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Slowing time by stretching the waves in special relativity

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Abstract

The time dilation of special relativity is considered as a pure relativistic phenomenon not deduced from wavelength distortion. Alternatively, it is suggested here that time distortion always originates from wavelength distortion, including in special relativity. A link between the special relativistic time and the quantum of time is first established to propose that wavelengths are adjustable uncertainty parameters allowing to maintain the invariance of the light speed and the number of periods. The light clock of Einstein is then used to develop a new Doppler analysis in the frame of an external observer and transformed to cancel the virtual effect between comoving points. This approach yields a conjectural Doppler formula with remarkable properties. Contrary to the previous Doppler equations characterized by their asymmetry, the new formula gives geometrically symmetrical Doppler effects in the whole space in front and behind the closest point from the source, whose center of gravity corresponds precisely to a global inter-frame dilation factor $1/\sqrt{1-(v/c)^2}$. These results extend the direct relation between wavelength and time distortion to special relativity.

1 Introduction

Time dilation, the keystone of special relativity, is conceived as a pure relativistic phenomenon and the time between wave crests is just expected to comply with this time dilation [1]. The inverse view proposed here is that time distortion results from wavelength distortion in all contexts. Special relativity does not escape the fundamental parallel between time and wavelengths which is much more general than special relativity. Using the light clock of Einstein, it will be shown that relativistic time dilation is analogous to a generalized Doppler effect. While doing so, an intriguing theoretical Doppler equation will be obtained by cancelling a virtual Doppler effect viewed from a moving frame.

2 A wave distortion is a time distortion

The most striking evidence that distortions of wavelengths exactly correspond to time distortions has not been provided by a Doppler effect, but by the cosmological redshift. Supernovae are star explosions remaining extremely bright for a few weeks, a short duration at the cosmological scale. This time window is relatively constant for comparable supernovae, but astronomers made a remarkable discovery: the apparent time window of brightness depends on the distance of the supernova, in exactly the same proportion that their redshift. For instance, a distant supernova with a redshift of $\lambda^{\text{app}}/\lambda = 1.5$, has precisely a 1.5-fold longer duration of brightness [2]. Hence, apparent time dilation corresponds to wavelength increase, or equivalently frequency decrease, whereas apparent time contraction corresponds to wavelength shortening and frequency increase. The decrease or increase of wavelengths is evaluated by comparison with their standard values measured in the comoving frame, using for example atomic rays identical in all inertial frames when viewed by comoving inertial observers. Strangely, although the Doppler effect is clearly a phenomenon of wavelength distortion, the scientific community did not establish a clear relationship between time dilation and a Doppler effect, perhaps because time dilation depends only on relative speeds whereas a Doppler effect is orientation-dependent. Certain authors stated that there is no necessary relation at all between the relativity theory and the Doppler effect [3]. But the opposite view is suggested here using a pseudo-Doppler analysis of the universal light clock of Einstein, that is not based on an atomic ray but on the constancy of light velocity.

3 The light clock of Einstein

The beat of the light clock of Einstein is the rebound of a photon between facing mirrors. This clock is placed vertically in a wagon rolling at constant speed v . For an external observer, the light path appears oblique when the train moves, whereas for an observer located inside the train, it appears always vertical (Fig.1). The reference time interval Δt corresponds to the frame comoving

with the light clock.

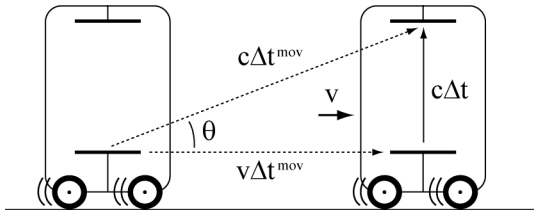


Figure 1. The famous triangular time-space diagram of the moving Einstein's clock. The horizontal scale is artificially stretched relatively to the vertical one for better visualization.

With respect to the angle θ of Fig.1, elementary trigonometry says

$$\sin \theta = \frac{c\Delta t}{c\Delta t^{\text{mov}}} = \frac{\Delta t}{\Delta t^{\text{mov}}} \quad (1a)$$

and

$$\cos \theta = \frac{v\Delta t}{c\Delta t^{\text{mov}}} = \frac{v}{c} \quad (1b)$$

which immediately gives, using the relationship $\sin^2 \theta + \cos^2 \theta = 1$,

$$\frac{\Delta t^{\text{mov}}}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1c)$$

