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# Slowing time by stretching the waves in special relativity

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## Abstract

The correction of the classical Doppler formula by the time dilation factor gives the currently admitted relativistic Doppler equation. When the source path is not collinear to that of the receiver, this correction gives a transverse wave dilation whose possible detection was considered by Einstein as an ideal mean to prove the time dilation of special relativity. It is suggested here that this elusive effect whose measurement remains controversial, could in fact be absent from relativistic as well as classical contexts, but that this absence paradoxically proves the time-dilation of special relativity. An intuitive reasoning using parallel Doppler effects is first proposed to conceive why the absence of a wave carrier medium imposes time dilation for electromagnetic waves. A rational reconstruction of the non-collinear Doppler effect is then presented using only reciprocal quantities such as the source-receiver distance. This rigorous analysis reveals a virtual equation whose transverse contraction effect is exactly cancelled by the relativistic time dilation. The candidate Doppler equation emerging from this treatment is remarkably elegant, geometrically symmetrical and centered on the closest point between the source and the receiver.

*Keywords:* Time dilation; Relativistic Doppler equation; Transverse contraction.

## 1 Introduction

The wave dilation of special relativity is a general phenomenon which gives, when surimposed to the classical Doppler effect, the relativistic Doppler formula. A Doppler effect is caused by the velocity of a wave source relatively to an observer. The classical doppler effect shortens the apparent wavelength  $\lambda^{\text{mov}}$  of an object approaching at speed  $v$  such that  $\lambda^{\text{mov}} = cT - vT$  where  $T$  is the period, giving  $\lambda^{\text{mov}}/\lambda = 1 - \frac{v}{c}$ . Conversely it stretches the apparent wavelength of a receding object such that  $\lambda^{\text{mov}}/\lambda = 1 + \frac{v}{c}$ . In the general case, when the velocity vector is not strictly collinear with the line of sight, these equations should be modified by replacing  $v$  by a smaller value. When using radial velocities which are the orthogonal projection of the velocity vector on

the source-observer line ( $v \cos \theta$ ), there is no transverse effect because when the source is at the closest point from the observer ( $\theta = \pi/2$ ), the radial speed is zero. But this intuitive reasoning using radial velocities makes uses of angles which are not equivalent between the source and the receiver. A new equation is built using only parameters symmetrical between the source and the receiver and where the non-reciprocal cosine velocities will be replaced by the reciprocal rate of source-receiver distance change. But let us start with thought experiments enlighting the importance of a wave-carrier medium in Doppler effects.

## 2 Medium-dependent and independent wave propagation

The classical Doppler effect holds for waves carried by the medium, like sound, whereas the relativistic Doppler effect applies to electromagnetic waves propagating in vacuum.

### 2.1 Longitudinal Doppler effect

A source and a receiver can move relative to one another and in addition, can move relative to an hypothetical static medium supporting wave propagation at speed  $c$ . The source and the receiver recede form each other at speed  $v$ . At time  $t_E$ , when spaced from the receiver by  $D_E$ , the source emits a wave pulse towards the receiver. Different results are expected depending on whether this is the source or the receiver which moves relatively to the background medium (Fig.1).

*The source is considered immobile relative to the background medium*

In this case, the wave is expected to reach the receiver after crossing a distance  $D_R$  (middle line of Fig.1). The duration of the wave travel is

$$t_R - t_E = D_R/c \quad (1a)$$

At  $t_R$ , the new spacing between the source and the receiver has become

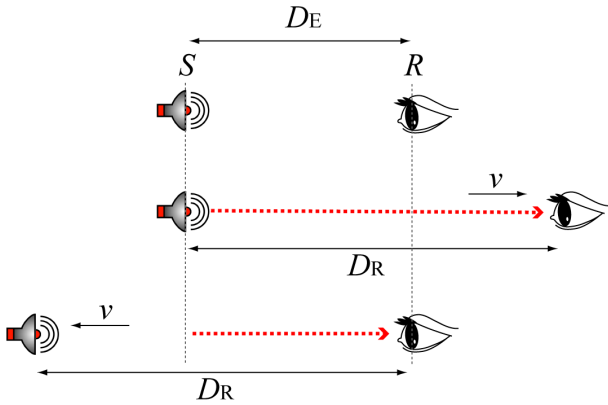
$$D_R = D_E + v(t_R - t_E) \quad (1b)$$

Replacing the duration in Eq.(1b) by the value given by Eq.(1a), yields a distance ratio corresponding to a classical Doppler effect

$$\frac{D_R}{D_E} = \frac{1}{1 - \frac{v}{c}} \quad (1c)$$

In case of collinear approach, the same reasoning gives

$$\frac{D_R}{D_E} = \frac{1}{1 + \frac{v}{c}} \quad (1d)$$



**Figure 1.** A wave pulse is emitted by a source ( $S$ ) when spaced from a receiver ( $R$ ) by  $D_E$ . Just like a ball thrown between two players, the wave travels through a static medium relatively to which either  $S$  (middle line) or  $R$  (bottom line), is considered immobile.

