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1 Disconcerting learning on climate sensitivity and the 2 uncertain future of uncertainty

3 Alexis Hannart · Michael Ghil ·
4 Jean-Louis Dufresne · Philippe Naveau

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7 **Abstract** How will our estimates of climate uncertainty evolve in the coming
8 years, as new learning is acquired and climate research makes further progress?
9 As a tentative contribution to this question, we argue here that the future path
10 of climate uncertainty may itself be quite uncertain, and that our uncertainty is
11 actually prone to increase even though we learn more about the climate system.
12 We term *disconcerting learning* this somewhat counter-intuitive process in which
13 improved knowledge generates higher uncertainty. After recalling some definitions,
14 this concept is connected with the related concept of *negative learning* that was
15 introduced earlier by Oppenheimer et al. [2008]. We illustrate disconcerting learn-
16 ing on several real-life examples and characterize mathematically certain general
17 conditions for its occurrence. We show next that these conditions are met in the
18 current state of our knowledge on climate sensitivity, as shown physically on hand
19 of an energy balance model of climate. Finally, we discuss the implications of these
20 results on developing policy for adaptation and mitigation.

21 **Keywords** Climate change uncertainty · Knowledge evolution · Learning models

22 1 Introduction and motivation

23 Strong scientific consensus prevails over the fact that Earth's climate is cur-
24 rently warming and will be warming further over the coming decades, as a con-
25 sequence of the radiative perturbations caused by anthropogenic greenhouse-gas
26 (GHG) emissions. The conclusions of the IPCC's Fourth Assessment Report (AR4:
27 [Solomon et al. (2007)], [AR4] hereafter) further buttress this consensus. There
28 is, however, substantial uncertainty regarding the extent of future warming, as
29 pointed out in the same report and in many of its references.

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This uncertainty renders decision making on appropriate adaptation and mitigation steps more difficult. In addition, the uncertainty level regarding future climate evolution has not decreased significantly over the past decades. This observation paves the way for climate-warming naysayers; it is sometimes used as an argument to discredit climate science as a whole and to slow down action on this issue. Lively scientific debate continues on the extent and the reasons for the uncertainty. This debate motivates us to revisit here the question of the future evolution of uncertainties.

Uncertainties regarding future climate warming are usually divided into three categories [Hawkins and Sutton (2009)]: (i) those regarding GHG increase scenarios [AR4]; (ii) those arising from the climate system’s internal variability [Ghil et al. (2008)]; and (iii) those inherent to the climate system’s long-term response to a given forcing. Because contribution (i) is part and parcel of humankind’s future course of action and the relative contribution of (ii) may vanish after a few decades, we focus on the third category, which we refer to henceforth simply as *climate uncertainty*. To quantify it, a widely used metric consists in the spread $\sigma_{\Delta T}$ associated with the probability density function (PDF) of *climate sensitivity*; the latter is defined here as the change ΔT in global equilibrium surface temperature T associated with a doubling in atmospheric CO₂ concentration.

This metric stems from the fact that the diversity of plausible long-term future climate states for a given emission scenario is determined, to a large extent, by the range of climate sensitivity ΔT . According to [AR4] — which compiled PDFs of ΔT obtained by various studies over the last few years — ΔT is likely to lie between 2°C and 4.5°C, a range which is still high. It is thus relevant for socio-economic and political decision making to ask how this range will evolve in the future, as climate research makes further progress.

To answer this question, one can find, on the one hand, numerous studies (e.g., [Stainforth et al. (2005), Knutti and Hegerl (2008), Roe and Baker (2007), Hannart et al. (2009)] and references therein) that focus on the reasons for the presently high range of ΔT . These studies identified a number of key research areas — such as cloud processes (e.g., [Soden and Held (2006), Dufresne and Bony (2008)]) or oceanic variability and response [Dijkstra and Ghil (2005), Ghil et al. (2008)] — whose better understanding and modeling may potentially lead to a reduction of the uncertainty in ΔT .

On the other hand, a vast body of literature addresses the question of learning at an epistemological level and that of uncertainty in the general context of scientific research. For instance, the very definitions of learning and scientific progress, as well as the question of the existence of truth, have been debated at length over millennia of philosophical tradition (e.g., [Aristotle (40 B.C.), Bacon (1605), Kuhn (1962)]). The interplay between learning, uncertainty, erroneous judgements and decision making has received increased attention in recent years, especially in the context of environmental policy (e.g., [Crutzen and Oppenheimer (2008), Keller and McInerney (2007), O’Neill et al. (2006)]). There are still but few studies, however, (e.g., [Oppenheimer et al. (2008), Webster et al. (2008)]) that address jointly the question of the uncertainty in ΔT — so often debated in the climatic literature — and the aforementioned, more general literature on learning and progress.

The [Oppenheimer et al. (2008)] paper ([ONW08] hereafter) not only included such a broader perspective, but also made several important points that we briefly

79 recall here. First and foremost, [ONW08] challenged the intuitive, and hence per-
80 vasive view that usually enters into decision making on environmental problems,
81 namely that “scientific research can be equated (...) with truer beliefs about the
82 outcomes of problems (...) thus providing a superior basis for crafting solutions.”
83 In formulating their challenge, these authors introduced the broad concept of *neg-*
84 *ative learning* to describe any situation where “new technical information leads to
85 scientific beliefs that diverge over time from the a posteriori right answer.”

86 [ONW08] illustrated the concept of negative learning on hand of four prominent
87 case histories, thus showing that negative learning did occur in the past. One of
88 these case histories dealt with advances in the understanding of ozone depletion
89 in the 1970’s and 80’s. In the latter case, the negative aspects of the learning
90 touched upon important facets of the problem under study, for reasons that were
91 similar to those involved in global warming, and did affect policy making. Finally,
92 [ONW08] showed that negative learning on climate sensitivity could well occur in
93 the future, for instance if an unknown radiative feedback is not incorporated into
94 climatic models, i.e. if the latter are subject to structural error.