Another straightforward way to obtain this result is to use the powerful geometrical tool of special relativity: the Minkowski spacetime. The oblique (hypotenuse) and vertical paths of light are necessarily identical in the Minkowski spacetime because light starts from and arrives to the same points. This common spacetime interval should reconcile the point of view of an observer in the train, for whom the clock appears immobile, and that of an outside observer for whom there is an additional translation. Hence,

$$\Delta s^2 = (c\Delta t)^2 = (c\Delta t^{\text{mov}})^2 - \Delta x_{\text{mov}}^2 \quad (2a)$$

giving

$$c^2 \left(\frac{\Delta t}{\Delta t^{\text{mov}}} \right)^2 = c^2 - \left(\frac{\Delta x^{\text{mov}}}{\Delta t^{\text{mov}}} \right)^2 \quad (2b)$$

and finally

$$\frac{\Delta t^{\text{mov}}}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2c)$$

In spacetime diagrams like the triangular scheme of Fig.1, the proper time of the clock is obtained when the distance to be crossed by light appears minimal. In Fig.1, it corresponds to the vertical path (Δt), whereas for the external observer moving at speed v relatively to the clock this path appears stretched by $\Delta t^{\text{mov}}/\Delta t = 1/\sqrt{1 - (v/c)^2}$.

4 Linking the relativistic and the quantum times

4.1 The quantum of time

The apparent paradox of special relativity is that the hypotenuse and the vertical side of the triangle shown in Fig.1 are simultaneously crossed by light at the same speed c . There is a very simple solution to explain this constancy, which consists in modifying the fundamental components of c . The speed of light can be written

$$c = \lambda \text{ meters}/T \text{ seconds} \quad (3)$$

where the spatial unit is the wavelength λ and the time unit is the period T . A quantum of time ε can be defined, which depends on the energetic status of the system: It is the time necessary to cross the length unit below which successive configurations cannot be distinguished because of the uncertainty principle. For a thermodynamic system, the length unit is the thermal wavelength of de Broglie λ and the mean particle velocity is the averaged Maxwell velocity, which gives [4]:

$$\varepsilon = \frac{\lambda}{\langle v \rangle} = \frac{h}{4k_B T} = 4 \times 10^{-14} \text{ s at } T = 300 \text{ K} \quad (4)$$

where T is the temperature. The corresponding value for photons ($k_B T = h\nu$), is

$$\varepsilon = \frac{1}{4\nu} = \frac{\lambda}{4c} = \frac{T}{4} \quad (5)$$

where T is the period. A period has internal symmetries and can be subdivided into 4 indivisible motifs ($\pi/2$ windows). Furthermore, the 4 is imposed as a condition for recovering the undulatory behaviour of light from the definition of ε as an "uncertainty window". Indeed, the conception of ε as the minimal time interval during which no evolution can be perceived, means that it can be included in a delay differential equation of the form

$$\frac{dy(t)}{dt} = -ky(t - \varepsilon) \quad (6a)$$

of which an undulatory solution is

$$k = \frac{\pi}{2\varepsilon} \quad (6b)$$

and

$$y(t) = A \sin\left(\frac{\pi}{2\varepsilon}t\right) \quad (6c)$$

In statistical mechanics,

$$\frac{\pi}{2\varepsilon} = \frac{k_B T}{h} \quad (6d)$$

(where T is the temperature) and for photons, if adding to this pulsation a uniform spatial translation along x at speed $\lambda/4\varepsilon$

$$y(x, t) = A \sin\left(\frac{\pi}{2\varepsilon} \left(\frac{4\varepsilon}{\lambda}x - t\right)\right) \quad (6e)$$

which is the traditional light wave function

$$y(x, t) = A \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \quad (6f)$$

(where T is the period). Time and energy can be adjusted in time-energy boxes of uncertainty, provided maintaining invariants. In other words, the velocity of light and the number of time units can be preserved from all uniformly moving viewpoints by compressing or dilating the waves.

4.2 Stretching the waves

The Einstein clock gives

$$\Delta t^{\text{mov}} > \Delta t \quad (7a)$$

Since the two paths of light in Fig.1 are the same, the number of periods, say n , is preserved so that Eq.(7a) is equivalent to

$$nT^{\text{mov}} > nT \quad (7b)$$

Hence

$$\nu^{\text{mov}} < \nu \quad (7c)$$

a phenomenon of wave stretching compensates space-time dilation in such a way that (i) the speed of light and (ii) the number of periods, are both maintained identical (Fig.2).

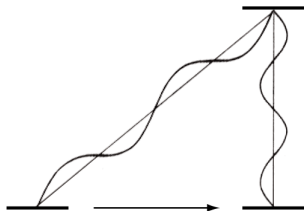


Figure 2. Wavelength dilation allows to maintain the invariance of the light speed and of the number of time units for the immobile and the moving observers.