*The receiver is considered immobile relative to the background medium*

The wave reaches the receiver at time  $t_R$  after crossing a distance  $D_E$  (bottom line of Fig.1). Hence, the duration of the wave travel is

$$t_R - t_E = D_E/c \quad (2a)$$

Replacing the duration in Eq.(1b) by its value given by Eq.(2a), yields

$$\frac{D_R}{D_E} = 1 + \frac{v}{c} \quad (2b)$$

Distance increases in the same ratio that the classical Doppler effect. In case of collinear approach, the same reasoning gives

$$\frac{D_R}{D_E} = 1 - \frac{v}{c} \quad (2c)$$

These Doppler effects are suitable for the sound that is carried by physical supports, but not for light travelling in vacuum. Since it is impossible in vacuum to

assign the relative movement to either the source or the observer, it seems natural to take the geometric mean of the two extreme situations of Fig.1 (Eqs (1c)/(2b) and Eqs (1d)/(2c)). This gives the relativistic Doppler effect. For the recession:

$$\left\langle \frac{D_R}{D_E} \right\rangle = \sqrt{\frac{c+v}{c-v}} \quad (3a)$$

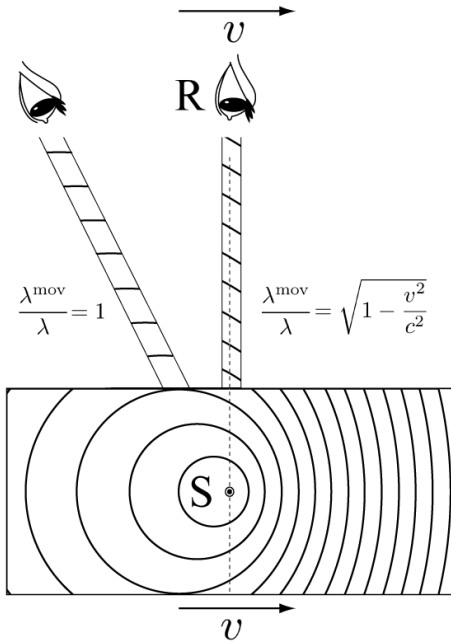
and for the approach

$$\left\langle \frac{D_R}{D_E} \right\rangle = \sqrt{\frac{c-v}{c+v}} \quad (3b)$$

Such a correspondence between wavelength distortion and source-receiver distance changes during light travel has also been formulated by Lemaître in the context of the cosmological redshift [1].

## 2.2 Parallel Doppler effect

The classical transverse Doppler effect is inexistent for a sound wave emitted by a moving source located at the closest point from an immobile receiver. But if the source and the receiver move in parallel and in the same direction, the wavelength appears shortened as represented in Fig.2, with a contraction between  $S$  and  $R$  precisely corresponding to the inverse of the relativistic dilation factor. The source and receiver do not move relative to each other, but both move relative to the medium which works as an apparent wind shifting back the wave crests. The second step of the reasoning is to switch from sound waves to electromagnetic waves propagating in vacuum. Once the medium is removed, it is no longer possible to assert that the source and the receiver have an absolute movement. Relativistically speaking, they belong to same reference frame, so that the previous apparent wind is inexistent. It is therefore necessary to cancel the wave contraction between  $S$  and  $R$ .



**Figure 2.** Classical Doppler effect predicted when the source (S) and the receiver (R) run in parallel at the same speed  $v$ . There is no relative movement of S and R, but both move relatively to the medium. The wave between S and R appears stably contracted by  $\sqrt{1 - \frac{v^2}{c^2}}$ . Now let us remove the medium and imagine that the wave self propagates in vacuum. The source and receiver then clearly belong to the same inertial frame and as a consequence, the previous wave distortion orthogonal to velocity vectors and now corresponding to the light path should be cancelled by multiplication by  $1/\sqrt{1 - \frac{v^2}{c^2}}$ .