95 The present article pursues the same line of questioning as [ONW08]. While
96 [ONW08] focused on the conditions of occurrence and on the damaging effects of
97 negative learning, they did not examine the detailed dynamics of learning in the
98 “non-negative” case, which they termed *progressive learning*. Progressive learning,
99 though, may still be problematic when it comes to uncertainty. Our main point in
100 the present paper is that, while progressive learning always leads to truer beliefs by
101 definition, it does not systematically imply that these truer beliefs are less uncer-
102 tain. We thus introduce the term *disconcerting learning* to describe this nonethe-
103 less counter-intuitive situation, in which new information leads to scientific beliefs
104 that are closer to the a posteriori right answer, while still being marked by greater
105 uncertainty. Conversely, we use the term *reassuring learning* for the more intuitive
106 situation in which progressive learning does lead to less uncertainty. These four
107 possibilities — of negative vs. positive, and of disconcerting vs. reassuring learning
108 — are illustrated in Fig. 1, and are explained more precisely in the next section.

109 Although the term “disconcerting learning” introduced here is novel, to the
110 best of our knowledge, earlier works in statistics, probability and economics have
111 already pointed out the existence of this situation (e.g., [Burdett (1996)] or
112 [Zidek and van Eeden (2003), Bagnoli and Bergstrom (2005), Chen et al. (2010)]
113 or [Chen (2011)]). These theoretical studies have also established a few rigorous
114 results concerning the conditions of occurrence of such a situation, but only under
115 some very restrictive conditions.

116 Hence, a general theory of disconcerting learning is lacking for the time being
117 and [Chen et al. (2010)] have even described such a theory as elusive. In any case,
118 research on this type of learning is still in its early days and more work is needed
119 to improve its understanding. The importance of uncertainty regarding climate
120 sensitivity motivates us to do so, and the relevance of this motivation will be
121 made clear in Section 4. Thus, our main contribution here is to further illustrate
122 and analyze why and how disconcerting learning occurs and to demonstrate that
123 it is prone to occur in learning about ΔT in the future.

124 The paper is organized as follows. In Section 2, we recall the definitions of
125 [ONW08] and introduce our own definitions and notation. In Section 3, discon-
126 certing learning is illustrated based on two real-life, biomedical problems that are
127 more insightful in our view than the climatological situation eventually at stake

128 here. Then, we introduce a simple Bayesian model of progressive learning and
 129 we use it to study the general properties of and conditions for the occurrence of
 130 disconcerting learning. In Section 4, we return to the physics and sensitivity of
 131 climate and we show that disconcerting learning may occur in studying ΔT , i.e.
 132 that climate uncertainty may persist or increase even though scientific research
 133 yields progressive results. We emphasize this finding in the idealized context of
 134 a linear energy balance model of climate and illustrate it more concretely with
 135 a real example. Section 5 comments on some policy implications of the present
 136 results, while Section 6 discusses some further aspects of our work and states our
 137 conclusions.

138 2 Definitions and notations

139 As in [ONW08], an *outcome* is any quantity, process or structure of interest, and
 140 we denote it by x . The state of knowledge on x , for a given observer at a given
 141 moment, consists in the set of informations relating to x that are available to
 142 the observer at that moment. We denote this set by \mathcal{I} and represent the state of
 143 knowledge on x in probabilistic terms by using the pdf $p(x | \mathcal{I})$ of x conditional
 144 on \mathcal{I} . Further considerations on the relevance of this probabilistic description of
 145 a state of knowledge, as well as on the underlying interpretation of probabilities,
 146 can be found in Supplemental Material A.

147 Learning on x is thus defined here by a change in the pdf of x subsequent
 148 to its update by some new information. Such changes may occur as a result of
 149 developments in theory, modeling, observations or experiments. We denote by \mathcal{I}_0 ,
 150 \mathcal{I} and $\mathcal{I}_1 = \mathcal{I}_0 \cup \mathcal{I}$, respectively, the a priori information, the new information
 151 learnt, and the a posteriori information.

152 With this notation, learning can be formalized in the Bayesian framework as
 153 follows:

$$154 \quad p_1(x) = \frac{p_0(x)\mathcal{L}(x | \mathcal{I})}{\int p_0(x)\mathcal{L}(x | \mathcal{I}) dx}. \quad (1)$$

155 In Eq. (1), the prior distribution $p_0(x) = p(x | \mathcal{I}_0)$ represents the initial state of
 156 knowledge on x and is multiplied by a likelihood function $\mathcal{L}(x | \mathcal{I})$ that summarizes
 157 the new information. This product yields, after normalization, the a posteriori
 158 distribution $p_1(x) = p(x | \mathcal{I}_1)$.

159 In this probabilistic definition, the level of uncertainty on x that is associated
 160 with a given state of knowledge \mathcal{I} is easily quantified by using $p(x | \mathcal{I})$. We do
 161 so using the standard deviation $\sigma_{\mathcal{I}}$ of this pdf as a metric. Other metrics for
 162 uncertainty are possible, i.e. Shannon entropy, but this choice is not critical for
 163 the present discussion; see Supplemental Material B.

164 In this Bayesian setting, the definition of disconcerting learning given in Section
 165 1 becomes simply

$$166 \quad \sigma_1 > \sigma_0, \quad (2)$$

167 i.e. the uncertainty level on the outcome increases even though more information on
 168 x was gained. Conversely, reassuring learning corresponds to a learning situation
 169 in which $\sigma_1 \leq \sigma_0$, while the definition of negative learning given by [ONW08]
 170 becomes

$$171 \quad p_1(x^*) < p_0(x^*), \quad (3)$$

172 whereas $p_1(x^*) \geq p_0(x^*)$ for progressive learning; here x^* denotes the true value of
 173 x . Figure 1 describes the four possibilities associated with the pair of inequalities
 174 between prior and posterior variance and between prior and posterior bias.

