} F_i(x) \quad \text{and} \quad F_i^{M} = \min_{x \in P} F_i(x) \]
By this way the solutions are “non-dimensionnalized” and get comparable values between 0 and 1.

**Step 10:** Level Diagrams and selection of a norm

By comparing the results it turns out that the designs with 6 blades give poorer results compared with the 7 and 8 blades cases. Therefore hereafter the level diagrams are presented only with the 7 and 8 blades cases for the sake of clarity. These diagrams present a norm metric corresponding to the distance with respect to the “Utopian Point” cumulating the best values (the minimal values of the three objectives in the present case).

Different norms are possible. Here the Euclidian norm is used as it is well adapted for calculating the distance of each solution on the Pareto front with respect to the “Utopian Point”, (with \( O \) the number of objectives):

\[
\left\| \mathbf{F}(X) \right\|_2 = \sqrt{\sum_{i=1}^{O} F_i(X)^2}
\]

The level diagrams are presented hereafter showing this metric (distance to the Utopian point) with respect to the three objectives and with respect to the design variables.

![Figure 7: Level diagrams using the Euclidian norm](image)

**Step 11:** Minimization of the distance wrt the “Utopian Point” by using the selected norm

The red point on these diagrams (Figure 7) is the solution point on the Pareto front the closest to the “Utopian Point”, i.e. with the minimal norm. With this concept defining the best compromise solution as the one the closest to the “Utopian Point”, the multi objective problem is converted into a mono criterion. Therefore the final calculation of the optimal solution can be done by minimizing the norm measuring this distance and that can be performed by a determinist algorithm.

This final local optimization can be dealt with either by using the process with the RSM or by using the MDA (complete actual chain). A Nelder-Mead algorithm is applied for this purpose. In the first option (the quickest), the minimization of the norm with the RSM gives the following results:
Table 2: results of the determinist optimization with the RSM

**Step 12:** final evaluation of the predesign solution

Then by simply recalculating the objectives for this set of design values with the MDA (presizing and evaluation chain), the following values are obtained:

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Objectives Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Wempty</td>
</tr>
<tr>
<td>c</td>
<td>Facou</td>
</tr>
<tr>
<td>U</td>
<td>Wfuel</td>
</tr>
<tr>
<td>b</td>
<td>Wdgw</td>
</tr>
</tbody>
</table>

Table 3: results of the final evaluation with the MDA from the RSM optimization

The objectives values are very close to the ones resulting from the RSM with less than 1% of differences indicating that the process with RSM gives here a very good approximation of the complete MDA chain. For a better accuracy, it is also possible to perform the final optimization with the MDA chain. It is computationally more costly, although by starting from one of the previous evaluations the computational time is reduced.

6 **DETERMINIST APPROACH**

The second approach presented in this part is an alternative process using a determinist approach to solve the three-objective optimization corresponding to the presizing of the helicopter HO90. The main characteristics of this approach are the following:

- only use determinist optimizations, based on RSM in order to reduce as much as possible the computational time,
- use Mixture of Experts (MOE) to consider mixed design variables (integer and real continuous ones), so that only one surrogate can be used to treat the three possible values of b (number of blades).
Mixture of Experts strategy

In a recent paper [24], a strategy, called Mixture of Experts (MOE), to approximate discontinuous or derivative discontinuous functions is proposed. This strategy uses a clustering of the learning basis into regions where the function to be approximated is expected to be continuous or at least more simple. The innovative idea here is to perform the clustering on the discrete variable (blade number) and to consider a local expert for each of its discrete values.

The method strongly relies on the EM algorithm (Expectation-Maximization), which is a classical algorithm in statistics to solve maximum-likelihood problem, such as density estimation. In this case, the learning data (input and output) is modelled as a finite mixture of multivariate Gaussians. The EM algorithm is first used to estimate the mixture parameters (weights of each mixture component, means and variance-covariance matrices of each Gaussian component), a clustering is performed on the data (by means of Maximum A Posteriori), a local expert is built over each region, marginal densities are then derived from the finite mixture parameters and a finite approximation model is built as:

\[ f(x) = \sum_{i=1}^{K} \frac{P(\kappa = i | X = x)}{\lambda} f_i(x) \]

where \( K \) is the number of Gaussian components, \( P(\kappa = i | X = x) \) is the probability to lie in cluster \( i \) knowing that \( X = x \) (analytic expression depending on the parameters of the Gaussian multivariate distribution) and \( f_i \) is the local expert built on cluster \( i \).

