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# Mechanical fluctuations suppress the threshold of soft-glassy solids : the secular drift scenario

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We propose a dynamical mechanism leading to the fluidization of soft-glassy amorphous material driven below the yield-stress by external mechanical fluctuations. The model is based on the combination of memory effect and non-linearity, leading to an accumulation of tiny effects over a long-term. We test this scenario on a granular packing driven mechanically below the Coulomb threshold. We bring evidences for an effective viscous response directly related to small stress modulations in agreement with the theoretical prediction of a generic secular drift.

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Numerous amorphous materials such as concentrated suspensions, colloidal glasses, foams or granular materials share common global features in their mechanical response to shear. They are characterized by a yield stress below which the material appears as a solid [1]. However, they usually display ageing properties manifesting in slow creep relaxation processes [2, 3]. Even though these materials are seen as almost athermal, ageing properties stem from a remaining thermal activation providing the possibility to cross enthalpic or entropic barriers and progressively go into deeper local minima where mechanical solidity is reinforced. In this sense, the behaviour of these amorphous solids is very close phenomenologically to molecular glass-formers obtained by thermal quenching. Moreover, when driven mechanically to the yield stress limit, these material display plastic relaxation processes in the form of local plastic events coupled elastically to form complex spatio-temporal avalanching patterns. When the fluid state is attained, the rheology is non-Newtonian usually of Herschel-Bulkey type. The direct visualization of local plastic events and the associated complex dynamics is supported by many experimental [4–7] or numerical [4, 8–10] studies.

As this behaviour is shared by so many different materials, a common conceptual and theoretical framework called “soft glassy rheology” (SGR) [11–14] was put forwards to provide a quantitative basis for the phenomenology, above and beyond the yield stress. It is based on a microscopic descriptions of the localized plastic processes and their dynamical coupling. Even though many other parallel approaches exist, sometimes at different level of description, they all share the underlying idea of local processes triggered by thermal or by mechanical activation. Some models, such as SGR, consider that the local events generate a mechanical noise leading to the gener-

alized concept of “effective temperature” [11–13].

The fact that mechanical perturbations can contribute to an effective temperature is debated [15] and an important problem is to access how small should be the amplitude perturbations to modify the ageing properties of the system [25]. Indeed, a lot of processes show that vanishingly small perturbations cannot be neglected in the dynamics of soft glassy materials. Since such effects may be cumulated over long times, it becomes problematic when a solid response is expected but uncontrolled mechanical noise would eventually lead to a significant fluidization. For example the effect of mechanical noise on soils is of major importance for the long-term stability of structure foundations [16]. It may play a determinant role in the context of earthquakes triggering [17]. Controlled mechanical fluctuations can also be used as an investigation tool, as for example in superposition rheology [18]. In this instance, understanding the system response to various forcing of different forms and amplitudes, is crucial for an accurate exploitation of the system dynamics.

In this letter, we propose a new conceptual picture to understand the fluidization processes that soft glassy materials may undergo in the solid phase, under mechanical noise. First, theoretical arguments are presented to describe the solid phase where ageing and shear activation processes are both present. We show that small stress modulations around a mechanical reference state lead to the dynamical destruction of the yield stress and to the emergence of a viscous response. An explicit derivation is presented on a simple model and thereafter the result’s generality is discussed. Second, we present experiments on granular packing sheared below the Coulomb threshold and we show that the response to small mechanical modulations is in agreement with the generic predictions of the model.

–*Model ingredients.* Models aiming at describing yield-stress fluids and amorphous materials in the solid phase, need to account for two fundamental features in their dy-

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namics [11, 14]: (i) ageing of the system with time and (ii) rejuvenation due to shear rate  $\dot{\gamma}$ . That rejuvenation can be seen microscopically as a local structural reorganizations induced by the strain. Such "reactivation" of the system has to depend on the strain rate amplitude only: when an amorphous material is submitted to a cyclic load, after a complete cycle the system is not back to its initial state [14]. Mathematically, it means that the variable describing the activity of the system has an even dependence on  $\dot{\gamma}$ . This non-linearity is the first key ingredient of the model. The other one is linked to memory effects. Because of the different time scales at play in the dynamics (typical time of a reorganization process compared to the ageing time), the description of glassy materials depends on the observation time scale. Choosing this scale can be problematic in creep experiments as the system exhibits no intrinsic time scale. In presence of noise or fluctuations, the macroscopic variables measured are averaged quantities giving the mean long-term behaviour. If the system is submitted to stress vibrations of typical amplitude  $\delta$ , very small compared to the yield stress  $\sigma_D$ , and displaying a characteristic time  $\tau_{vib}$ , a pertinent observation time is given by the number of perturbations of amplitude  $\delta$  necessary to cumulate an equivalent stress of order  $\sigma_D$ :  $T_{obs} = \frac{\sigma_D}{\delta} \tau_{vib}$ . Because of the positive non-linear dependence of the rejuvenation term, we expect that those perturbations will indeed dynamically stack, giving rise after a time of order  $T_{obs}$  to an effect equivalent to a stress of order  $\sigma_D$ .