According to this scheme, relative times simply emerge from relative frequencies. The principle of uncertainty $\Delta E \Delta t \geq h/4\pi$, where $\Delta E = h\Delta\nu$, can be expressed as $\Delta t \Delta\nu \geq 1/4\pi$, showing that time and frequencies are mutually constrained. It is interesting to note that the Doppler effect obtained here by stretching a fictitious space, somewhat resembles the cosmological redshift [5]. In this respect, special relativity appears as a phenomenon of virtual space expansion.

4.3 Time dilation of special relativity is a perspective effect

In the experiment of the light clock inside a train described above, the distinction between moving and non-moving frames is irrelevant as they can be permuted.

The wagon and the station platform are two equivalent systems of reference and the situation is simply inverted if the clock is put on the platform and if the observer inside the train considers that it is the platform that moves relative to the train in the opposite direction. A perspective effect is naturally reciprocal.

The best known perspective effect is the apparent contraction of the size of a person standing far away from us compared to a person standing close to us. We are very accustomed to this familiar effect and easily understand that this person should have exactly the inverse perception. Distance is a symmetrical notion, as is uniform motion. The apparent size reduction effect is so well integrated in our mind that it is unconsciously corrected. Moreover, it is used inversely to estimate the distance. The same operation can be adapted to time dilation to deduce the speed of the source from the degree of wavelength stretching. Eq.(2a) can be modified as follows: On the one hand, $c = \lambda/T$ and on the other hand, the time intervals can be replaced by a given number (n) of periods $\Delta t = n\lambda/c$. The invariance of this number from any viewpoint allows to rewrite Eq.(2a) as

$$\frac{\Delta x^{\text{mov}}}{\Delta t^{\text{mov}}} = v = c \sqrt{\frac{\lambda^{\text{mov}} - \lambda}{\lambda^{\text{mov}}}} \quad (8a)$$

which can be transformed into

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8b)$$

Eq.(8b) is typically the equation of a Doppler effect. Accordingly, it is shown below that the time dilation of special relativity can be derived from a virtual Doppler effect.

5 Analogy between time dilation and a Doppler effect

In spite of its apparent simplicity, this original section is completely heterodox because the relativistic Doppler formula is assumed to be not a matter of compression or elongation of wavelengths [1, 3]. Conversely, it is suggested here that the phenomenon of time dilation of special relativity is simply based on a wavelength distortion which can be recovered using the classical Doppler formula. A Doppler effect is caused by the velocity of a wave source relatively to an observer. The classical doppler effect shortens the apparent wavelength λ^{mov} of an object approaching at speed v such that $\lambda^{\text{mov}} = cT - vT$ where T is the period, giving $\lambda^{\text{mov}}/\lambda = 1 - \frac{v}{c}$. Conversely it stretches the apparent wavelength of a receding object such that $\lambda^{\text{mov}}/\lambda = 1 + \frac{v}{c}$. In the general case, when the velocity vector is not strictly collinear with the line of sight, these equations should be modified by replacing

v by a smaller value. When using radial velocities which are the orthogonal projection of the velocity vector on the source-observer line ($v \cos \theta$), there is no transverse effect because when the source is at the closest point from the observer, $\theta = \pi/2$ and the radial speed is zero. But let us develop a new Doppler approach using the classical Doppler formula in which radial velocities defined with angles, are replaced by time-dependent Doppler-generating speeds. Since the light speed is finite and invariant, this new approach is completely reciprocal between the sources and the observer, contrary to the angles which are subject to aberration effects. The resulting conjectural Doppler formula will prove surprisingly elegant.

5.1 A Doppler effect derived from the light clock of Einstein

Classical Doppler effects mix longitudinal and transverse effects and range between the two asymptotes $1 - \frac{v}{c}$ and $1 + \frac{v}{c}$. The intermediate values are currently defined using the angle θ between the motion line and the receiver. In fact, since θ varies with time and generates certain problems such as aberration effects, it seems more

rational to skip it and to calculate the Doppler effect directly as a function of time. A Doppler-generating speed h can be defined using the Pythagorean theorem. When a moving source at distance H_0 from the observer reaches the closest point from this observer, the triangle shown in Fig.3 evolves such that the hypotenuse reduces from H_0 to D while the source path reduces from L_0 to 0, with speeds h and v respectively, related to each other with a couple of simple equations.