175 Since the key idea associated with negative learning is to describe a situation
 176 in which “scientific beliefs diverge over time from the a posteriori right answer,”
 177 one could consider an alternative, but very closely related definition of negative
 178 learning as an increase in bias — i.e. $|\mu_1 - x^*| > |\mu_0 - x^*|$; such a definition
 179 would more closely parallel the definition of disconcerting learning as an increase
 180 in uncertainty, where μ is the distribution mean. The purpose of this article is
 181 primarily to study the situation of disconcerting learning, $\sigma_1 > \sigma_0$, in the context
 182 of progressive learning, which we choose for simplicity to be defined as $p_1(x^*) \geq$
 183 $p_0(x^*)$.

184 3 Conditions of occurrence of disconcerting learning

185 3.1 Two illustrations of disconcerting learning

186 With the quantitative definitions formulated in Section 2 in hand, we now proceed
 187 to exhibit two typical situations of disconcerting learning that occur in the medical
 188 context, before turning to our main climatic applications in Section 4.

189 *Medical screening test.* Suppose one is interested in whether or not an individual
 190 is affected by a disease. The outcome x here is a binary variable with $x = 1$ if the
 191 individual is affected by the disease and $x = 0$ if not. We assume that a medical
 192 screening test is available for the detection of the disease. The result of the test z
 193 can also be treated as a binary variable with $z = 1$ if the test is positive and $z = 0$
 194 if it is negative.

195 Our initial state of knowledge consists simply in the mean frequency of occur-
 196 rence q_0 of the disease in the population. We thus have $p_0(x) = q_0^x(1 - q_0)^{1-x}$,
 197 and the a priori standard deviation is given by $\sigma_0 = \sqrt{q_0(1 - q_0)}$. Then, the med-
 198 ical screening test is conducted on the individual, and we assume that it gives a
 199 positive result, $z = 1$, thus suggesting illness. However, the test is known to be
 200 imperfect: it has a false positive frequency q and a false negative frequency q' .

201 The new probability q_1 and standard deviation σ_1 , after learning the test result,
 202 are equal to

$$203 \quad q_1 = \{1 + (1 - q_0)/q_0\beta^2\}^{-1}, \quad \sigma_1 = \sigma_0\{\beta q_0 + \beta^{-1}(1 - q_0)\}^{-1}, \quad (4)$$

204 where $\beta = \sqrt{(1 - q')/q}$. It thus follows immediately that, whenever the prior
 205 probability q_0 is smaller than $(1 + \beta)^{-1}$, learning the positive result of the test leads
 206 to an increase of the uncertainty level, i.e. to disconcerting learning. Furthermore,
 207 the increase is largest for $q_0 = q = q'$.

208 In the present context, q_0 is typically small but nonzero — i.e., illness is a priori
 209 possible, but remains the exception and health the rule — and so are q and q' , since
 210 medical tests are reasonably trustworthy, although not completely so. We are thus
 211 often in a situation in which the condition $q_0 < (1 + \beta)^{-1}$ could be met, and where
 212 $q_0 \simeq q \simeq q'$ is also perfectly plausible. For instance, [Humphrey et al. (2002)]
 213 gives $q_0 = 0.06$, $q = 0.13$ and $q' = 0.02$. For these values, learning a positive result
 214 almost doubles the standard deviation from 0.23 to 0.45, i.e. it is an instance of
 215 strongly disconcerting learning.

216 It should be emphasized that disconcerting learning in such a situation is possible, but not unavoidable. For instance, learning a negative test result ($z = 0$) will
 217 result in a sharp decrease in the standard deviation, from 0.23 to 0.16, i.e. reassuring learning. Learning the positive test result ($z = 1$) could also be reassuring
 218 in two situations.
 219
 220

221 First, if the test were much more reliable, the resulting probability of illness
 222 q_1 would be closer to one and the standard deviation closed to zero. In the above
 223 example, this would require $q < 0.004$; for $q = 0.001$, the standard deviation σ_1
 224 would then decrease from 0.23 to 0.12. Second, if the prior probability of illness q_0
 225 was close to 0.5, the prior uncertainty level would nearly equal its maximal value
 226 and its posterior value would thus necessarily decrease. In the above example, this
 227 would require $q_0 > 0.26$ for the given test reliability.

228 To summarize the insights gained from this example, one can state the following:
 229 For a binary outcome with contrasted a priori probabilities, $0 < q \ll 0.5 \ll$
 230 $1 - q < 1$, new information that favors the unexpected modality tends to be disconcerting,
 231 as long as the new information is not conclusive. One can thus speculate
 232 that in general, disconcerting learning occurs when surprising but inconclusive
 233 evidence is found.

234 *Disease incidence rate.* We focus next on a slightly different, but connected,
 235 real-life situation. We are interested this time in the frequency of occurrence x of
 236 the disease in a given population. In this case, learning is obtained by observing
 237 whether a new individual is ill or not.

238 Let us suppose that this new individual is found to be affected by the disease,
 239 i.e. $z = 1$. Consider, for definiteness, that at the time z is observed, $n = 15$
 240 individuals were already observed and that $k = 3$ of them are ill and $n - k = 12$
 241 are healthy. Finally, suppose that prior to this initial observation of n individuals,
 242 x was assumed to be uniform on $[0, 1]$. In this situation, we find from Eq. (1)
 243 that $p_0(x)$ is the beta distribution $\mathcal{B}(k, n - k)$ and that the posterior $p_1(x)$, after
 244 learning $z = 1$, is $\mathcal{B}(k + 1, n - k)$. (Supplemental Material C). Hence:

$$245 \quad \sigma_1 = \sigma_0 \{(1 - (k + 1)^{-1})(1 + n^{-1})(1 + 2n^{-1})\}^{-\frac{1}{2}}. \quad (5)$$

246 Equation (5) yields $\sigma_1/\sigma_0 = 1.05$ and we find ourselves in a situation of disconcerting
 247 learning as well.