### 3 A new reciprocal Doppler treatment

The Doppler effect will be recalculated using as a speed the rate of distance change between the source and the receiver, that is completely symmetrical. Classical Doppler effects mix longitudinal and transverse effects and range between the two asymptotes  $1 - \frac{v}{c}$  and  $1 + \frac{v}{c}$ . The intermediate values are currently defined using the angle  $\theta$  between the motion line and the receiver. In fact, since  $\theta$  varies with time and generates certain problems such as aberration effects, it seems more rational to skip it and to calculate the Doppler effect directly as a function of variation of distance between a source and an observer that are unique and reciprocal. A Doppler-generating speed will be defined using the Pythagorean theorem, which remains valid in the Euclidean relativistic space. Let us define virtual rates of simultaneous shortening of the hypotenuse (rate  $h$ ) and of one side (rate  $v$ ), while maintaining the other side constant and the triangle rectangle.  $h$  and  $v$  are related to each other with a couple of simple equations. If denoting  $H_0$  the starting length of

the hypotenuse,  $L_0$  that of the shortening side and  $D$  the constant side,

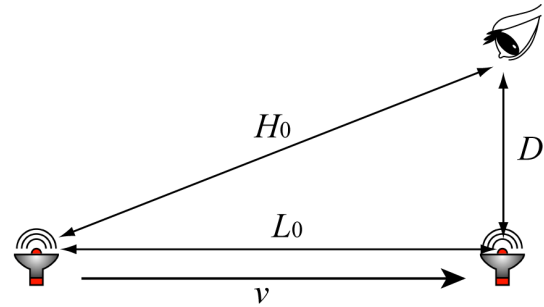
$$H_0^2 = D^2 + L_0^2 \quad (4a)$$

and

$$(H_0 - ht)^2 = D^2 + (L_0 - vt)^2 \quad (4b)$$

whose subtraction allows to eliminate  $D$  and yields

$$ht = H_0 - \sqrt{H_0^2 + (vt)^2 - 2L_0vt} \quad (4c)$$



**Figure 3.** A source moving at constant speed  $v$  starts from a distance  $H_0$  from the immobile observer. The shortest distance between the source and the observer is  $D$ .

Now let us apply this general result, which does not correspond to a specific physical situation, to the particular case represented in Fig.3, of a the source moving non-collinearly relative to an observer and reaches at speed  $v$  the closest point from this observer. The triangle of Fig.3 evolves such that the hypotenuse reduces from  $H_0$  to  $D$  while the source path reduces from  $L_0$  to  $0$ .  $H_0$  and  $L_0$  are precisely adjusted such that the wave front reaches the receiver when the source reaches the closest point, following a time delay  $\Delta t$ . In this case,  $H_0$  and  $L_0$  can be replaced by  $c\Delta t$  and  $v\Delta t$  respectively and Eq.(4c) becomes

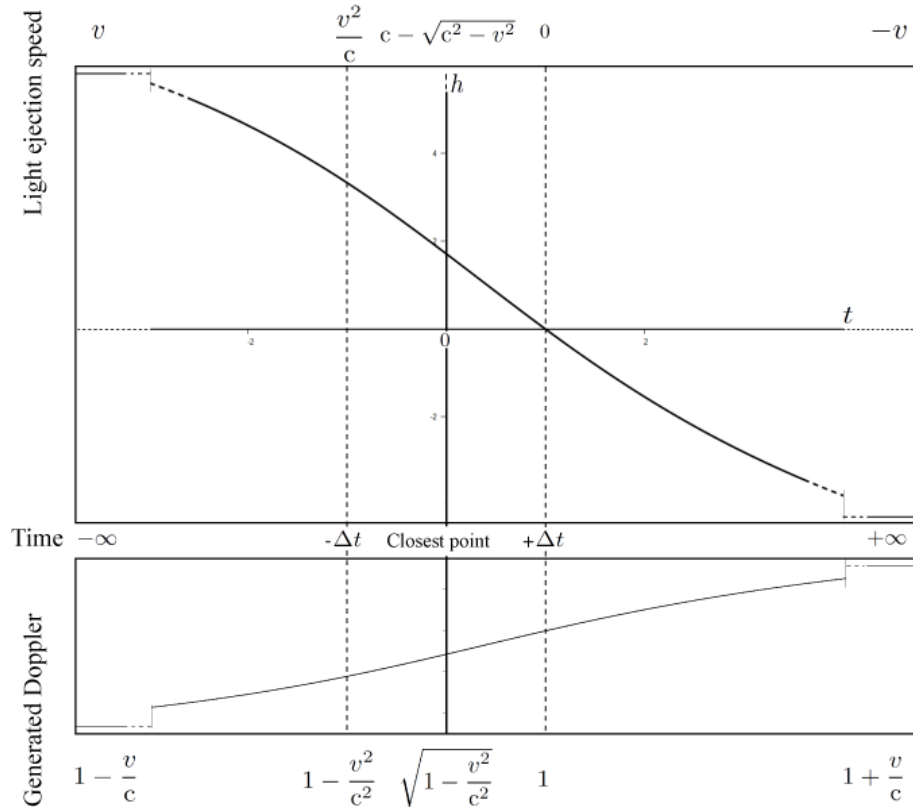
$$h = \left( c\Delta t - \sqrt{(c\Delta t)^2 + (vt)^2 - 2v^2t\Delta t} \right) / t \quad (5)$$

When inserted in the classical Doppler formula, this speed gives the results presented in Fig.4. The signal received when the observer is at right angle to the motion line was emitted at  $h = v^2/c$ . This value, giving a Doppler effect of  $\lambda^{\text{mov}}/\lambda = 1 - \frac{v^2}{c^2}$  is not defined and calculated as a series expansion limit. More interestingly, The signal is emitted at right angle to the observer at  $h = c - \sqrt{c^2 - v^2}$ , giving a Doppler effect of  $\lambda^{\text{mov}}/\lambda = \sqrt{1 - \frac{v^2}{c^2}}$ .