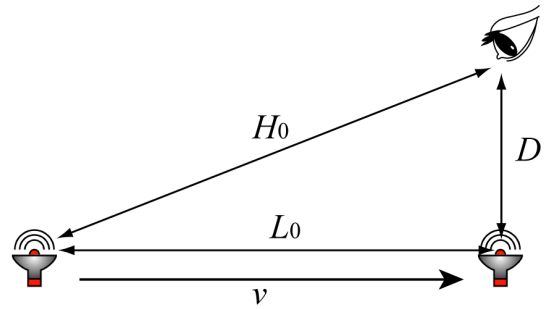


Figure 3. A source moving at constant speed v starts from a distance H_0 from the immobile observer. The shortest distance between the source and the observer is D .

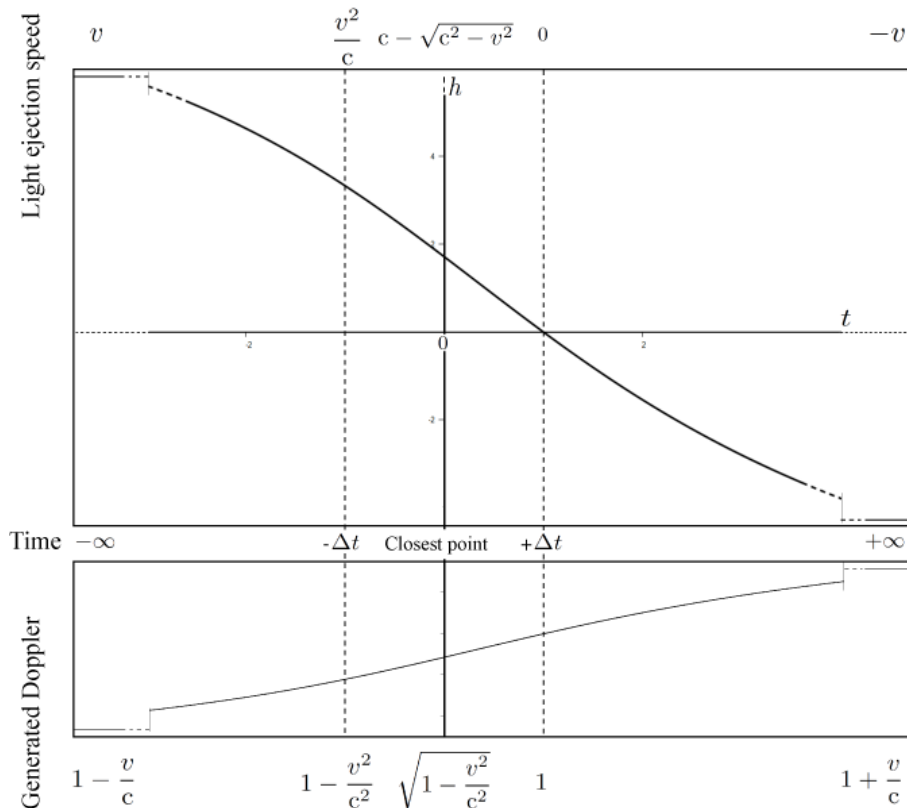


Figure 4. Evolution of the Doppler-generating speed calculated using Eq.(10). The time unit Δt is the travel time of the signal reaching the receiver when the source is the closest to it. The origin of time $t = 0$ is centered at this closest position.

$$H_0^2 = D^2 + L_0^2 \quad (9a)$$

and

$$(H_0 - ht)^2 = D^2 + (L_0 - vt)^2 \quad (9b)$$

whose subtraction allows to eliminate D and yields

$$ht = H_0 - \sqrt{H_0^2 + (vt)^2 - 2L_0vt} \quad (9c)$$

Eq.(9c) also holds in the particular case where H_0 and L_0 are precisely adjusted such that the wave front reaches the receiver when the source reaches the closest point, following a time delay Δt . In this case, H_0 and L_0 can be replaced by $c\Delta t$ and $v\Delta t$ respectively and Eq.(9c) becomes

$$h = \left(c\Delta t - \sqrt{(c\Delta t)^2 + (vt)^2 - 2v^2t\Delta t} \right) / t \quad (10)$$

When inserted in the classical Doppler formula, the speeds calculated with Eq.(10) give the results presented in Fig.4. The signal received when the observer is at right angle to the motion line was ejected towards the observer at rate $h = v^2/c$. This value is calculated as the limit of a series expansion of Eq.(10) that is not defined at this point. The resulting Doppler effect is $\lambda^{\text{mov}}/\lambda = 1 - \frac{v^2}{c^2}$. Of course, this effect is artificially generated and should now be cancelled to impose the expected absence of Doppler effect between the comoving mirrors of the light clock of Einstein (Fig.1). This cancellation yields a conjectural Doppler equation with very interesting properties compared to the genuine relativistic Doppler equation.