248 As in the previous example, disconcerting learning happens here because the
 249 new information is simultaneously surprising — i.e., the observation of a new case
 250 of disease was rather unexpected, due to the fact that most previous observations
 251 were of healthy people — and yet inconclusive — i.e., one extra case of disease
 252 is insufficient to properly estimate the frequency of disease occurrence over the
 253 population. Conversely, had the observation been unsurprising, i.e. had z been
 254 equal to its expected modality of zero, $z = 0$ for $n = 16$, the spread would have
 255 decreased. Likewise, had the observation been surprising but conclusive, i.e. had
 256 we observed a very large number (say 500) of cases of disease instead of one single
 257 case, the spread would also have decreased.

258 3.2 Disconcerting learning and shape of the prior distribution

259 In this subsection, we now focus on scalar, continuous outcomes x , and we address
 260 the following two questions: are there characteristics inherent to the prior pdf $p_0(x)$

261 that increase the chances for disconcerting learning to occur; and if so, which? We
 262 address these questions based on a review of results available in recent literature
 263 and on hand of a detailed simulation study designed for this purpose.

264 The simulation study relies on a Bayesian learning model applied to a variety
 265 of prior distributions that combine several shape features, namely: skewness, from
 266 fully symmetric to pronounced asymmetry; kurtosis, from leptokurtic to platykur-
 267 tic; tail size, from bounded to heavy-tailed; and multimodality, from one to two
 268 modes. Distributions combining these features were generated based on the Pear-
 269 son family. The literature review as well as the simulation study are described and
 270 illustrated in detail in Supplemental Material D and E, respectively.

271 This exploration yielded three main findings. First, there are essentially two
 272 characteristics that enhance the likelihood of disconcerting learning to occur,
 273 namely that $p_0(x)$ (i) is highly skewed, and (ii) that it possesses heavy tails,
 274 cf. Figs. 2 and SM1; when combining these two characteristics, the incidence of
 275 disconcerting learning tends to increase substantially. This result sheds light on
 276 the illustrative examples given in Section 3.1: in each instance, when disconcerting
 277 learning occurred, the prior distribution did indeed have significant skewness. For
 278 instance, the skewness of the Bernoulli prior $p_0(x) = q_0^x (1-q_0)^{1-x}$ was equal to 3.7
 279 for $q_0 = 0.06$, and that of the Beta distribution $\mathcal{B}(3, 12)$ was equal to 1.5; for com-
 280 parison purposes, the skewness of the highly asymmetric exponential distribution
 281 is equal to 2.

282 Second, disconcerting learning in this model is always associated with a large
 283 swing in the value of the mean; see Figs. 2 and SM2. This result further supports
 284 the validity of the speculation in Section 3.1, according to which disconcerting
 285 learning occurs whenever surprising evidence is found, as shown by large shifts in
 286 the expected value of the outcome. Note that the two findings summarized so far
 287 are perfectly consistent with, and shed light on, each other. Indeed, skewed and
 288 heavy-tailed distributions share a property that symmetric, light-tailed distribu-
 289 tions do not have: They assign high probabilities to the occurrence of values that
 290 are remote from the “central core” of the distribution — i.e., unexpected values —
 291 which are precisely those that give rise to large swings and disconcerting learning.

292 Third, disconcerting learning is systematically associated with a large disper-
 293 sion of the trajectories of the uncertainty (Figure SM2b). This finding can be
 294 understood qualitatively by considering the fact that no distribution can generate
 295 surprises in a systematic manner — otherwise they would not be surprises. In
 296 other words, a distribution that is compatible with the occurrence of surprises—
 297 i.e., that is skewed or heavy tailed or both — still generates unsurprising evidence
 298 most of the time. Accordingly, a distribution that is compatible with the occur-
 299 rence of disconcerting trajectories still generates reassuring trajectories most of
 300 the time, resulting in a widespread range of trajectories.

301 4 Disconcerting learning and climate sensitivity

302 4.1 Implications from recent PDFs of climate sensitivity

303 At present, most PDFs obtained for climate sensitivity ΔT are skewed and heavy-
 304 tailed (Fig. 3). There is ongoing debate and discussion on the reasons for the
 305 redundancy of skewness in these PDFs (see for instance [Allen et al.(2006)] or

[Zaliapin and Ghil (2010), Roe and Baker(2011)]) but these interesting discussions are beyond our scope, which is merely to analyze the implications of these factual features for disconcerting learning. In this purpose and as a starting point, we first applied directly the general learning model used in Section 3.2 to several PDFs of climate sensitivity sampled from recent studies (references in Supplemental Material). Doing so, we obtained a set of future trajectories for the uncertainty in climate sensitivity (Fig. 3). Unsurprisingly, we find that (i) disconcerting learning on climate sensitivity is prone to occur in the future; that (ii) it is most severe when the prior distribution is highly skewed; and that (iii) the future trajectory of the uncertainty is itself quite uncertain.

The interpretation of these findings is quite straightforward by using the insights gained from Sections 3.1 and 3.2: because of skewness, high values of climate sensitivity are unlikely but cannot be discarded altogether thus our present state of knowledge allows surprises to occur as we learn more. More specifically, skewness here implies that the shape of the PDF is flatter for high values than it is for medium values, i.e. high values tend to be more evenly distributed than medium values. This means that our knowledge is more imprecise in the upper range than it is in the medium range. Therefore, if a new piece of information shifts our beliefs with respect to climate sensitivity upwards, this will take us into a domain of values about which we know less. The new information will thereby raise more questions and doubts than it will bring answers and certainties. Uncertainty will thus increase and the learning will be disconcerting.

4.2 An illustration of disconcerting learning on climate sensitivity

We now discuss and illustrate more concretely how learning on climate sensitivity may occur. For this purpose, we adapt our general Bayesian learning framework to the case of climate sensitivity, by following an approach similar to that of [Kelly and Kolstad (1999), Leach (2007), Webster et al. (2008)] in which the new knowledge on climate sensitivity ΔT is obtained from a new temperature observation T . The latter is interpreted in terms of climate sensitivity by means of a climate model that establishes a probabilistic connection between ΔT and T , summarized by $p(T | \Delta T)$. Then, the Bayesian update Equation (1) yields:

$$p_1(\Delta T) = \frac{p_0(\Delta T) p(T | \Delta T)}{\int p_0(\Delta T) p(T | \Delta T) dT}. \quad (6)$$

Equation (6) describes a learning process on ΔT which combines two types of information: (i) a climate observation T indirectly linked to ΔT ; and (ii) a climate model representing the available physical knowledge with respect to the indirect link between T and ΔT . In the following, we focus on the observational learning (i) only. Note, though, that an improvement of our theoretical understanding of climate physics subsequently affecting the model (ii) can also be accounted for using this framework. But in any case, even though the learning process is assumed to be purely observational here, the climate model is at the core of this process, because it entirely defines the distribution $p(T | \Delta T)$ of the new observation conditional on climate sensitivity used in Equation (6).