In order to demonstrate simply this mechanism, we build on the local rheological model proposed by Derec *et al.* [14]. This generic model introduces a phenomenological variable, the *fluidity* describing the stress rate of relaxation, in the spirit of Maxwell visco-elasticity. Note importantly, that fluidity can take different definitions and interpretations whether the material is in the liquid phase or in the solid phase [19–21]. Here, fluidity is an inverse time scale characterizing the material visco-elastic response as it ages and rejuvenates. Interestingly, for granular matter and shear stresses not too close to  $\sigma_D$ , this variable was identified experimentally, as the rate of occurrence of local rearrangements called "hot-spots" [6]. For larger stresses, one has to account for avalanching processes leading to partial failure and material structural reorganisation. To provide a comprehensive analytical understanding of how a steady fluidity can appear below  $\sigma_D$ , we study the response of this model to small stress perturbations.

–*Fluidization as a secular drift.* Derec *et al.* [14] have proposed a generic visco-elastic scalar equations for the description of the soft-glassy materials rheology that we use here in its simplest non trivial form:

$$\dot{\sigma} = G\dot{\gamma} - f\sigma \quad (1)$$

$$\dot{f} = -af^2 + r\dot{\gamma}^2, \quad (2)$$

with  $\sigma$  the applied shear stress,  $\dot{\gamma}$  the shear rate and  $G$

the shear elastic modulus. Fluidity,  $f(t)$ , is an inverse characteristic stress relaxation time. Dimensionless parameters  $a$  and  $r$  represent respectively ageing and shear-induced rejuvenation processes. This generic model was used to describe the logarithmic creep process occurring in a granular packing when submitted to a constant shear stress below the dynamical yield stress  $\sigma_D$  [3]. Those experiments have shown that the parameters  $a$  and  $r$  depend on stress for intermediate shear stress values. For clarity and simplicity, we consider here  $a$  and  $r$  as constant. The stationary solutions of those equations depend on the value of the stress compared to  $\sigma_D = G\sqrt{\frac{\sigma}{r}}$ . For a constant  $\sigma < \sigma_D$ , the fluidity  $f$ , as well as the shear rate  $\dot{\gamma}$ , decreases to 0 as the inverse of time, thus leading to a logarithmic creep process.  $\sigma_D$  is the dynamical yield threshold : above this value, the material behaves as a fluid with a finite fluidity value. We consider the case of a mean imposed stress  $\sigma_0$  below the threshold  $\sigma_D$  combined with a modulation of small amplitude  $\delta \ll \sigma_0$ , leading to an imposed stress  $\sigma(t) = \sigma_0 + \delta \sin(\omega t)$ . By construction, the fluidity model has no time scale. When imposing a modulation, one can study the response of the system over a time of the order of  $\tau_{vib} = \frac{2\pi}{\omega}$  to study the rheological in-phase and out-of-phase response [14]. Here we aim at understanding the long-term behaviour, given by the number of cycles of amplitude  $\delta$  necessary to be equivalent to a stress of order  $\sigma_D$ :  $T_{obs} = \tau_{vib}/\epsilon$ , with  $\epsilon = \delta/\sigma_D$ . We then adimensionalize the equations using the following scales:  $1/(\epsilon\omega)$  for the time,  $\sigma_D$  for the stress and  $\gamma_0 = \sigma_D/G$  for the deformation. We write the adimensionalized variables with a tilde. Then, the adimensionalized equations with derivatives being in  $\tilde{t}$  are:

$$\dot{\tilde{\sigma}} = \dot{\tilde{\gamma}} - \tilde{f}\tilde{\sigma} \quad (3)$$

$$\dot{\tilde{f}} = -a(\tilde{f}^2 - \dot{\tilde{\gamma}}^2) \quad (4)$$

with  $\tilde{\sigma} = \tilde{\sigma}_0 + \epsilon \sin\left(\frac{\tilde{t}}{\epsilon}\right)$ , Eq. 3 gives

$$\dot{\tilde{\gamma}} = \cos\left(\frac{\tilde{t}}{\epsilon}\right) + \tilde{f}\tilde{\sigma}_0 + \epsilon\tilde{f}\sin\left(\frac{\tilde{t}}{\epsilon}\right) \quad (5)$$

