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Field Theoretical Approach to Deformation and Fracture

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ABSTRACT Dynamics of plastic deformation and fracture is discussed based on the field theoretical approach introduced by a recent gauge theory called physical mesomechanics. The Maxwell type field equation derived by physical mesomechanics is interpreted as the equation of motion that describes the dynamics of a volume element in plastically deforming material. Theoretical analysis indicates that this dynamics can be argued in terms of energy conservative and dissipative forces. The energy conservative force is identified as a restoring force associated with rotational equilibrium and responsible for the oscillatory nature of plasticity. The energy dissipative force is identified as being associated with a drift of a quantity equivalent to the electric charge. Experimental observations that support these ideas are presented.

Introduction

Much remains unknown in the dynamics of plastic deformation. Its full understanding is important not only for scientific interests but also for various applications in the contemporary engineering. It is of our particular interest to formulate the dynamics on the same basis as elasticity. In this regards, a recent theory called physical mesomechanics (PMM) [1] has great potential. Based on a fundamental physical principle known as the gauge symmetry, PMM derives a field equation [2] that describes the spatiotemporal characteristics of the displacement field of a plastically deforming object. Formulaically, this field equation is quite similar to Maxwell equation of electrodynamics. In terms of Newtonian mechanics, this dynamics can be understood as the interaction among local volume elements within which the deformation is elastic. In this interaction, material rotation plays an important role. In the elastic regime, while differential translational displacement causes deformation, rotation is common to the whole object. When a defect is created in the material, however, the surrounding regions gain rotational degrees of freedom. Thus in the plastic regime, differential rotation causes rotational deformation. The above-mentioned field equation can be viewed as describing the synergetic interaction between the translational and rotational modes of deformation.

It is interesting to argue the above dynamics in terms of the force that the local volume elements exert on one another. As reported in ref. [3], this force consists of the energy conservative and energy dissipative parts. The former is a restoring force associated with the local rotational displacement and responsible for the recovery nature of the dynamics [4]. The latter is a damping force associated with a quantity equivalent to the electric charge and responsible for the irreversible nature of the dynamics [3]. The energy dissipation caused by plastic deformation can be considered as a process analogous to the energy dissipation caused by conduction currents in electricity. In this study, an attempt is made to explain temperature rise observed in plastically deforming metal samples in connection with the heat associated with this energy dissipative process. A quantity equivalent to the electric charge is identified and the temperature rise is discussed in conjunction with its appearance.

Theoretical formulation

The field equation derived by PMM can be put in the following form [1, 3].

$$\nabla \cdot \vec{v} = \frac{q}{\varepsilon} \quad (1)$$

$$\nabla \times \vec{\omega} = -\varepsilon\mu \frac{\partial \vec{v}}{\partial t} - \vec{j}, \quad (2)$$

where \vec{v} is the velocity at (x, y, z) and $\vec{\omega}$ is the angle of the local volume element from its rotational equilibrium. \vec{v} and $\vec{\omega}$ are connected as

$$\nabla \times \vec{v} = \frac{\partial \vec{\omega}}{\partial t}. \quad (3)$$

Rearrangement of the terms of eq. (2) as below indicates that this equation is an equation of motion that describes the dynamics of the unit volume.

$$\varepsilon \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\mu} \nabla \times \vec{\omega} - \frac{\vec{j}}{\mu}. \quad (4)$$

Here the left-hand side of eq. (4) is the product of the mass and the acceleration of the unit volume, and the right-hand side represents the total external force on the unit volume. The first term on the right-hand side is the recovery force per unit volume. To understand it more clearly, multiply the volume $dxdydz$ and consider the x component of the resultant force,

$$f_x = \frac{1}{\mu} (\nabla \times \vec{\omega})_x dxdydz = \frac{1}{\mu} \frac{\partial \omega_z}{\partial y} dydxdz = \frac{1}{\mu} d\omega_z dxdz = \frac{1}{\mu} \frac{\partial \xi_x}{\partial y} dxdz.$$

Here $\xi_x(y)$ is the displacement in the plane perpendicular to the y -axis at y . By dividing by the lateral area $dxdz$, we can see that this equation relates the shear stress τ_x and shear strain $\partial \xi_x / \partial y$ as

$$\tau_x = \frac{f_x}{dxdz} = \frac{1}{\mu} \frac{\partial \xi_x}{\partial y},$$

and $1/\mu$ represents the shear modulus. Fig. 1 illustrates the situation schematically. The other components of the shear stress can be argued in the same fashion.