The speed  $h$  may reflect the transverse contraction factor  $\sqrt{1 - \frac{v^2}{c^2}}$  of Voigt's transformations, but the Doppler effect generated by this speed will be called virtual Doppler effect (VD), because on the one hand, it is not relativistic since it does not take into account the relativistic time dilation, but on the other hand it is not more classical as it is calculated without taking into account a reference

medium. When applied to the classical Doppler effect with a reference medium (CD), the time dilation factor gives the traditional relativistic Doppler formula (TRD). Application of the same Lorentz factor to the new vir-

tual Doppler effect (VD), turns to exactly compensates its transverse contraction effect and gives an interesting candidate relativistic Doppler effect (CRD).



**Figure 4.** Evolution of the Doppler-generating speed  $h$  given by Eq.(5). The time unit  $\Delta t$  is the travel time of the signal reaching the receiver when the source is the closest to it. The origin of time  $t = 0$  is centered at this closest position. The Doppler effect generated by light emitted at this position (at right angle to the receiver) is shifted blue by  $\sqrt{1 - (v/c)^2}$ .

## 4 Comparison of the different Doppler approaches

### 4.1 Normalization of the different Doppler formulas with respect to distances

To compare the different Doppler formulas, they should be comparable for any relative configuration of the source and the observer. The comparison with the relativistic equation described with angles [2] is delicate because several equations are possible depending on the angle used: either the original angle between the velocity vector and the source-observer connection line ( $\theta$ ) or the reception angle ( $\theta'$ ).

$$\frac{\lambda_{\text{mov}}}{\lambda} = \frac{1 + \frac{v}{c} \cos \theta}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c} \cos \theta'}, \quad (6a)$$

the two angles of this identity are related to each other through the so-called aberration formula [2]

$$\cos \theta = \frac{\cos \theta' - \frac{v}{c}}{1 - \frac{v}{c} \cos \theta'} \quad (6b)$$

Table 1: The aberration effect in special relativity is related to the time points of wave emission.

$t$	$\cos \theta$	$\cos \theta'$	$\lambda^{\text{mov}}/\lambda$
$-\Delta t$	$-\frac{v}{c}$	0	$\sqrt{1 - \frac{v^2}{c^2}}$
0	0	$\frac{v}{c}$	$1/\sqrt{1 - \frac{v^2}{c^2}}$

Approaches using angles are confusing because if one assumes that the transverse effect is obtained when the cosinus is 0, the first formula of Eq.(6a) predicts a wave dilation, whereas the second formula gives the inverse wave contraction, because  $\theta$  and  $\theta'$  cannot be simultaneously equal to  $\pi/2$  [3]. This subtlety is a matter of delay of wave travel  $\Delta t$  (Table.1). The non-collinear Doppler formulas contain two variables: the speed  $v$  and an angle. This angle varies along the

wave path and can be expressed as a function of time, such that  $\theta(t) = \tan^{-1}(D/vt)$ . Hence, on the one hand  $\cos\theta(t) = 1/\sqrt{1+(D/vt)^2}$ , and on the other hand the distance  $D$  can itself be defined as a function of  $\Delta t$  ( $D = \Delta t\sqrt{c^2 - v^2}$ ), thereby allowing to make the formulas functions of distances only. There are other ambiguities in the literature about the sign of the velocity ( $-v$  and  $+v$ ) in Doppler equations.