Any choice of climate model — deterministic, stochastic, from low to high complexity — is in theory fit within this framework (Supplemental Material F).

350 For the present, learning-theoretic purposes, we chose a stochastic version of a
 351 zero-dimensional energy balance model, in discrete time:

$$352 \quad \kappa \frac{\delta \bar{T}_t}{\delta t} = -\frac{\Delta R_0}{\Delta T} \bar{T}_t + R_t, \quad T_t = \bar{T}_t + \varepsilon_t. \quad (7)$$

353 where R_t is the radiative forcing at t ; and κ , ΔR_0 and σ are climate parameters
 354 that are assumed to be known, i.e. ΔT is assumed to be the only uncertain pa-
 355 rameter here. The model and its assumptions are described in detail and solved
 356 in Supplemental Material G; it yields a closed form expression of $p(T_t | \Delta T)$, the
 357 distribution of T_t seen from time $t-1$ conditional on ΔT . For observed trajectories
 358 of the forcing R_t and of the temperature response T_t , we can thus use $p(T_t | \Delta T)$
 359 and Eq. (6) to perform iterative updates and obtain the successive PDFs $p_t(\Delta T)$
 360 of ΔT at each instant t , as well as the corresponding successive values of the
 361 standard deviations σ_t — i.e. the trajectories of the uncertainty in ΔT .

362 We applied the latter procedure by using an initial prior distribution $p_0(\Delta T)$
 363 that synthesizes the [AR4] inferences on PDFs of climate sensitivity, i.e. a mean
 364 equal to 3.2°C , a likely range of 2°C – 4.5°C , and a positive skewness that we
 365 assume equal to 0.7. We stopped this updating process at present time t , and we
 366 studied the sensitivity of $p_{t+1}(\Delta T)$ to T_{t+1} , the new temperature observation for
 367 year 2013, which is assumed to be still unknown at time t . We found that, for
 368 the simulated value T_t , whenever $T_{t+1} - T_t \leq 0.24^\circ\text{C}$ — i.e., for a δT in Eq. (7)
 369 that corresponds to either cooling or to a moderate warming between t and $t+1$
 370 — we will always have $\sigma_{t+1} \leq \sigma_t$ and, if so, the new observation corresponds to
 371 reassuring learning. Conversely, when $T_{t+1} - T_t > 0.24^\circ\text{C}$, i.e. for a more intense
 372 warming between t and $t+1$, then $\sigma_{t+1} > \sigma_t$ and the new observation corresponds
 373 to disconcerting learning; see Fig. 4.

374 These findings match our previous results and conclusions. Indeed, according
 375 to the information available at t , a moderate warming between t and $t+1$ is to
 376 be expected. If a moderate warming materializes, this will be in line with the
 377 expected value of climate sensitivity and will confirm this value; thus learning
 378 in this case will be reassuring. Conversely, if an intense warming materializes,
 379 this will be unexpected and come as a surprise. This surprising observation will
 380 tend to indicate that climate sensitivity is higher than expected. The indication,
 381 though, will be inconclusive: first, because the unexpected observation could still be
 382 explained to a large extent by short-term fluctuations caused by internal variability,
 383 rather than characterizing the climate system’s long-term response; and second,
 384 because the a priori PDF $p_0(\Delta T)$ is skewed towards high values. The role of the
 385 latter skewness is particularly important here. To further emphasize it, we applied
 386 the same update procedure of Eq. (7) on the same simulated values of temperature,
 387 but initializing this time with a Gaussian, symmetric prior distribution. We find
 388 that under such an a priori, learning is always reassuring no matter the value of
 389 the new observation T_{t+1} ; this is the case even for unexpected, intense warming
 390 between t and $t+1$ (not shown).

391 We end this subsection with an example of an actual recent observation that,
 392 in line with our above illustration, may arguably be considered as disconcerting.
 393 In 2007, the yearly minimal extent of Arctic sea ice has started to decline abruptly
 394 and faster than expected by climate models ([Stroeve et al. (2012)]). All observa-
 395 tions after 2007 were consistent with an abrupt change, especially the latest to date

(September 2012). Such a situation is indeed surprising, yet it is clearly inconclusive: On the one hand, it tends to indicate that climate sensitivity may be higher than expected. It might even suggest that the climate system has passed a tipping point ([Lenton et al. (2008), Abbot et al.(2011), Livina and Lenton (2012)]), even though this possibility is still actively debated ([Tietsche et al. (2011)]). On the other hand, internal variability is high in the polar regions [Ghil et al. (1987), Darby and Mysak (1993)], and it could explain this situation without requiring a high sensitivity. So, it can be argued that the recent decline in Arctic sea ice raises more questions than it provides answers, and it is therewith a disconcerting observation. To settle the matter would require applying the procedure described in Section 4.2 to a more detailed model than our linear model of Eq. (7). Such a model would have to explicitly represent sea ice and allow for the presence of tipping points [Ghil (2001)]; it is thus beyond the scope of the present paper.

410 5 Policy implications

411 The key finding of the previous section is that the future trajectory of uncertainty
412 with respect to climate sensitivity is itself uncertain and that this uncertainty
413 could well increase. Such a finding may have implications for the development
414 of climate change mitigation policy. In the present section, we merely discuss
415 whether and how disconcerting learning may affect policy, but stop short of any
416 recommendations on this matter.