The main interest of this method is to handle easily mixed variables and to provide a global approximation model which can be used in a gradient-based method for the optimization.

Figure 8 illustrates the three clusters relative to the three discrete values of blade number (b=6, b=7 and b=8). Based on this clustering, a local expert (Artificial Neural Network, Radial Basis Function, Polynomial approximation, etc.) is built in each area and a global model (denoted in the following by MOE) depending on the 4 mixed design variables (R,C,U,b) can be obtained as detailed above.

In this context, a MOE is built for each output of interest: \( W_{fuel}, W_{empty}, Facou \). To assess the goodness of the fit, we use the relative mean error \( E_{\text{rel}} \):

\[ E_{\text{rel}} = \frac{1}{N} \sum_{i=1}^{N} \frac{\hat{y}_i - y_i}{y_i} \]

where \( N \) is the number of points, \( \hat{y}_i \) corresponds to the surrogate model applied to the design variable \( x_i \) and \( y_i \) is the associated output (resp. \( W_{fuel}, W_{empty} \) and \( Facou \)) given by the full process (MDA).
Description of the determinist approach

This alternative process is divided into 4 main steps.

- The first step is the generation of a database (591 points) by using the complete MDA process with the inputs (design variables) $x=(R,C,U,b)$ and the outputs $y=(W_{fuel}, W_{empty}, Facou)$. This step is similar to the previous approach (phase 1, preparation of the optimization, step 1 to 4).
- The second step concerns the generation of a RSM by using a mixture of experts (MOE), each output getting its specific MOE. The proposed MOE uses a specific clustering relative to each value of $b$ ($b=6$, $b=7$ or $b=8$).
- The third step is dedicated to mono-objective optimization using the MOEs. For each output, a determinist algorithm based on the Sequential Quadratic Programming method is applied to determine the minimum. Each of the three optimizations is done with a multi-start technique initialized by several points distributed over the whole optimization region in order to increase the confidence in the obtained solution.
  - $W_{empty_{min}}=\min_{(R,C,U,b)}MOE_{W_{empty}}$
  - $W_{fuel_{min}}=\min_{(R,C,U,b)}MOE_{W_{fuel}}$
  - $Facou_{min}=\min_{(R,C,U,b)}MOE_{Facou}$

An utopian point [25] composed on the three minima is then defined

$\left(W_{empty_{min}}, W_{fuel_{min}}, Facou_{min}\right)$

- The fourth and last step is a distance optimization (SQP algorithm) using MOEs, performed to find the optimal design values ($R^*,C^*,U^*, b^*$)

$\left(R^*,C^*,U^*, b^*\right)=\arg\min_{(R,C,U,b)}\left[||W_{empty_{min}}-MOE_{W_{empty}}||^2 + ||W_{fuel_{min}}-MOE_{W_{fuel}}||^2 + ||Facou_{min}-MOE_{Facou}||^2\right]$

Analysis of intermediate results

One of the main interests of the approach was to benefit from MOE strategy to obtain a single surrogate model per objective for the mixed integer design variables of the problem. The first result to be analysed is the relative error obtained by MOE after the second step of the approach. For the experiments, the initial database is made of 591 learning points (similar to the previous strategy). This database has been reduced to respectively 559 and 396 points by removing "aberrant points" (i.e. points were the convergence criteria in the presizing process was considered as too high). Additional 30 validation points, located in the optimal re-
gion, have been run on the overall process in order to calculate the relative error: these three
developed MOE have been tested on these new set of points. The mean relative error $E_{rel}$ (here
$N=30$) has been computed for each objective and is given below in Table 4.

<table>
<thead>
<tr>
<th>E_rel_mean 30 points</th>
<th>MOE - 591 points</th>
<th>MOE - 559 points</th>
<th>MOE -396 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wempty</td>
<td>0.0087</td>
<td>0.0116</td>
<td>0.0061</td>
</tr>
<tr>
<td>Wfuel</td>
<td>0.0173</td>
<td>0.0354</td>
<td>0.0176</td>
</tr>
<tr>
<td>Facou</td>
<td>0.0037</td>
<td>0.0047</td>
<td>0.0036</td>
</tr>
</tbody>
</table>

Table 4: relative mean error of the different MOEs for the local validation database (30 points)

One can check that these errors are quite small (less than 1% to 3.5%) even with some
reduced number of points for the database. Notice that these MOE combine either artificial
neural networks or second order polynomials. The efficiency of our mixture of experts is thus
demonstrated on these three objective functions.