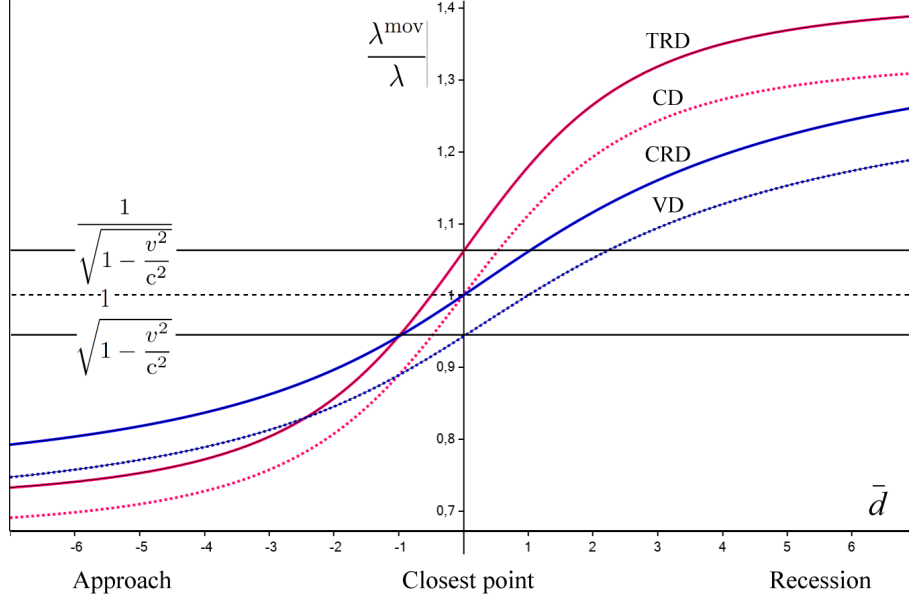
Table 2: Doppler effects generated by a wave emitted at the normalized distance  $\bar{d}$  from the closest point and calculated using the different formulas. CD= Classical doppler effect; TRD= traditional relativistic Doppler effect, VD= virtual doppler formula based on the rate  $h$ ; CRD= candidate relativistic formula. The new formulas are built using the classical Doppler framework  $1 - v/c$ , in which  $v$  is replaced by the speed  $h$  given by Eq.(5). The unit of distance is  $v\Delta t$  where  $\Delta t$  is the time-of-flight of the wave reaching the observer when located at the closest point from the source. The circled 1 indicate the inversion points where the Doppler effects cancel. The distance  $\sigma$  corresponds to the inversion point for the TRD, but has no particular meaning for the other equations.

Doppler.	$\frac{\lambda^{\text{mov}}}{\lambda}$	$\bar{d} = -\infty$	$\bar{d} = -1$	$\bar{d} = \sigma$	$\bar{d} = 0$	$\bar{d} = +1$	$\bar{d} = +\infty$
CD	$1 + \frac{v^2 \bar{d}}{c^2 \Phi}$	$1 - \frac{v}{c}$	$1 - \frac{v^2}{c^2}$		⊙	$1 + \frac{v^2}{c^2}$	$1 + \frac{v}{c}$
TRD	$\frac{\text{CD}}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$	$\sqrt{1 - \frac{v^2}{c^2}}$	⊙	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\frac{1 + \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$
VD	$\frac{\Phi + \bar{d}}{1 + \bar{d}}$	$1 - \frac{v}{c}$	$1 - \frac{v^2}{c^2}$		$\sqrt{1 - \frac{v^2}{c^2}}$	⊙	$1 + \frac{v}{c}$
CRD	$\frac{\text{VD}}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$	$\sqrt{1 - \frac{v^2}{c^2}}$		⊙	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$

$$\text{with } \bar{d} = \frac{t}{\Delta t}, \quad \Phi = \sqrt{1 - \frac{v^2}{c^2}(1 - \bar{d}^2)} \quad \text{and} \quad \sigma = -\frac{c}{v\sqrt{2}}\sqrt{\sqrt{1 - \frac{v^2}{c^2}} - 1 + \frac{v^2}{c^2}}$$

To eliminate all these sources of confusion, the equations are composed here without referring to angles and using speeds always positive, irrespective of the relative location of the observer, by transferring the sign to the time  $t$  ranging from  $-\infty$  and  $+\infty$ . To "synchronize" the formulas at the time points of wave emission, in the new formula,  $t$  should be replaced by  $t + \Delta t$ . Finally, a dimensionless normalized distance is defined for all source paths relatively to the closest point ( $\bar{d} = vt/v\Delta t = t/\Delta t$ ). A little algebra satisfying all these requirements gives the

equations compiled in Table.2. For  $\bar{d} = -1$  where the virtual function is not defined, the Doppler effect takes the limit value  $1 - \frac{v^2}{c^2}$ . These different Doppler equations describe general combinations of longitudinal and transverse Doppler effects for any relative position of the source and the observer. As these normalized equations can now be compared, their profiles as functions of  $\bar{d}$  are superposed for visualization in Fig.5.



**Figure 5.** Comparative profiles of Doppler effects predicted for  $v/c = 1/3$ , by the different formulas. The switch between the contraction and the dilation occurs at the closest point for the CD and CRD, just before the closest point for the TRD (between -1 and 0, see the text) and at  $\bar{d} = +1$  for the VD.

**Table 3:** Arithmetic and geometric means of Doppler effects expressed using either wavelengths or frequencies.  $\bar{d}$  and  $\Phi$  are defined in Table.2. Note that for each type of averaging of CRD, the same result is obtained for wavelengths and frequencies.