Dynamically, we have a two-times system:  $T = \tilde{t}$ , the time of observation in creep experiments, is a *slow* time. The modulation time is a *fast* time  $\tau = \tilde{t}/\epsilon$ . Note that the modulation frequency can still be low enough to be described in the framework of SGR models. A multiple scale analysis can be done [22], with  $\tilde{f}(\tau, T) = \tilde{f}^{(0)}(\tau, T) + \epsilon\tilde{f}^{(1)}(\tau, T)$  and  $\frac{d}{d\tilde{t}} = \frac{1}{\epsilon}\frac{\partial}{\partial\tau} + \frac{\partial}{\partial T}$ . We then obtain for  $(1 - \tilde{\sigma}_0^2) = O(1)$ :

$$\frac{1}{\epsilon}\frac{\partial\tilde{f}^{(0)}}{\partial\tau} + \frac{\partial\tilde{f}^{(1)}}{\partial\tau} + \frac{\partial\tilde{f}^{(0)}}{\partial T} + O(\epsilon) = -a\left[-\frac{1 - \cos 2\tau}{2} + (1 - \tilde{\sigma}_0^2)\left(\tilde{f}^{(0)}\right)^2 + 2\tilde{f}^{(0)}\tilde{\sigma}_0 \sin \tau + O(\epsilon)\right] \quad (6)$$

From the order  $O(\frac{1}{\epsilon})$ , one deduces that  $\tilde{f}^{(0)}(T)$  is a function of the slow time only. Cancelling the secular

term at the order  $O(1)$  [22], one obtains:

$$\frac{\partial \tilde{f}^{(0)}}{\partial T} = a \left[ \frac{1}{2} - (1 - \tilde{\sigma}_0^2) \left( \tilde{f}^{(0)} \right)^2 \right] \quad (7)$$

This equation corresponds to the normal form of a saddle-node bifurcation  $\dot{x} = \mu - x^2$ . For  $\mu > 0$  the solution  $+\sqrt{\mu}$  is the only stable stationary solution. The dimensional expression of the stationary value for the mean fluidity is thus:

$$f^* = \frac{\omega \frac{\delta}{\sigma_D}}{\sqrt{2 \left( 1 - \frac{\sigma_0^2}{\sigma_D^2} \right)}} \quad (8)$$

which is finite for non-vanishing values of  $\delta$ , the stress modulation, even when  $\sigma_0 \ll \sigma_D$ . Consequently, even far below the yield threshold, the long-term behaviour tends to a liquid-like response, with constant mean strain rate  $\dot{\gamma}^{(0)} = \frac{f^*}{G} \sigma_0$  which corresponds to the emergence of a finite viscosity:

$$\eta = G/f^* = G \frac{\sqrt{2 \left( 1 - \frac{\sigma_0^2}{\sigma_D^2} \right)}}{\omega \frac{\delta}{\sigma_D}} \quad (9)$$

Note that this viscous response is linearly related to the inverse of the stress modulation rate. Therefore, it does not appear as an activated process where mechanical fluctuations are described as an effective temperature as it was suggested previously to extend the SGR conceptual vision [24].

–*Stress modulation experiments.* The theoretical outcomes are now tested experimentally on a granular packing under a confinement pressure that sets a scale for the Coulomb dynamical yield stress. Granular materials are often seen as rigorously athermal. Indeed, in most numerical approaches, granular contacts are modelled as elastic repulsive forces and a Coulomb solid friction threshold. Consequently, for infinitesimal deformations around a reference state, a granular packing has a true elastic response and displays no ageing. However for real granular materials, the actual pressures at contact are generically high and contacts may creep plastically. Therefore, the contact status will be intrinsically coupled to a thermally activated process [23]. Moreover, the contact status can also be extremely sensitive to the ambient mechanical noise. In fact, macroscopically, real granular packing in the solid phase, display generically ageing and shear rejuvenation [3] and this, in association with the apparition of localized plastic events [6]. As a consequence, experiments on granular packing in the solid phase, can be considered as of general relevance to the class of soft-glassy materials that display similar phenomenology [27].

