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► **To cite this version:**

Evans Gouno, Lise Guérineau. FAILURE RATE ESTIMATION IN A DYNAMIC ENVIRONMENT. FLINT INTERNATIONAL STATISTICAL CONFERENCE, Jun 2014, Flint, Michigan, United States. hal-01093721

HAL Id: hal-01093721

<https://hal.science/hal-01093721>

Submitted on 11 Dec 2014

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FAILURE RATE ESTIMATION IN DYNAMIC ENVIRONMENT

EVANS GOUNO AND LISE GUÉRINEAU

ABSTRACT. We present a method to assess the reliability of a material operating in a dynamic environment. The dynamic environment is represented as a sequence of shocks governed by a self-exciting point process. Different distribution models for the time-to-failure of the material are suggested. The case of a piecewise exponential model where a Cox's model is integrated to take into account the effect of the stress is investigated. Maximum likelihood estimates of the model parameters are studied.

Keywords: Reliability, failure rate, self-exciting point process, piecewise exponential distribution, Cox's model, maximum likelihood,

1. INTRODUCTION

Many models have been suggested to describe the life distribution of a device subjected to a sequence of shocks occurring randomly. Esary and al. [1] consider a Poisson process. Singpurwalla [2] reviews the stochastic-process-based approach to failure modeling. Gaudoin et al. [3] present a theoretical study of different classes of stochastic models to depict the influence of stress on lifetime. In the situation considered here, we assume that shocks undergone by the device result in damage (not necessarily a failure) so that its failure rate increases. We also assume that the device keeps memory of the previous shocks. We consider that the shocks process is a stochastic self-exciting point process (SEPP) also called *Hawkes' process* [4, 5]. The jumps of this SEPP define the steps of a piecewise exponential model [6] which is assumed to be the distribution of the time-to-failure. Thus the failure rate is supposed to be constant in each step and related to the environment according to a Cox's model [7, 8, 9]. We address the problem of fitting a SEPP and estimating the Cox's model parameters. Some tests of hypotheses procedures are suggested to assess the shocks effect.

2. THE SHOCKS PROCESS

We consider that the shocks process is a self-exciting point process (SEPP). Self-exciting point processes are counting processes with an intensity depending on all or part of the history of the process itself. The intensity of SEPP is not only a function of time but it is also a function of the number of jumps previously occurred on the process. In other words, a given number of more recent events will influence the evolution of the process; the process is said to be *self-excited processes*. This type of processes can be especially suitable to model some dynamic environments; earthquakes, thunderstorm for example.

Let $\{N(t), t \geq 0\}$ be the shocks process, its intensity is considered to be of the form :

$$(1) \quad \mu(t) = \mu_0 + \int_0^t g(t-s)dN(s) \text{ with } g(t) = \eta e^{-\gamma t}, 0 < \eta < \gamma.$$

If the sequence t_1, \dots, t_n is available, one can easily check the goodness-of-fit to a SEPP using the residual method ([10], p.258). Maximum likelihood estimate of η and γ can be obtained with the method suggested by Ozaki [11]. Some other response functions $g(t)$ can be considered. In particular, one can consider a link between the covariate characterising the environment and the shocks process. The SEPP can also be used to define the time-to-failure distribution.

3. THE TIME-TO-FAILURE DISTRIBUTION

One can consider that after a shock, the instantaneous probability of failure is high and then decreases. A Weibull distribution can be assumed with a shape parameter depending on the history of the shocks process. The shape parameter can be assumed to be $\mu(t)$, the intensity of the SEPP and the level of the shock can be integrated in the function g . In the present work, we consider a sample of time-to-failure (X_1, \dots, X_n) . We assume that the X_i 's are independent and follow non-identical piecewise exponential distributions with change-points equal to the dates of the shock.

Thus, the failure rate for X_i , the time-to-failure of the device i is of the form:

$$(2) \quad \lambda_i(x) = \sum_{j=0}^{+\infty} \lambda_{i,j} \mathbb{1}_{[t_{i,j}; t_{i,j+1}[}(x).$$

where $t_{i,j}$ is the date of the j -th shock undergone by the device i , $t_{i,0} = 0$ and $\lambda_{i,j}$ is the failure rate after the j -th shock and before the $(j+1)$ -th shock.