Mean	$\lambda$ vs $\nu$	CD	TRD	VD	CRD
Arithmetic	$\lambda$	1	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\frac{\Phi - \bar{d}^2}{1 - \bar{d}^2}$	$\frac{\Phi - \bar{d}^2}{(1 - \bar{d}^2)\sqrt{1 - \frac{v^2}{c^2}}}$
	$\nu$	$\frac{\Phi^2}{\left(1 - \frac{v^2}{c^2}\right)\left(1 + \frac{v^2}{c^2}\bar{d}^2\right)}$	$\frac{\Phi^2}{\sqrt{1 - \frac{v^2}{c^2}}\left(1 + \frac{v^2}{c^2}\bar{d}^2\right)}$	$\frac{\Phi - \bar{d}^2}{\Phi^2 - \bar{d}^2}$	$\frac{\Phi - \bar{d}^2}{(1 - \bar{d}^2)\sqrt{1 - \frac{v^2}{c^2}}}$
Geometric	$\lambda$	$\frac{1}{\Phi}\sqrt{\left(1 - \frac{v^2}{c^2}\right)\left(1 + \frac{v^2}{c^2}\bar{d}^2\right)}$	$\frac{1}{\Phi}\sqrt{1 + \frac{v^2}{c^2}\bar{d}^2}$	$\sqrt{1 - \frac{v^2}{c^2}}$	1
	$\nu$	$\frac{\Phi}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)\left(1 + \frac{v^2}{c^2}\bar{d}^2\right)}}$	$\frac{\Phi}{\sqrt{1 + \frac{v^2}{c^2}\bar{d}^2}}$	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	1

## 5 Properties of the new Doppler equations

### 5.1 Comparative symmetry

The mean values of the Doppler effects generated at symmetrical distances from the closest point, depend on the modes of averaging, which are, when expressed using wavelengths,

- the arithmetic mean:  $\frac{1}{2} \left( \frac{\lambda_{(-d)}^{\text{mov}}}{\lambda} + \frac{\lambda_{(+d)}^{\text{mov}}}{\lambda} \right)$
- and the geometric mean:  $\left( \frac{\lambda_{(-d)}^{\text{mov}}}{\lambda} \frac{\lambda_{(+d)}^{\text{mov}}}{\lambda} \right)^{\frac{1}{2}}$ .

The appropriate tool is logically the geometric mean, because it is expected to hold both for periods and frequencies such that  $\langle T_{(-d)}, T_{(+d)} \rangle = 1 / \langle \nu_{(-d)}, \nu_{(+d)} \rangle$  and satisfies the rule of color reflectance fusion. The new formulas display a perfect geometric symmetry, in such a way that the geometric mean of the Doppler effects before and after the midpoint are independent of time and always  $\sqrt{1 - \frac{v^2}{c^2}}$  for the VD and 1 for the CRD (Table 3).

### 5.2 Switching points

The four different Doppler equations compared in Fig.5, have different switching points between contracted and dilated waves:

- for the CD, at  $\bar{d} = 0$ ,
- for the VD at  $\bar{d} = 1$  (or  $t = +\Delta t$ ),
- for the TRD, the traditional relativistic formula yields the most complicated result, close to the transverse line

$$\bar{d} = -\frac{c}{v\sqrt{2}} \sqrt{\sqrt{1 - \frac{v^2}{c^2}} \left( 1 - \sqrt{1 - \frac{v^2}{c^2}} \right)} \quad (7)$$

- for the CRD, at the closest point  $\bar{d} = 0$ . We get rid of the weird result of the TRD.

### 5.3 Comparative angle-dependence

The time-dependent profiles of the different formulas have been compared in Fig.5. It is also interesting to compare their shape using an angular representation. The angle  $\theta$  of Fig.6 will be used while keeping the speed  $v$  always positive, such that

$$\cot(\pi - \theta) = \frac{t}{\Delta t} \frac{\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The TRD is

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\sqrt{1 + \cot^2 \theta} - \frac{v}{c} \cot \theta}{\sqrt{1 - \frac{v^2}{c^2}} \sqrt{1 + \cot^2 \theta}} \quad (8a)$$

multiplying by  $\sin \theta$  allows to recover the traditional, but no longer modulo- $\pi$ , formula

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{1 - \frac{v}{c} \cos \theta}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8b)$$