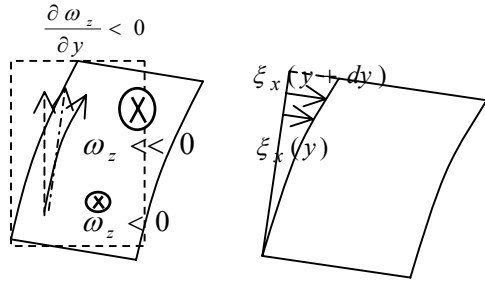


Fig. 1 Schematic illustration of recovery force

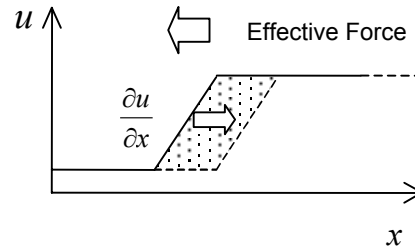


Fig. 2 Schematic illustration of damping force

The second term on the right-hand side of eq. (4) represents the damping force. Eqs. (2) and (3) lead to a continuous equation, which allows us to put $\vec{j}/\mu = \rho W_d$, where W_d is the drift velocity of the charge. Fig. 2 illustrates schematically a simplified situation where a charge represented by smooth variation $\partial u / \partial x$ from the lower to higher level of u drifts in the positive x direction. Here u is the x component of the velocity. As this drift takes place, the volume corresponding to the lower velocity level increases and the volume corresponding to the higher velocity level decreases. Hence, overall the material loses its total momentum, which is equivalent that a damping force is exerted in the opposite direction to the drift of the charge.

Experiment

Experiments were conducted with electronic speckle pattern interferometry [5] and temperature measurement by thermistors. Fig. 3 shows the experimental arrangement. A dual-beam optical interferometer sensitive to in-plane,

horizontal displacement was set up in front of a tensile machine that held a thin plate sample. The sample materials tested were aluminum and brass. The sample had a notch on a side near the vertical center so that fracture necessarily occurred there. A thermistor was placed near the notch and an additional thermistor was placed near the top grip of the tensile machine. A CCD (charge coupled device) camera captured the whole image of the sample at a preset constant interval, and the image data were sent to a computer where image subtraction was made for fringe formation. Temperature data collected by the thermistors were sent to the computer via a data acquisition board along with the load and elongation data collected by the tensile machine. The tensile machine was operated at a constant crosshead speed. .