Successive shocks increase the instantaneous probability of failure by a multiplicative factor depending on the covariate characteristics associated with the shocks. We assume a Cox's model such that the failure rate after the j -th shock is constant equal to:

$$(3) \quad \lambda_{i,j} = \exp\{\beta' z_{i,j}\} \lambda_{i,j-1}$$

where $z_{i,j} \in \mathbb{R}^q$ is a covariate associated with the j -th shock sustained by device i and $\lambda_{i,0} = \lambda_0$, the baseline failure rate.

By induction, we have:

$$(4) \quad \lambda_{i,j} = \exp\{\beta' Z_{i,j}\} \lambda_0 \text{ where } Z_{i,j} = \sum_{\ell=1}^j z_{i,\ell}.$$

β and λ_0 are the parameters to be estimated on the basis of the observation of a sequence $(t_{i,j}, z_{i,j})$ representing the dates and the levels of shocks experienced by the device i , associated with X_i , the time-to-failure of the device.

4. LIKELIHOOD

Let us denote $\theta_{i,\ell}(\beta) = \exp\{\beta' Z_{i,\ell}\}$ and $\Delta_{i,\ell+1} = t_{i,\ell+1} - t_{i,\ell}$, $\ell = 0, \dots, k_i$ with k_i the total number of shocks occurring on device i .

From (2), for $x_i \in [t_{i,j}; t_{i,j+1}[$, the reliability for the device i is:

$$(5) \quad R_j(x_i) = \exp\{-\lambda_0 A_{i,j}(x_i, \beta)\},$$

where $A_{i,j}(x_i, \boldsymbol{\beta}) = \sum_{\ell=0}^{j-1} \theta_{i,\ell}(\boldsymbol{\beta}) \Delta_{i,\ell+1} + \theta_{i,j}(\boldsymbol{\beta})(x_i - t_{i,j})$.

The probability density function of X_i is thus:

$$(6) \quad f_j(x_i) = \lambda_0 \theta_{i,j}(\boldsymbol{\beta}) \exp \{-\lambda_0 A_{i,j}(x_i, \boldsymbol{\beta})\}, \text{ for } x_i \in [t_{i,j}; t_{i,j+1}[.$$

Let $\delta_{i,j} = 1$ if the event (failure or censoring) for device i occurs in $[t_{i,j}; t_{i,j+1}[$, 0 else. Let $d_i = 1$ if the device i fails, 0 else. Then the expression of the likelihood is:

$$\begin{aligned} L(\lambda_0, \boldsymbol{\beta}) &= \prod_{i=1}^n \prod_{j=1}^{k_i} [f_j(x_i)^{d_i} R_j(x_i)^{1-d_i}]^{\delta_{i,j}} \\ &= \prod_{i=1}^n \prod_{j=1}^{k_i} [\lambda_0 \theta_{i,j}(\boldsymbol{\beta})]^{\delta_{i,j} d_i} \exp \{-\delta_{i,j} \lambda_0 A_{i,j}(x_i, \boldsymbol{\beta})\} \end{aligned}$$

The log-likelihood is:

$$(7) \quad \log L(\lambda_0, \boldsymbol{\beta}) = k \log \lambda_0 + \sum_{i,j} \delta_{i,j} d_i \boldsymbol{\beta}' Z_{i,j} - \lambda_0 \sum_{i,j} \delta_{i,j} A_{i,j}(x_i, \boldsymbol{\beta})$$

where $\sum_{i,j}$ stands for $\sum_{i=1}^n \sum_{j=1}^{k_i}$ and $k = \sum_{i,j} \delta_{i,j} d_i$.