417 It is clear that climate change may seriously affect humankind’s socio-economic
418 well-being in the future. The extent and cost of any future damages, though, are
419 quite uncertain, in particular because of uncertainty concerning climate sensitiv-
420 ity. It is also clear that uncertain future damages can be mitigated by actions
421 taken today — e.g., a CO₂ abatement achieved by various means, including a
422 carbon tax, for instance. Unlike the cost of climate damages, which lies in future,
423 these mitigation actions have a cost that is immediately incurred and is also fairly
424 accurately known.

425 The crucial issue at the heart of mitigation policies is thus one of defining the
426 right trade-off between uncertain, future damages and certain, present costs. The
427 issue can thus be posed as a risk management problem, and there is an abundant
428 literature in which it is tackled within this framework, using the concepts and tools
429 of decision under uncertainty ([Arrow and Fisher (1974)]). In spite of the common
430 analysis framework used to tackle mitigation policy design, policy recommenda-
431 tions range from very substantial ([Rahmstorf (1999)]) to very low ([Tol (1997)])
432 near-term CO₂ abatement. Such a degree of divergence may relate to the fact that
433 optimal policy design depends critically on a number of key assumptions built
434 into both the economic and the climate model involved in the analysis. A lack of
435 consensus prevails on these assumptions, e.g. on the assumption of reversibility
436 used in the climate model as well as in the target criterion ([Keller et al. (2004)]).

437 In the framework of decision making under uncertainty, one wishes to hedge
438 against an undesirable future outcome. Thus the present level of uncertainty on
439 the future outcome obviously influences the hedge level chosen at the present time.
440 In a learning situation in which uncertainty is expected to evolve in the future, it

441 is not as obvious whether and how the expected trajectory of uncertainty should
442 affect the present decision.

443 Several studies ([Keller et al. (2004), Webster et al. (2008)]) have explicitly ap-
444 proached this question by comparing the optimal policy found under static uncer-
445 tainty (i.e., no learning) and under decreasing uncertainty (i.e., reassuring learn-
446 ing, in the terminology proposed herein). In spite of differences in assumptions
447 and methods, the conclusions of these studies are qualitatively consistent: the ex-
448 pectation of a future decrease of uncertainty is found to influence policy when its
449 aim is to avoid a dangerous threshold, and to have a negligible influence when the
450 cost-benefit objective function is smooth and has no such threshold. In the former
451 case, the level of abatement is significantly reduced if the uncertainty is expected
452 to decrease.

453 So far, though, there have been no studies that investigated the influence on
454 optimal mitigation policy of an expectation of increasing uncertainty — i.e., of
455 disconcerting learning in the present terminology — or even given a more general
456 expectation of uncertain future uncertainty. Nevertheless, it is quite plausible,
457 given the results of previous studies, that such expectations should greatly affect
458 optimal policy design.

459 Indeed, since we might expect — given a continued IPCC learning process with
460 a 6–7-year cycle — that the uncertainty may get higher before it gets lower, one
461 could argue that it is worthwhile to “buy some time” for this hectic learning process
462 to reach its final target of full certainty, at which time wiser, optimally informed
463 decisions are more likely to be reached. In the present context, buying some time
464 could, however, mean one of two things: either (i) enforcing higher abatements so
465 as to delay an irreversible climate catastrophe [Ghil (2001), Lenton et al. (2008),
466 Zaliapin and Ghil (2010)]; or, (ii) to the contrary, take care of other, possibly more
467 urgent problems while the learning goes on, with still-growing or fluctuating uncer-
468 tainties [Hillerbrand and Ghil (2008)]. Given the divergence of opinions on such a
469 momentous decision, it is imperative to go beyond the speculative reasoning in this
470 section and apply systematically the learning-theoretical framework introduced in
471 Sections 2–4, in combination with the risk-management type of analyses cited in
472 the present section.

473 6 Discussion and conclusions

474 [Oppenheimer et al. (2008)] (cited as [ONW08] throughout the present paper) in-
475 troduced a probabilistic definition of learning in the context of scientific research
476 on environmental problems. These authors showed that learning does not neces-
477 sarily lead to truer beliefs, a situation they termed *negative learning*.

478 We have extended this analysis here to show that learning does not necessarily
479 lead to more certain beliefs either, a situation for which we introduced the term of
480 *disconcerting learning*. Negative learning corresponds to an increase in PDF bias,
481 disconcerting learning corresponds to an increase in PDF dispersion. We have
482 shown that the latter differs from, and is not tied to, the occurrence of the former.
483 In other words, learning may well result in a state of knowledge which is closer to
484 the truth and yet more uncertain, cf. Fig. 1.

485 We have shown that this rather counter-intuitive situation typically arises when
486 a surprising but inconclusive piece of evidence is found. In Section 3.1, we used

487 the simple example of a medical screening test that gives a positive result as an
488 illustration of this fact. Such medical evidence is definitely informative but it is
489 surprising because a negative result is a priori more likely; at the same time, it is
490 also inconclusive because false positives are quite common in screening tests. Thus
491 the patient, once informed of the test result, definitely knows more about his or
492 her health but is still left with greater uncertainty than before the test.

493 Motivated by this simple example, we introduced in Section 3.2 a probabilistic
494 model based on reasonable assumptions about learning, and used it to confirm that
495 disconcerting learning in general occurs as a result of surprising but inconclusive
496 evidence at a particular step in the learning process. Furthermore, we narrowed
497 in on this situation arising when the PDF that reflects the state of knowledge
498 is asymmetric or has heavy tails (Fig. 2). We have shown that the dispersion of
499 the trajectories of uncertainty as learning occurs — i.e. the uncertainty on the
500 uncertainty — is high when disconcerting learning is prone to happen.

501 Finally, because pronounced asymmetry appears to be a pervasive feature of
502 the PDF of climate sensitivity in our current state of knowledge [AR4], climate
503 uncertainty is thus prone to remain high or to increase — even if and as climate
504 science makes steady progress — and thus its future trajectory is itself highly
505 uncertain. Whether or not this is good news remains to be seen.