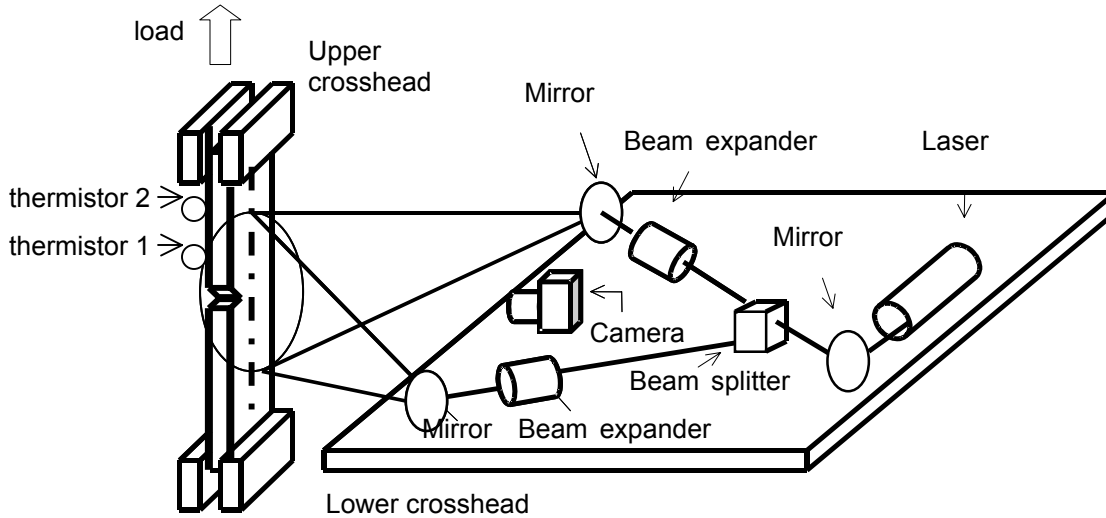


Fig. 3 Experimental arrangement.

Results and discussions

Charge-like fringe pattern

Fig. 4 shows typical fringe patterns obtained before and after the crack starts to grow. The horizontal fringes observed above and below the horizontal line passing through the notch indicate that the sample experiences bodily rotation (clockwise/counterclockwise above/below the notch). The wedge-shaped fringe pattern circled in (b) represents non-zero $\partial u / \partial x$, hence can be viewed as a charge [eq. (1)]. A charge observed during a crack growth usually drifts at a constant velocity as the four images labeled (c) indicate. This type of motion of the charge is still under investigation, but it possibly represents the dynamics associated with the damping (energy dissipative) force described in Fig. 2.

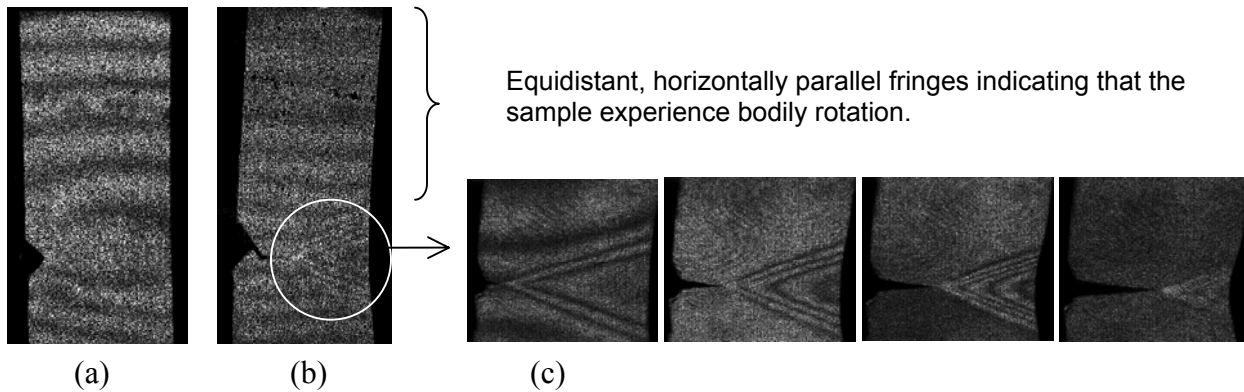


Fig. 4 Typical fringe patterns before (a) and after (b and c) the crack starts to grow. The wedge shaped fringe pattern circled in (b) is considered to be a charge. Four images in (c) show a similar charge observed in a different experiment indicating that it propagates to the right as the crack grows.

Temporal variation of load and temperature

The temporal variations in temperature and load were recorded as the tensile load is applied at a constant crosshead speed of 20 $\mu\text{m/s}$. Fig. 5 shows typical data for the brass sample (top) and the aluminum sample (bottom). The more oscillatory plots of the temperature data are the temperature measured near the notch and the less oscillatory plots were near the upper grip of the tensile machine. The former is more oscillatory because of the higher sensitivity of the thermistor used near the notch. 25 samples in total were tested. Thorough analysis of the obtained data led us to the following findings.