6 Comparison of the different Doppler approaches

6.1 Normalization of the different Doppler formulas with respect to time

To compare the different Doppler formulas, the conjectural and existing equations should be comparable for any relative configuration of the source and the observer. The comparison with the relativistic equation [6] is delicate because several equations are possible depending on the angle used: either the original angle between the velocity vector and the source-observer connection line (θ) or the reception angle (θ').

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{1 + \frac{v}{c} \cos \theta}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c} \cos \theta'}, \quad (11a)$$

the two angles of this identity are related to each other through the so-called aberration formula [6]

$$\cos \theta = \frac{\cos \theta' - \frac{v}{c}}{1 - \frac{v}{c} \cos \theta'} \quad (11b)$$

This situation is somewhat confusing because if one assumes that the transverse effect is obtained when the cosine is 0, the first formula of Eq.(11a) predicts a wave dilation, whereas the second formula gives the inverse wave contraction. The former solution is the right one in special relativity [7]. In fact, θ and θ' cannot be simultaneously equal to $\pi/2$. This subtlety is a matter of delay of wave travel Δt (Table.1).

Table 1: The aberration effect in special relativity is related to the time points of wave emission.

t	$\cos \theta$	$\cos \theta'$	$\lambda^{\text{mov}}/\lambda$
$-\Delta t$	$-\frac{v}{c}$	0	$\sqrt{1 - \frac{v^2}{c^2}}$
0	0	$\frac{v}{c}$	$1/\sqrt{1 - \frac{v^2}{c^2}}$

The relativistic Doppler formula contains two variables: the speed v and an angle. This angle varies along the wave path and can be expressed as a function of time, such that $\theta(t) = \tan^{-1}(D/vt)$. Hence, on the one hand $\cos \theta(t) = 1/\sqrt{1 + (D/vt)^2}$, and on the other hand the distance D can itself be defined as a function of Δt ($D = \Delta t\sqrt{c^2 - v^2}$), thereby allowing to make the relativistic formula a function of time only. There are ambiguities in the literature about the sign of the velocity ($-v$ and $+v$) in Doppler equations. To eliminate this source of confusion, all the equations can be composed to make this speed always positive, irrespective of the relative location of the observer, by transferring the sign to the time t ranging from $-\infty$ and $+\infty$. To "synchronize" the formulas at the time points of wave emission, in the new formula, t should be replaced by $t + \Delta t$. Finally, a dimensionless normalized time holding for all source paths relatively to a receiver is defined as $\bar{t} = t/\Delta t$. A little algebra satisfying all these requirements gives the equations compiled in Table.2.

Table 2: Doppler effects generated by a wave emitted at the normalized time \bar{t} and calculated using the different formulas. The virtual and conjectural formulas are built using the classical Doppler framework $1 - v/c$, in which v is replaced by the Doppler-generating speed h calculated previously. $t = 0$ is the time point at which the distance between the source and the observer is minimal and Δt is defined as the time-of-flight of the wave reaching the observer when located at the closest point from the source.

Treatment	Doppler effect	$\bar{t} = -\infty$	$\bar{t} = -1$	$\bar{t} = 0$	$\bar{t} = +1$	$\bar{t} = +\infty$
Classical	$\frac{\lambda^{\text{mov}}}{\lambda} = 1 + \frac{v^2 \bar{t}}{c^2 \Theta}$	$1 - \frac{v}{c}$	$1 - \frac{v^2}{c^2}$	1	$1 + \frac{v^2}{c^2}$	$1 + \frac{v}{c}$
Relativistic	$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\text{Classical}}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$	$\sqrt{1 - \frac{v^2}{c^2}}$	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\frac{1 + \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$
Virtual	$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\Theta + \bar{t}}{1 + \bar{t}}$	$1 - \frac{v}{c}$	$1 - \frac{v^2}{c^2}$	$\sqrt{1 - \frac{v^2}{c^2}}$	1	$1 + \frac{v}{c}$
Conjectural (λ)	$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\text{Virtual}}{1 - \frac{v^2}{c^2}}$	$\frac{1}{1 + \frac{v}{c}}$	1	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\frac{1}{1 - \frac{v^2}{c^2}}$	$\frac{1}{1 - \frac{v}{c}}$
Conjectural (ν)	$\frac{\nu^{\text{mov}}}{\nu} = \frac{\lambda}{\lambda^{\text{mov}}}$	$1 + \frac{v}{c}$	1	$\sqrt{1 - \frac{v^2}{c^2}}$	$1 - \frac{v^2}{c^2}$	$1 - \frac{v}{c}$

with $\bar{t} = \frac{t}{\Delta t}$ and $\Theta = \sqrt{1 - \frac{v^2}{c^2}}(1 - \bar{t}^2)$