506 At first, the news that substantial research efforts dedicated to improving our
507 understanding of the climate system could potentially result in an increased un-
508 certainty on the outcome of future climate change may sound rather discouraging.
509 On the other hand, the present article also provides a rational justification for
510 the fact that constant or even increasing uncertainty is perfectly compatible with
511 steady scientific progress and improved knowledge of the climate system. In other
512 words, our results suggest that the uncertainty on climate sensitivity should not
513 be considered as an appropriate metric to monitor progress in climate science, as
514 has sometimes been suggested.

515 Our discussion here emphasizes two characteristics of disconcerting learning.
516 First, disconcerting learning is a possibility, not a fatality. Since the occurrence of
517 a surprise is by nature unexpected — and thus unlikely — so is the occurrence
518 of disconcerting learning. Second, when it does occur, disconcerting learning is a
519 transient state that eventually ends, at least in our model of Section 3.

520 Indeed, the initial increase of uncertainty is caused by the inconclusive nature
521 of the surprising evidence. As more reassuring evidence confirms what was at first
522 a surprise, uncertainty will eventually decrease. Still, the process of uncertainty
523 getting worse before it gets better is intrinsic to the progressive learning model
524 that we introduced: a sufficiently large surprise can occur only once during pro-
525 gressive learning, because a second surprise of large amplitude cannot occur unless
526 expectation moves away from the true value, i.e. unless we are engaged in negative
527 learning. In other words, one cannot be surprised twice without being wrong at
528 least once.

529 Once we allow for the possibility of negative learning, though, successive sur-
530 prises — progressive and negative — become possible, causing the repetition in
531 time or the lengthening of disconcerting learning episodes. While this was beyond
532 our scope here, studying how uncertainty will evolve when taking negative learning
533 into account is both interesting and relevant. As Fig. 4 shows, the model intro-
534 duced in Section 4 is capable of exhibiting such behavior; it may thus be a good

535 starting point to investigate the more complex learning dynamics that can occur
536 when negative learning is a possibility.

537 Finally, we have considered in Section 4 a situation in which climate sensitivity
538 is the only uncertain quantity at stake in the evolution of climate, and in which
539 learning is driven by the mere observation of global temperature. It turns out that
540 surprises may occur in the future evolution of our assessment of climate sensitivity,
541 even in such a simple situation of linear deviations from a radiative equilibrium.

542 This being said, the framework applied in Section 4 here for illustration pur-
543 poses only can be made more realistic. Indeed, in more detailed climate models
544 there are various uncertain parameters and processes that are either independent
545 of climate sensitivity — e.g., ocean heat take-up or aerosol forcing — but do in-
546 fluence the climate response, or actually determine climate sensitivity itself, e.g.
547 cloud-radiative feedbacks.

548 On the other hand, global temperature is certainly not the only variable one can
549 use to constrain climate sensitivity and additional observations should be added
550 into the learning-process analysis. It would therefore be of interest to investigate
551 in a probabilistic framework, like that of Section 4 here, how the combined and
552 possibly nonlinear effect of simultaneous learning on these various uncertain quan-
553 tities, by means of multiple observations, plays out. It is quite possible that the
554 results of such studies might affect our conclusions with respect to the uncertain
555 future of climate uncertainty, as well as lead to more definitive policy implications,
556 as discussed in Section 5.

557

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567

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	$p_1(x^*) < p_0(x^*)$ ~ increasing bias	$p_1(x^*) > p_0(x^*)$ ~ decreasing bias
$\sigma_1 < \sigma_0$ = decreasing uncertainty	Reassuring Negative Learning	Reassuring Progressive Learning
$\sigma_1 > \sigma_0$ = increasing uncertainty	Disconcerting Negative Learning	Disconcerting Progressive Learning

Fig. 1 Schematic diagram of the four different learning situations that result from the definitions introduced in Oppenheimer et al. [2008] — i.e., negative vs. progressive — and in the present article — i.e., disconcerting vs. reassuring. These four situations are mapped here with respect to the evolution of the bias and of the uncertainty in the probability density function (PDF), as they reflect the state of knowledge, according to Eqns. (2) and (3).

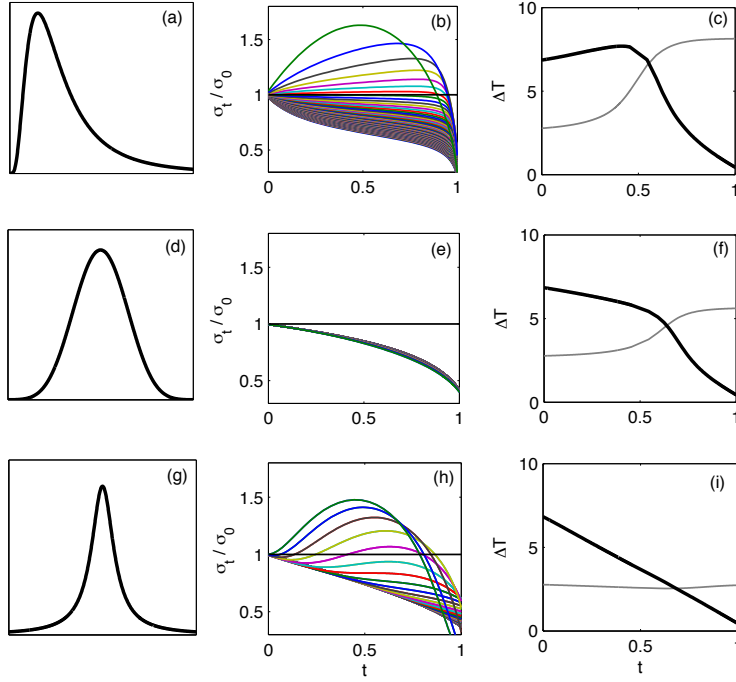


Fig. 2 Evolution of the PDF $p(x)$ for three typical shapes of the initial PDF $p_0(x)$ — shown in panels (a, d, g) — to final convergence, when the spread vanishes. The spread σ of $p(x)$ is normalized in panels (b, e, h) to its initial value σ_0 as learning occurs. For a given initial PDF, each trajectory is associated with a different true value towards which the progressive learning model of Eq. (SM3) converges; one hundred trajectories are thus plotted for each initial PDF, using its percentiles as true values. (a, b) Gamma PDF, asymmetric, with exponential tail; (d, e) Gaussian PDF, symmetric, with thin tails; and (g, h) Cauchy PDF, symmetric, with heavy tails. Evolution of the PDF parameters is plotted for the initial PDFs of Frame [2005] (c, f, i): percentile range 5%–95% (solid line) and median (gray line) of the climate sensitivity ΔT , for three different true values ΔT^* : (c) $\Delta T^* = 8^\circ\text{C}$, (f) $\Delta T^* = 5.5^\circ\text{C}$, and (i) $\Delta T^* = 3^\circ\text{C}$.