- (1) In all 25 samples tested, commonly to the brass and aluminum, a crack started to grow from the notch when the load starts to decrease after reaching the maximum value.
- (2) In all 25 samples tested, commonly to the brass and aluminum, the temperature near the notch was higher than near the grip.
- (3) The load decrease after the maximum was more gradual in brass than aluminum, indicating that brass is more ductile than aluminum.
- (4) In most cases, a temperature decrease was observed at the beginning of loading.
- (5) In all the aluminum samples except for one case, the temperature started to rise when the crack started to grow. In the only exceptional case, the temperature rise was prior to the initiation of the crack growth, i.e., while the load was still rising, as is the case of the brass data in Fig. 5.
- (6) In all brass samples, on the other hand, the beginning of the temperature rise was prior to the initiation of the crack growth, as seen in the brass data of Fig. 5.

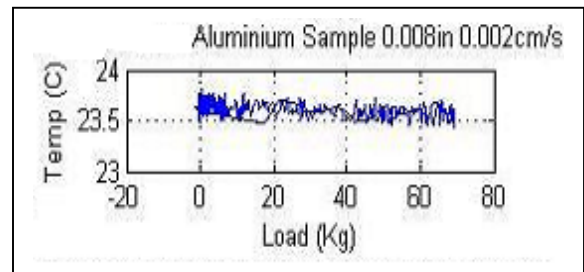
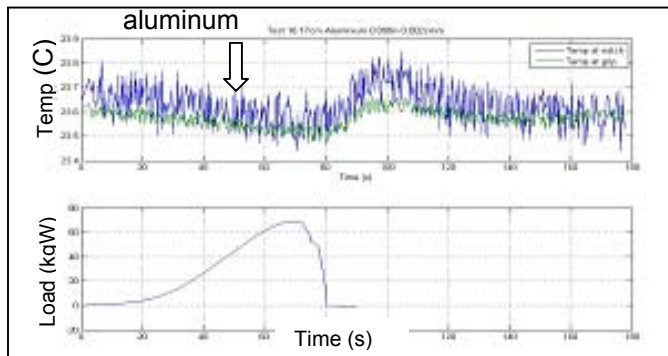
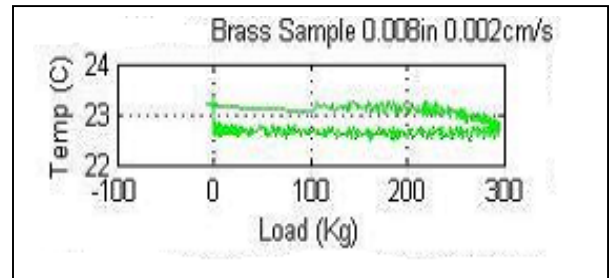
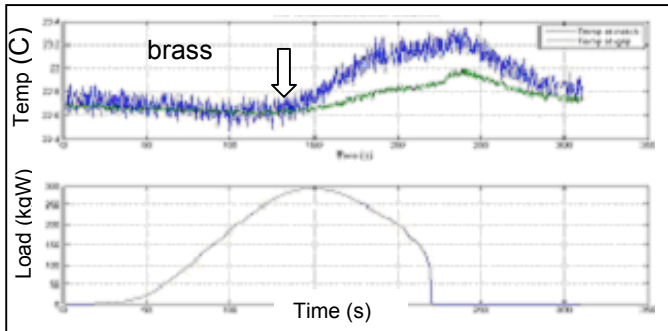


Fig. 6 Load vs. temperature for the same samples as Fig. 5

Fig. 5 Temperature and load as functions of time

Fig. 6 shows the same data as Fig. 5 in the form of the load change vs. the temperature change. In the brass data (top), the load increases along the lower path reaching the maximum value of 290 kgW, and returns to zero along the upper path. The gradual decrease in temperature in the lower path indicates the above finding (4), the initial temperature decrease. The slight temperature rise in the lower path near the maximum load indicates the above finding (6), the onset of temperature rise prior to the crack initiation. In the aluminum data (bottom), the forward path and return path are almost overlapping each other, while the return path is much smoother than the forward path indicating the fact that the load decrease is much faster than brass. There is no temperature rise near the maximum load in the forward path, indicating that the onset of the temperature rise is at the same time as the

onset of load decrease. The smoothness observed in the return path indicates the fact that the temperature rise is faster in aluminum. The temperature difference at the zero load point between the forward path and return path indicates the total heat release. It is clear that the brass sample released much more heat than the aluminum sample.