An experimental key point here is to achieve shear stress modulations around a nominal value without introducing uncontrolled mechanical perturbations. Besides

residual external noise, always present, even in quiet environments, a substantial source of mechanical noise comes from motorized elements. This is why we designed the experimental system as an Atwood machine. The setup is shown on Fig. 1(a). It consists in a shear cell (Radius  $R = 5\text{cm}$ , height  $H = 10\text{cm}$ ) filled with glass beads of density  $\rho = 2500\text{ kg/m}^3$  and mean diameter  $d = (200 \pm 30)\ \mu\text{m}$ . A well-defined packing fraction  $\phi = 0.605 \pm 0.005$  is obtained by a procedure described elsewhere [3].

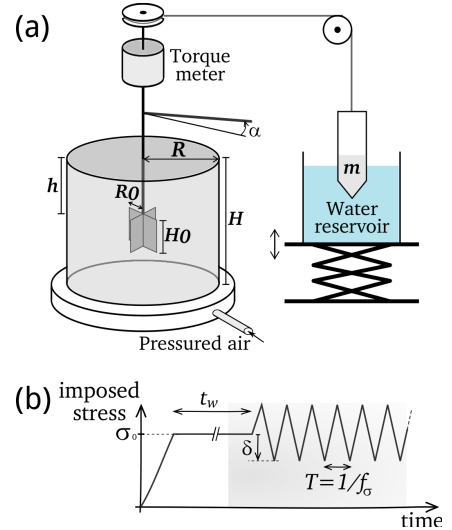


FIG. 1: (a) Experimental set-up. (b) Imposed stress during an experiment: stress ramp to reach mean stress,  $\sigma_0$ , constant stress during  $t_w$  and stress modulation characterized by a frequency  $f_\sigma$  and an amplitude  $\delta$ .

Shear is obtained by applying a torque on a four-blade vane ( $R_0 = 1.27\text{ cm}$ ,  $H_0 = 2.54\text{ cm}$ ) using a mass  $m$  suspended from a pulley (see fig. 1(a), vane penetration  $h = 5\text{ cm}$ ). A torque probe measures the applied torque  $\mathcal{T}$  and the angle of rotation of the vane  $\alpha$  is measured by an induction probe. We defined the mean stress and the mean strain as  $\sigma = \frac{\mathcal{T}}{2\pi R_0^2 H_0}$  and  $\gamma = \frac{\alpha R_0}{R - R_0}$  respectively. In conditions of the present experiment, the Coulomb threshold was determined at a value  $\sigma_0 = 2300\text{ Pa}$ . When a constant stress  $\sigma_0$  smaller than the yield stress is applied on the granular packing, a creep behaviour is observed with a logarithmic dependence of the strain with time (red curve of Fig. 2). This behaviour was studied in [3] and the fluidity model discussed in the previous part describes accurately the observed response.

Modulation of the applied torque is obtained by vertical oscillatory displacements of a water tank in which the mass  $m$  partially hangs. This motion modulates the torque by variation of the Archimede's forces on the mass. The protocol (fig. 1(b)) is then: 1) stress ramp at constant stress rate ( $\dot{\sigma} = 5\text{ Pa/s}$ ) up to the desired mean stress value  $\sigma_0$ ; 2) constant shear  $\sigma_0$  applied during  $t_w = 1500\text{ s}$ ; 3) modulation of the stress around  $\sigma_0$  for at

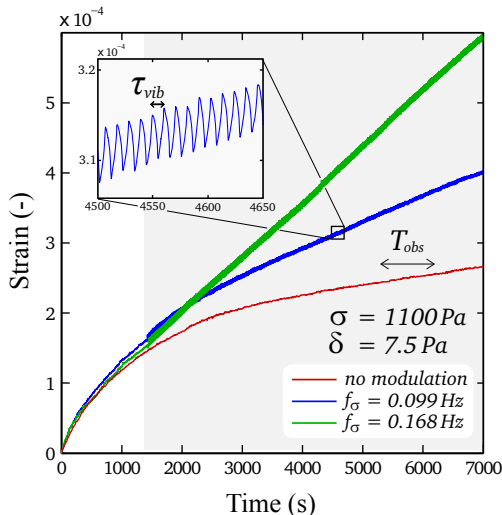


FIG. 2: Strain as a function of time for three experiments performed at  $\sigma_0 = 1100$  Pa, and  $\delta = 7.5$  Pa, and for various oscillations frequencies. The oscillations start at  $t = 1500$ s (grey area).

least 2 hours. The modulations are triangular oscillations of amplitude  $\delta$  and frequency  $f_\sigma$ .