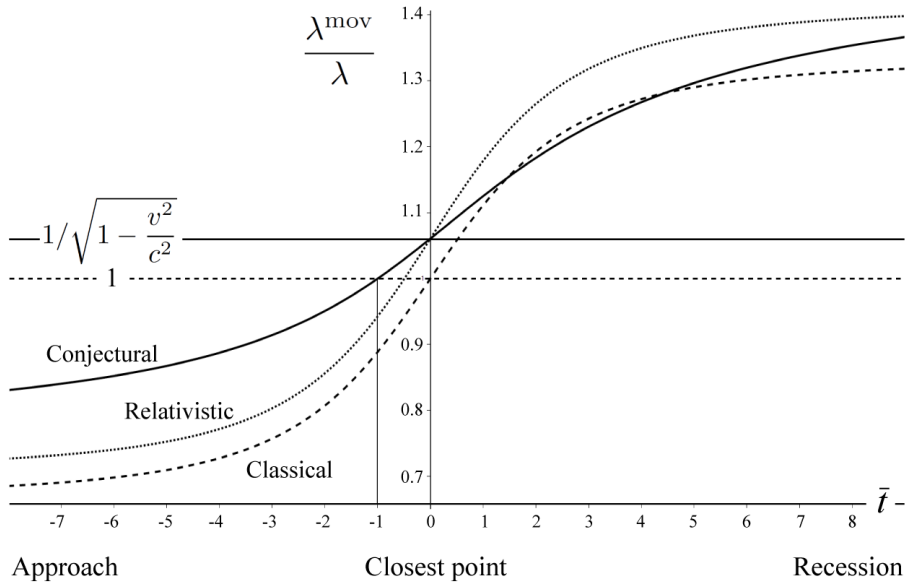


Figure 5. Comparative profiles of Doppler effects predicted for $v/c = 0.3$, by the relativistic equation (dotted line), the classical equation (dashed line) and the conjectural Doppler equation calculated here (plain line). The time point 0 corresponds to wave emission. The switch between the contraction and the dilation occurs at the closest point for the classical formula and before the closest point for the relativistic formula (between -1 and 0, see the text) and for the conjectural function, precisely at $\bar{t} = -1$.

For the time point $\bar{t} = -1$ where the conjectural function is not defined, the Doppler effect takes the limit value $1 - \frac{v^2}{c^2}$. These different Doppler equations describe general combinations of longitudinal and transverse Doppler effects for any relative position of the source and the observer. As these normalized equations can now be compared, their profiles are superposed for visualization in Fig.5, for the same arbitrary values of c and v and using \bar{t} units.

6.2 Comparative symmetry of the different Doppler formulas

The mean values of the Doppler effects generated at time points symmetrically located on both sides of the closest point ($t = 0$) depend on the modes of averaging, which are, when expressed using wavelengths,

- the arithmetic mean: $\frac{1}{2} \left(\frac{\lambda_{(-t)}^{\text{mov}}}{\lambda} + \frac{\lambda_{(+t)}^{\text{mov}}}{\lambda} \right)$
- and the geometric mean: $\sqrt{\frac{\lambda_{(-t)}^{\text{mov}}}{\lambda} \frac{\lambda_{(+t)}^{\text{mov}}}{\lambda}}$, respectively.

The appropriate tool to evaluate the symmetry on both sides of the closest point is logically the geometric mean, because it holds both for periods and frequencies such that $\langle T_{(-t)}, T_{(+t)} \rangle = 1 / \langle \nu_{(-t)}, \nu_{(+t)} \rangle$ and satisfies the rule of color reflectance fusion. The Doppler equations based on radial velocities, classical and relativistic, are characterized by their absence of symmetry whereas the conjectural formula displays a perfect geometric symmetry, in such a way that the geometric mean of the Doppler effects before and after the midpoint are independent of time and always $1/\sqrt{1 - \frac{v^2}{c^2}}$ (Table 3).

Table 3: Arithmetic and geometric means of Doppler effects expressed using either wavelengths or frequencies. \bar{t} and Θ are defined in Table.2.