Another result to be analyzed deals with the behavior of the surrogate model for the do-
main of definition. On Figure 9 are plotted the database objective values (here $W_{\text{fuel}}$) for the
3 continuous variables $(R,C,U)$ and the equivalent MOE objective. The global surrogate model
provides a good representation of the database behavior.

The last interesting analysis is linked to the ability of MOE to handle the mixed integer
design variables. In other words, within the optimisation process, on the fourth step, the
approach assesses that the selected configurations end up on a realistic number of blades (6, 7 or
8). Notice that the optimal blade number can differ from the initial starting point, this property
is due to the global approximate model which has been built with MOE where each cluster is
associated to a discrete value of $b$.

7 COMPARISONS

This paragraph present the global optimum obtained by the determinist approach using
MOE surrogate modeling. After the fourth step of optimization, aiming at minimizing the dis-
tance with the “Utopian Point”, an optimum is found for each database. The behaviour of the objective function is presented on Figure 10, for 2 fixed values of the input variables (U=200 m.s⁻¹, b = 8). It appears that the configuration selected by the optimiser is well located in the centre of the objective function surface. One can also note that the optimum area is quite flat in terms of blade radius and chord with very low gradients around the selected solution.

![Figure 10: Localisation of the optimal rotor design in R and C (for U=200 m.s⁻1, b = 8).](image)

In order to compare the results obtained with this methodology with the previous approach, the optimal configuration (in terms of design values as inputs) has been reevaluated in the HO-90 MDA process for assessing the “real” values. The Table 5 summarizes the optimal configurations (4 mixed design variables) and their performance for both methodologies, denoted by "multi-obj" (part 5) or "mono-obj" (part 6).

<table>
<thead>
<tr>
<th>Full process- Model Center</th>
<th>Wempty (kg)</th>
<th>Wfuel (kg)</th>
<th>Facou (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X* (multi-obj) (R=16.49, C=0.88, U=200, b=8)</td>
<td>26894</td>
<td>9782</td>
<td>69.81</td>
</tr>
<tr>
<td>X*(mono-obj)-591 pts (R=16.17, C=0.91, U=200, b=8)</td>
<td>26940</td>
<td>10012</td>
<td>69.94</td>
</tr>
<tr>
<td>X*(mono-obj)-559 pts (R=16.14, C=0.89, U=201.8, b=8)</td>
<td>26736</td>
<td>10050</td>
<td>69.84</td>
</tr>
<tr>
<td>X*(mono-obj)-396 pts (R=16.17, C=0.91, U=200, b=8)</td>
<td>26939</td>
<td>10014</td>
<td>69.94</td>
</tr>
</tbody>
</table>

Table 5: Comparison of the optimal configuration between the two methodologies

First of all, all the determinist optimisations converged towards very similar configurations, even though the database size was different. All configurations have 8 blades and a speed at tip close to minimum boundary value (200 m.s⁻¹). The differences are really slight on the 2 other variables (Radius and Chord). As consequence, the differences in terms of performances are negligible, reaching less than 1%.

When comparing the optimal configuration obtained with this approach to the previous one, the localisation of both in the design variables domain is very close with only a difference on the blade radius. Moreover, the results in terms of objectives are rather close, although a little worse compared with the initial optimisation performed with a genetic algorithm.
8 CONCLUSIONS

Two methodologies have been presented for addressing the multi objectives optimization corresponding to the presizing of a helicopter.

The first one is a hybrid approach combining a genetic algorithm for a first global assessment and a determinist algorithm for the final local optimization. The genetic algorithm is able of dealing with the multi objectives, multi design variables under constraints problem. One interest of this approach is to provide to the design engineer a Pareto front corresponding to a first assessment of different optimal solutions. By analysing this Pareto front with the level diagrams, he can have a view of the trade-off between the objectives and the sensitivity with respect to the design variables. The minimization of the distance with respect to the “Utopian Point” is a good concept for helping the designer to select one best compromise solution when all the objectives have the same weight.

The second one is a mono-objective determinist approach thanks to the use of Mixture of Experts surrogate models. A global surrogate model can be built to deal with mixed integer design variables, the mixture of experts can handle this kind of heterogeneity with efficiency.

The mono-objective determinist approach can quickly provide few points of interest given by a multi-start technique. So, this method can be used for fast preliminary design in order to detect a zone of interest. Then, this preliminary phase might be followed by a sequential enrichment with new points of calculation or by a use of high fidelity models more expensive but restrained in a small research area. In this second phase, a multi-objective approach may be used to detect a Pareto front and offer a larger choice of potential solutions to the design engineer.

REFERENCES