Figure. 2 shows typical deformations for two experiments performed at the same mean stress ( $\sigma_0 = 1100$  Pa) and oscillation amplitude ( $\delta = 7.5$  Pa) but for various oscillation frequencies. During the constant stress phase, a slow increase of the deformation,  $\gamma(t)$ , is observed corresponding to the beginning of the logarithmic creep. Then, when submitted to oscillations, the system will transit to a linear creep regime characterized by a constant mean strain rate,  $\dot{\gamma}_\infty$ , which increases with the oscillation frequency. The slope of this linear creep allows to define an effective viscous response:  $\eta = \sigma_0 / \dot{\gamma}_\infty$ .

Figure 3 shows the values obtained for the mean strain rate  $\dot{\gamma}_\infty$  as a function of the modulation stress rate  $R_\sigma = 2\omega\delta/\pi$ , for a given value of the applied mean stress  $\sigma_0$ . The observed linear dependences are in agreement with the normalization parameters chosen. Indeed, the finite viscosity that we expect to arise from the modulation should vary as  $\eta \propto \frac{1}{\delta\omega}$  (see eq. (9)) leading to a strain rate  $\dot{\gamma}_\infty \propto \omega\delta$ .  $\sigma_0$  corresponds to an applied shear stress far enough from the dynamical threshold. Experimentally, when this limit is approached one observes a strong increase of the strain rate. The results are then much less reproducible and may be quite sensitive to uncontrolled external perturbations.

Those experimental results confirm the following points: when the stress imposed to a granular solid is combined to a small perturbation, the mechanical response switches from a logarithmic creep to a fluid-like behaviour with constant mean shear rate. The effective viscosity of this fluid is then inversely proportional to the modulation stress rate  $R_\sigma$  and decreases when approach-

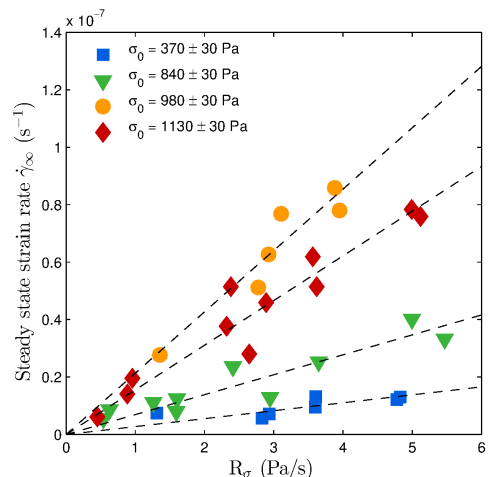


FIG. 3: Steady state strain rate,  $\dot{\gamma}_\infty$ , as a function of modulation stress rate  $R_\sigma$  for four different values of mean stresses,  $\sigma_0$ . The straight lines are linear fits.

ing the dynamical threshold.

–*Summary and discussion.* In this letter, we propose a new fluidization pathway that applies to soft glassy materials in the solid phase. Under external shear stress fluctuations and on a long-term basis, the combination of two generic features, memory effects and non-linear flow-induced rejuvenation, destroys the yield stress and creates a steady Newtonian-like flow. Even though the derivation was explicitly done on a simple phenomenological model, the existence of a secular term bringing out a steady-state fluidity, is a generic feature coming out from any model that would mix ageing and non-linear rejuvenation process [27]. We have only studied here the stationary states which needs a very long time to be reached, especially when the perturbation is small in amplitude and rate. Consequently, a logarithmic-like transient creep can still be observed in similar configurations over long periods of time, which might be the case for granular packing driven acoustically [25]. The underlying mechanism at work in our model is very different from a thermal activation or any equivalent mechanism accounting for stress fluctuations as an effective temperature [26]. Fluidization stems here from a dynamical bifurcation of the rheological equations which is a very general feature of dynamical systems where processes at very different time scales are taking place. The accumulation of tiny effects over long time may lead to large, *secular*, drifts of the long-term response [22]. The induced viscosity is related quantitatively to the inverse of the stress modulation rate and cancels out at the dynamical threshold value. The effect is observed and quantified for a granular packing submitted to controlled stress modulations below the Coulomb threshold. Note that in spite of resemblances, this is quite different from another fluidization process occurring when a granular packing is

placed in contact with a fluidized shear band [28–30]. In the last case, theoretical analysis and numerical simulations show that the induced creeping process comes from a non-local relaxation from the flowing part to the material bulk [31]. Moreover, at steady-state, the granular packing is essentially in a fluid state whereas in our case, the fluidization happens in the solid phase. This new fluidization process may be of practical importance to assess the stability and reliability of structures strained externally in their environment over very long time scales.

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