Mean	λ vs ν	Classical	Relativistic	Virtual	Conjectural
Arithmetic	λ	1	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\frac{\Theta - \bar{t}^2}{1 - \bar{t}^2}$	$\left(\frac{\Theta - \bar{t}^2}{1 - \bar{t}^2} \right) / \left(1 - \frac{v^2}{c^2} \right)$
	ν	$\frac{\Theta^2}{\left(1 - \frac{v^2}{c^2} \right) \left(1 + \frac{v^2}{c^2} \bar{t}^2 \right)}$	$\frac{\Theta^2}{\sqrt{1 - \frac{v^2}{c^2}} \left(1 + \frac{v^2}{c^2} \bar{t}^2 \right)}$	$\frac{\Theta - \bar{t}^2}{\Theta^2 - \bar{t}^2}$	$\left(\frac{\Theta - \bar{t}^2}{\Theta^2 - \bar{t}^2} \right) / \left(1 - \frac{v^2}{c^2} \right)$
Geometric	λ	$\frac{1}{\Theta} \sqrt{\left(1 - \frac{v^2}{c^2} \right) \left(1 + \frac{v^2}{c^2} \bar{t}^2 \right)}$	$\frac{1}{\Theta} \sqrt{1 + \frac{v^2}{c^2} \bar{t}^2}$	$\sqrt{1 - \frac{v^2}{c^2}}$	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
	ν	$\frac{\Theta}{\sqrt{\left(1 - \frac{v^2}{c^2} \right) \left(1 + \frac{v^2}{c^2} \bar{t}^2 \right)}}$	$\frac{\Theta}{\sqrt{1 + \frac{v^2}{c^2} \bar{t}^2}}$	$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\sqrt{1 - \frac{v^2}{c^2}}$

A frame is not a unique object but is made of an infinite number of mutually synchronized clocks and of comoving points in infinite space, which can all be considered as sources. As a consequence, the global Doppler effect perceived by a single point moving relatively to this

frame is the geometric mean of all these sources which distribute equally in front of and behind the moving point. With the conjectural Doppler formula, the mean interframe Doppler effect is $\langle \lambda^{\text{mov}} / \lambda \rangle = 1 / \sqrt{1 - \frac{v^2}{c^2}}$.

This result suggests a correspondence between the time dilation effect and a generalized interframe Doppler effect described by the conjectural Doppler equation.

7 Can the conjectural formula be regarded as a putative Doppler equation?

The conjectural Doppler equation built here appears remarkably elegant in many aspects. (i) As shown above, it is completely symmetric and predicts a global interframe effect equal to the Lorentz factor. (ii) By construction, its inversion point between the blue and red shifts is simply obtained at $\bar{t} = -1$, which seems to be the only expectable value based on the light clock device of Fig.1. For comparison, it is of interest to determine the inversion points of the different formulas.

7.1 Switching points

The three different Doppler equations compared in Fig.4, have different switching points between contracted and dilated waves:

- for the classical formula, at $\bar{t} = t = 0$,
- for the conjectural formula at $\bar{t} = -1$ or $t = -\Delta t$,
- and finally, the most complicated result is obtained for the traditional relativistic formula, for which the switching point is close to the transverse line, at

$$\bar{t} = -\frac{c}{v\sqrt{2}} \sqrt{\sqrt{1 - \frac{v^2}{c^2}} \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)} \quad (12a)$$

or

$$t = -\frac{D}{v\sqrt{2}} \sqrt{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1} \quad (12b)$$

where D is the minimal distance between the source and the observer.

7.2 Coincidence between the conjectural Doppler effect at speed v and distance increase at speed v

Imagine that a receiver recedes from a source (considered immobile) at speed v . The source emits a light beam propagating at speed c towards the receiver at time t_E , when they are spaced by D_E . Light reaches the receiver at time t_R after crossing a distance D_R . The duration of the light travel is

$$t_R - t_E = D_R/c \quad (13a)$$

The new spacing between the source and the receiver has become

$$D_R = D_E + v(t_R - t_E) \quad (13b)$$

Replacing the duration in Eq.(13b) by its value given by Eq.(13a), yields

$$\frac{D_R}{D_E} = \frac{1}{1 - \frac{v}{c}} \quad (13c)$$

This classical treatment gives the conjectural Doppler value $\lambda^{\text{mov}}/\lambda$ (Table.2). In case of collinear approach, the same reasoning gives

$$\frac{D_R}{D_E} = \frac{1}{1 + \frac{v}{c}} \quad (13d)$$

7.3 Comparative angle-dependence of the relativistic and conjectural Doppler equations

The time-dependent profiles of the different formulas have already been compared in Fig.5. It is also interesting to compare their angular behaviour. To this end, the angle θ of Fig.6 will be used, ranging from 0 to π while keeping the speed v always positive, such that

$$\cot(\pi - \theta) = \frac{t}{\Delta t} \frac{\frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The relativistic equation becomes