- and the CRD becomes

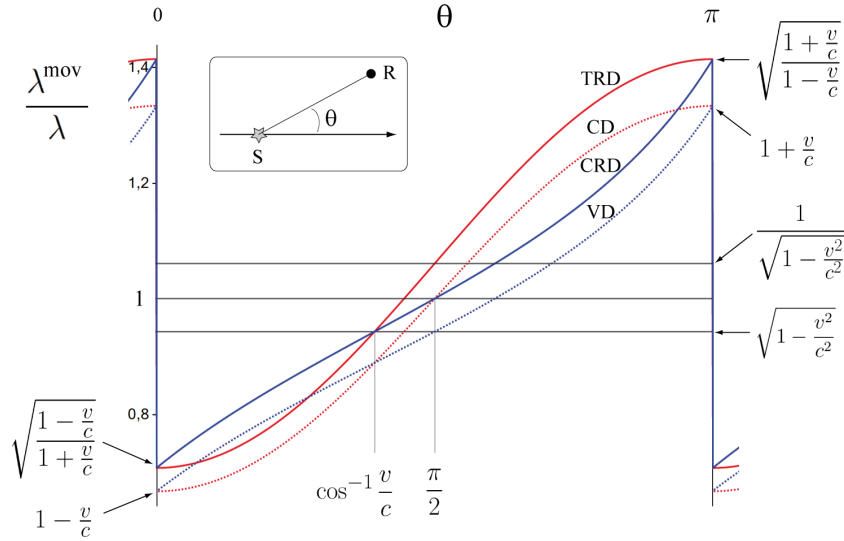
$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\frac{v}{c} \sqrt{1 + \cot^2 \theta} - \cot \theta}{\frac{v}{c} - \sqrt{1 - \frac{v^2}{c^2}} \cot \theta} \quad (9a)$$

multiplying by  $\sin \theta$ ,

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\frac{v}{c} - \cos \theta}{\frac{v}{c} \sin \theta - \sqrt{1 - \frac{v^2}{c^2}} \cos \theta} \quad (9b)$$

As shown in Fig.6, the profiles drawn to Eqs.(8a) and (9a) strikingly differ by their slopes, those of the new formula being closer to the tangent function.





**Figure 6.** Comparative profiles of Doppler effects predicted for  $v/c = 1/3$ , by the classical (dotted lines), and relativistic (plain lines) Doppler equations, either traditional or modified here, as functions of the emission angle  $\theta$  shown in the inset.

## 6 Correspondence between time and wave distortion effects.

Several experiments have been conducted to test the theory of special relativity through the relativistic Doppler formula of Einstein [4], following a suggestion of Einstein himself [5]. But the present analysis suggests that the time dilation is in fact responsible for the absence of transverse Doppler effect. Time and wave distortions are two facets of the same principle, illustrated in Appendix A and evidenced by a remarkable observation of supernovae, unfortunately too recent for Einstein to be aware of. The time window of brightness is relatively constant for comparable supernovae, but astronomers made an expected but nevertheless striking finding: the apparent time window of brightness depends on the distance of the supernova, in exactly the same proportion that their redshift. For instance, a distant supernova with a redshift of  $\lambda^{\text{app}}/\lambda = 1.5$ , has precisely a 1.5-fold longer duration of brightness [6]. Hence, time dilation corresponds exactly to wavelength increase, or equivalently frequency decrease, whereas apparent time contraction corresponds to wavelength shortening and frequency increase. Reciprocally, spacing wavelength crests automatically cause apparent time slowing, in the same manner that projecting at 12 frames per second a movie scheduled for 24 frames per second, shows abnormally slow scenes.

## 7 Test of the new Doppler equations

### 7.1 Longitudinal effects

The celebrated experiment of Ives and Stilwell [7] and its descendants [8] focused on the longitudinal Doppler effect. Ives and Stilwell recovered the time dilation factor by measuring the arithmetic means of the shifted wavelengths in front and behind moving atoms [7]. This result is however not discriminating since it is also obtained with the candidate formula.

### 7.2 Transverse effects

The existence of the transverse Doppler effect has been supported in a single study [9]. But more recently, using microwaves which exhibit a high spatial purity and a precise polarization plane, a sensitive apparatus failed to detect any transverse effect [10], which led the author to question the principle of dilation of special relativity. It is suggested here that this seemingly negative result could be indirectly considered as a verification of time dilation.

### 7.3 Relevance of the CRD equation

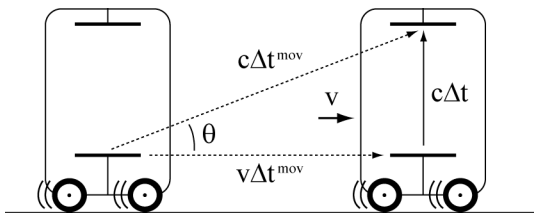
The test of the new candidate equation CRD is problematic considering the notorious difficulty of transverse tests and because it satisfies the Ives-Stilwell type longitudinal tests as well as the previous equation. In fact, its best asset for now is its elegance, that is often a sign of accuracy in science, and its fundamental symmetry illustrated by the geometric means of the non-collinear paths.