The likelihood equations are:

$$(8) \quad \frac{\partial}{\partial \lambda_0} \log L(\lambda_0, \boldsymbol{\beta}) = \frac{k}{\lambda_0} - \sum_{i,j} \delta_{i,j} A_{i,j}(x_i, \boldsymbol{\beta}) = 0.$$

For $u = 1, \dots, q$,

$$(9) \quad \frac{\partial}{\partial \beta_u} \log L(\lambda_0, \boldsymbol{\beta}) = \sum_{i,j} \delta_{i,j} d_i Z_{i,j,u} - \lambda_0 \sum_{i,j} \delta_{i,j} \frac{\partial}{\partial \beta_u} A_{i,j}(x_i, \boldsymbol{\beta}) = 0.$$

Existence and unicity of the maximum likelihood estimate (m.l.e.) for λ_0 and $\boldsymbol{\beta}$ can be shown applying the theorem 2.2 from Mäkeläinen et al. [12]. One has to prove that the Hessian matrix of the log-likelihood is negative definite. The proof requires then the computation of the second partial derivatives of (7). The Hessian matrix is negative definite if and only if odd upper-left minors are negative and even upper-left minors are positive. In our case, the first upper-left minor is obviously negative. Conditions on vectors \mathbf{Z} can be elicited to satisfy the conditions on the other minors. An expression of λ_0 depending on $\boldsymbol{\beta}$ can be obtained from (8). Replacing λ_0 with this expression in (9), the m.l.e. for $\boldsymbol{\beta}$ is computed using a numerical method.

5. TESTS OF HYPOTHESES

It is possible to test whether the shocks have an effect on the reliability device by considering the null hypothesis $H_0 : \boldsymbol{\beta} = \mathbf{0}$. A first possibility is to use the score statistic. Let $\mathbf{I}^{*-1}(\lambda_0, \boldsymbol{\beta})$ denotes the inverse of minus the matrix of the second partial derivatives of (7). The score statistic to test $\boldsymbol{\beta} = \mathbf{0}$ is: $\mathbf{U}(\tilde{\lambda}_0, \mathbf{0})' \mathbf{I}_q^*(\tilde{\lambda}_0, \mathbf{0})^{-1} \mathbf{U}(\tilde{\lambda}_0, \mathbf{0})$ where $\mathbf{I}_q^*(\cdot, \cdot)$ is the suitable submatrix of $\mathbf{I}^{*-1}(\cdot, \cdot)$, $\tilde{\lambda}_0$ is the m.l.e. of λ_0 under H_0 , and $\mathbf{U}(\mathbf{0}) = \left. \frac{\partial}{\partial \boldsymbol{\beta}} \log L(\tilde{\lambda}_0, \boldsymbol{\beta}) \right|_{(\tilde{\lambda}_0, \mathbf{0})}$ is the vector of the first partial derivatives of (7). The distribution of this statistic converges to a $\chi^2(q)$

distribution. The test can also be based on the asymptotic normal approximation: $(\hat{\lambda}_0, \hat{\beta})' \sim \mathcal{N}((\lambda_0, \beta)', [\mathbf{I}^*(\hat{\lambda}_0, \hat{\beta})]^{-1})$. Thus under $H_0 : \beta = 0$, $(\hat{\lambda}_0, \hat{\beta})' \mathbf{I}^*(\hat{\lambda}_0, \hat{\beta})(\hat{\lambda}_0, \hat{\beta})$ is approximately $\chi^2(q)$.

A third approach is to consider the likelihood ratio statistic that is

$$\Lambda = -2 \log \left(\frac{L(\hat{\lambda}_0, \mathbf{0})}{L(\hat{\lambda}_0, \hat{\beta})} \right).$$

Again this statistics is asymptotically $\chi^2(q)$ under H_0 . Remark that other tests can be considered to evaluate which shocks characteristics have an influence; the test is conducted on a portion of β .

6. CONCLUDING REMARKS

We have described a methodology to assess the failure rate of equipment under a dynamic environment. We have considered a piecewise exponential model with random steps combined with a Cox's model. An application of this approach has been conducted to study the reliability of equipments exposed to thunderstorm. The shocks are lightning strikes. It is possible to model the occurrence of lightning strikes in a given area with a self-exciting point process. Thus the technique described here can be used as a simulation tool to obtain chronicles of strikes and to assess the parameters of the time-to-failure distribution leading to a better understanding of the electrical surge effect on materials and allowing to elaborate strategy to manage a fleet of equipments subjected to such constraint.

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