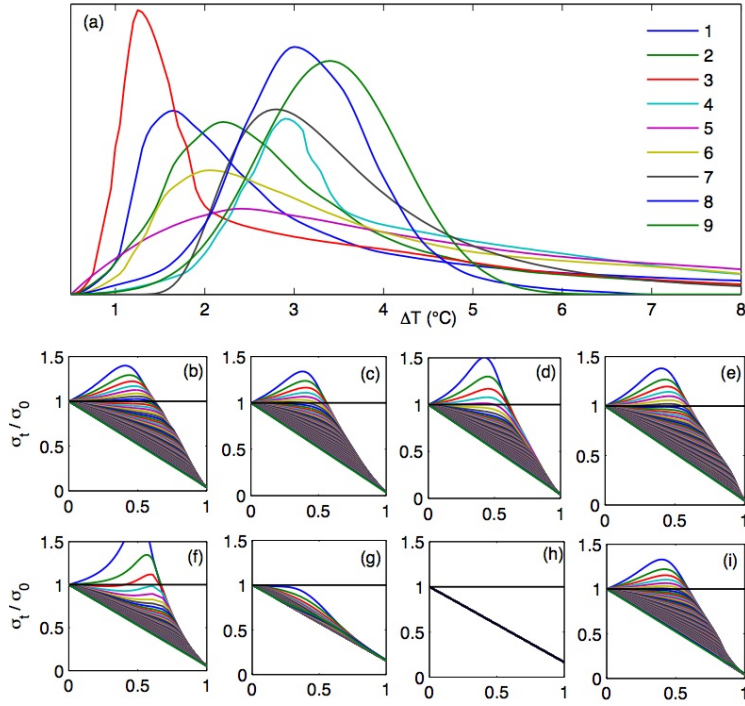


Fig. 3 (a) Nine PDFs of climate sensitivity reported by [AR4], obtained by different teams and using diverse data and methods: (1 through 5) PDFs constrained by the transient evolution of the atmospheric temperature, radiative forcing and ocean heat uptake; (6,7) constrained by present-day climatology; and (8, 9) unweighted or fitted distributions from different models or from perturbing parameters in a single model; see text for details. Evolution of the PDFs $p(x)$ for the nine initial PDFs $p_0(x)$; same treatment as in Fig. 2. For a given initial PDF, each trajectory is associated to a different true value towards which the progressive learning model of Eq. (SM3) converges. One hundred trajectories are plotted for each initial pdf using its percentiles as true values. (b, 1) [Forster and Gregory (2006)], (c, 2) [Gregory et al. (2002)], (d, 3) [Frame et al. (2005)], (e, 4) [Hegerl et al. (2006)], (f, 5) [Andronova and Schlesinger (2001)], (g, 6) [Forest et al. (2006)], (h, 7) [Roe and Baker (2007)], (i, 8) [Knutti et al. (2002)], (9) [Raisanen (2005)].

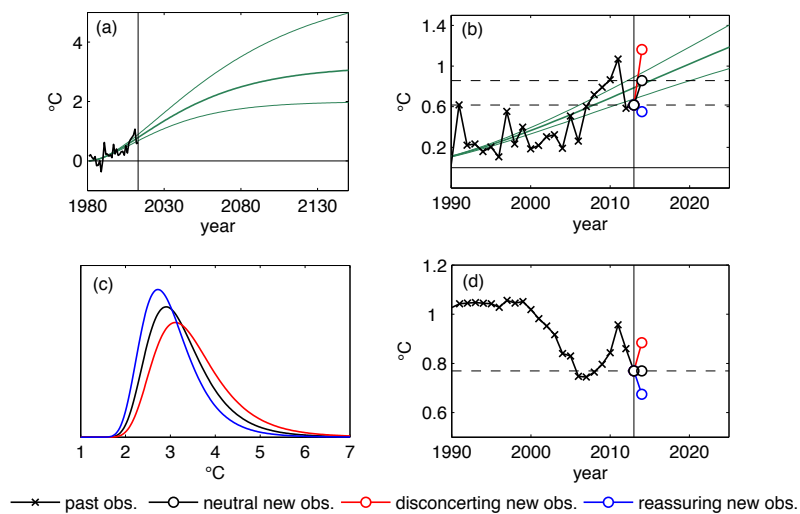


Fig. 4 (a) Trajectories of Earth's global temperature anomaly: smooth component \bar{T}_t simulated by the energy balance model of Eq. (7) over the time interval 1980–2150, for $\Delta T = 2^\circ\text{C}$, 3.2°C and 6°C (green lines); and observations T_t simulated over the interval 1980–2012 for $\Delta T = 3.2^\circ\text{C}$ (black line). (b) Same as (a) zoomed on the time period 1980–2025, with three possible new observations added in 2013: strong warming (red line and circle, disconcerting learning), moderate warming (dark line and circle, reassuring learning), moderate cooling (blue line and circle, reassuring learning). (c) Posterior PDFs of climate sensitivity after updating based on each of the three new 2013 observations. (d) Trajectory of the uncertainty on climate sensitivity σ_t over the observational interval 1990–2012 (black line), and new uncertainty value, after updating based on each of the three new 2013 observations.