# Appendices

For electromagnetic waves, the so-called time-dilation effect should be taken into account to yield the relativistic Doppler effect. This is not a Doppler effect strictly speaking, but another type of a wave stretching naturally obtained when the source and the receiver belong to different inertial frames. It has been clearly and entirely deduced from the invariance of light velocity by Einstein in his seminal paper of 1905 on special relativity [2]. This dilation can be even more intuitively conceived using the light clock of Einstein described below.

## A The light clock of Einstein

The beat of the light clock of Einstein is the rebound of a photon between facing mirrors. This clock is placed vertically in a wagon rolling at constant speed  $v$ . For an external observer (with an excellent view!), the light path appears oblique when the train moves, whereas for an observer located inside the train, it appears vertical (Fig.A.1). The reference time interval  $\Delta t$  corresponds to the frame comoving with the light clock.



**Figure A.1** The famous triangular time-space diagram of the moving Einstein's clock. The horizontal scale is artificially stretched relatively to the vertical one for better visualization.

With respect to the angle  $\theta$  of Fig.A.1, elementary trigonometry says

$$\sin \theta = \frac{c\Delta t}{c\Delta t^{\text{mov}}} = \frac{\Delta t}{\Delta t^{\text{mov}}} \quad (\text{A.1a})$$

and

$$\cos \theta = \frac{v\Delta t^{\text{mov}}}{c\Delta t^{\text{mov}}} = \frac{v}{c} \quad (\text{A.1b})$$

which immediately gives, using the relationship  $\sin^2 \theta + \cos^2 \theta = 1$ ,

$$\frac{\Delta t^{\text{mov}}}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (\text{A.1c})$$

Another way to obtain this result is to use the straightforward geometrical tool of special relativity: the Minkowski spacetime. The oblique (hypotenuse) and vertical paths of light start from and arrive to the same points. This common spacetime interval should reconcile the point of view of an observer in the train, for whom the

clock appears immobile, and that of an outside observer for whom there is an additional translation. Hence,

$$\Delta s^2 = (c\Delta t)^2 = (c\Delta t^{\text{mov}})^2 - \Delta x_{\text{mov}}^2 \quad (\text{A.2a})$$

giving

$$c^2 \left( \frac{\Delta t}{\Delta t^{\text{mov}}} \right)^2 = c^2 - \left( \frac{\Delta x^{\text{mov}}}{\Delta t^{\text{mov}}} \right)^2 \quad (\text{A.2b})$$

and finally

$$\frac{\Delta t^{\text{mov}}}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (\text{A.2c})$$

In spacetime diagrams like the triangular scheme of Fig.A.1, the proper time of the clock is obtained when the distance to be crossed by light appears minimal. In Fig.A.1., this is the vertical path ( $\Delta t$ ), whereas for the external observer moving at speed  $v$  relatively to the clock this path appears stretched by  $\Delta t^{\text{mov}}/\Delta t = 1/\sqrt{1 - (v/c)^2}$ .

The light clock of Einstein is illuminating in that there is obviously no relative movement between the facing mirrors. In this respect, the wavelength dilation perceived by the external observer is not a Doppler effect but just an interframe perspective effect.

## B Time dilation by wave stretching

In the light clock experiment, the distinction between moving and immobile frames is irrelevant as they can be permuted. The train and the station platform are two equivalent systems of reference and the situation is simply inverted if the clock is put on the platform and if the observer inside the train considers that it is the platform that moves relative to the train in the opposite direction. A perspective effect is naturally reciprocal. The best known perspective effect is the apparent contraction of the size of a person standing far away from us, compared to a person standing close to us. We are very accustomed to this familiar effect and easily understand that this person should have exactly the inverse perception. Distance is a symmetrical notion, as is uniform motion. The apparent size reduction effect is so well integrated in our mind that it is unconsciously corrected. Moreover, it is used inversely to estimate the distance. The same operation can be applied to time dilation to deduce the relative frame velocity from the degree of wavelength stretching. Eq.(A.2a) can be modified as follows: On the one hand,  $c = \lambda/T$  and on the other hand, the time intervals can be replaced by a given number ( $n$ ) of periods  $\Delta t = n\lambda/c$ . The invariance of this number from any viewpoint allows to rewrite Eq.(A.2a) as

$$\frac{\Delta x^{\text{mov}}}{\Delta t^{\text{mov}}} = v = c \sqrt{\frac{\lambda^{\text{mov}} - \lambda}{\lambda^{\text{mov}}}} \quad (\text{B.1a})$$

which can be transformed into

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (\text{B.1b})$$

Eq.(B.1b) is typically the equation of a Doppler effect, showing that time flows follows wave frequencies. If the successive images of a film are two-fold spaced, the film would naturally run 2 times slower.

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