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\sqrt{1 + \cot^2 \theta} - \frac{v}{c} \cot \theta}{\sqrt{1 - \frac{v^2}{c^2}} \sqrt{1 + \cot^2 \theta}} \quad (14a)$$

multiplying by $\sin \theta$ gives a result simpler but no longer modulo- π

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{1 - \frac{v}{c} \cos \theta}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (14b)$$

and the conjectural formula is

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\frac{v}{c} \sqrt{1 + \cot^2 \theta} - \cot \theta}{\sqrt{1 - \frac{v^2}{c^2}} \left(\frac{v}{c} - \sqrt{1 - \frac{v^2}{c^2}} \cot \theta \right)} \quad (15a)$$

multiplying by $\sin \theta$ as previously,

$$\frac{\lambda^{\text{mov}}}{\lambda} = \frac{\frac{v}{c} - \cos \theta}{\sqrt{1 - \frac{v^2}{c^2}} \left(\frac{v}{c} \sin \theta - \sqrt{1 - \frac{v^2}{c^2}} \cos \theta \right)} \quad (15b)$$

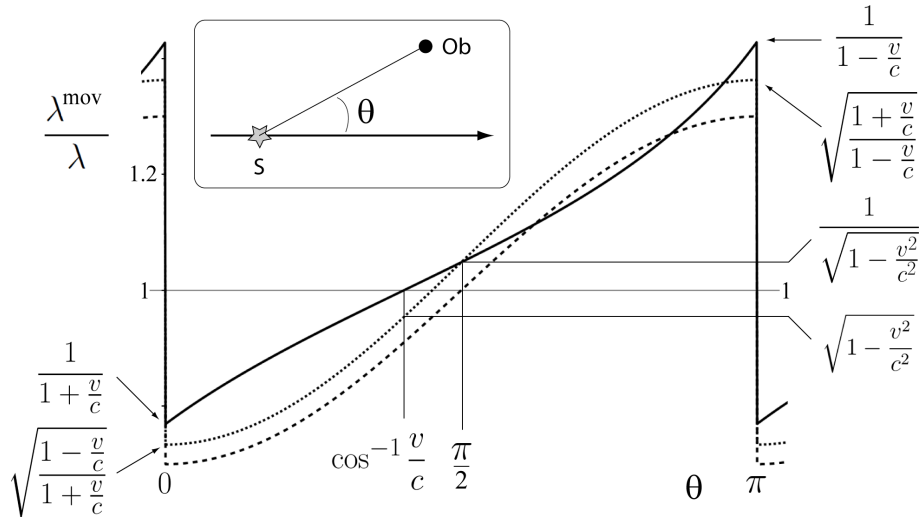


Figure 6. Comparative profiles of Doppler effects predicted for $v/c = 0.3$, by the relativistic equation (dotted line), the classical equation (dashed line), and the conjectural Doppler equation (plain line), as functions of the emission angle θ shown in the inset.

The profiles corresponding to Eq.(14a) and Eq.(15a) are shown in Fig.6. These curves mainly differ through their opposite relative slopes. A lower variation of Doppler effects around $\pi/2$ is specifically obtained for the conjectural formula.

7.4 Test of the relativistic Doppler equation

The conjectural Doppler equation naturally emerged from a Doppler analysis aimed at verifying if the time dilation of special relativity corresponds to a phenomenon of wave stretching. This approach yields a seemingly plausible Doppler equation, which was not intended to supplant the traditional relativistic formula already tested experimentally. Many experiments have indeed been conducted to test the theory of special relativity through verifying the validity of the relativistic Doppler formula of Einstein [8], through:

- *Transverse effects.* Following a suggestion of Einstein himself [9], the existence of the transverse Doppler effect has been experimentally observed in [10]. This test does not allow distinguishing between the equation of Einstein and the conjectural equation found here, which predict the same transverse effect.
- *Longitudinal effects.* The celebrated experiment of Ives and Stilwell [11] and its descendants [12] focused on the longitudinal Doppler effect. Ives and Stilwell recovered the time dilation factor by measuring the arithmetic means of the shifted wavelengths in front and behind moving atoms [11]. This result is not obtained with the conjectural

formula. Note that Ives and Stilwell did not use (i) frequencies although they would have been easier to study and (ii) the geometric mean. If their device is not strictly collinear (to allow visualizing simultaneously an atom and its reflected image), the time and angle-dependent values calculated here would be expected.

8 Conclusion

Contrary to the relativistic Doppler equation which did not result from a Doppler analysis [6], a Doppler approach is presented which suggests that the interframe time dilation is a matter of wave stretching, in agreement with the parallel between time and redshift observed for supernovae [2]. The global interframe time dilation factor $1/\sqrt{1 - v^2/c^2}$ corresponds exactly to the geometric average of this perfectly symmetric conjectural Doppler equation.

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