Stress concentration

The high intensity regions circled in Fig. 7 show where the fringe density is much higher than the surrounding area indicating localized strain due to a stress concentration. As seen in Fig.7, the stress concentration is spot-like in brass and line-like in aluminum. The arrows in Fig. 5 indicate when these patterns started appearing. In the case of brass, the spot-like pattern moves along with the tip of the crack at a few mm ahead of the crack tip. In the case of aluminum, the pattern extended as the load increases until it reaches the maximum point and the sample fractures.

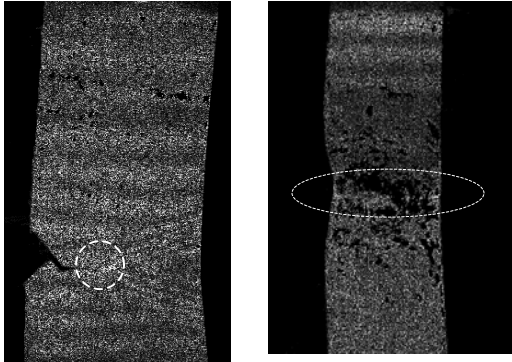


Fig. 7 Spot-line and line-like pattern observed in brass (left) and aluminum (right)

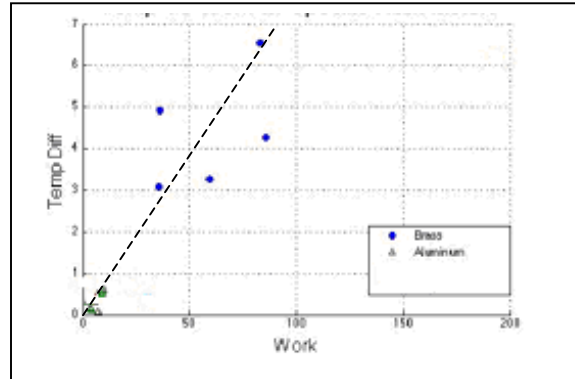


Fig. 8 Total temperature rise vs. work done by tensile machine

Interpretation of the observation

The initial temperature decrease [finding (4)] is apparently due to the phenomenon known as the thermo-elastic effect. The higher temperature rise near the notch than the grip [finding (2)] clearly indicates that the near notch region is the heat source. The temperature rise observed after the crack initiation is understood as dissipation of the material’s internal energy as heat. At the beginning of deformation when the restoring force represented by the first term on the right-hand-side of eq. (4) is dominant, the material stores the elastic energy (the spring potential energy) as the internal energy. When the crack starts to grow, the material becomes discontinuous and loses its capability of storing this internal energy. Consequently, it releases the energy as heat. When the sample is brittle, like the aluminum case in Fig. 5, these energy storing phase and heat releasing phase are sharply divided. That is why the onset of the heat release coincides with the onset of load decrease in the aluminum data in Fig. 5

The temperature rise prior to the onset of load decrease observed in more ductile samples [findings (5) and (6)] can be explained as heat release associated with the energy dissipative (damping) force. When the energy dissipation due to the damping force becomes substantial, the material can release heat while the restoring force is still effective; thus the load increase due to the restoring force and heat release due to the damping force can coexist. This is consistent with previous observations⁶ in various materials that when charge-like patterns drift in the samples the oscillation in the displacement field decays (the oscillation is damped). The fact that it is observed both in brass and aluminum indicates that it is not a phenomenon that some materials uniquely show but rather a phenomenon any material shows when it behaves ductile.

The ultimate energy source of the releases heat is the work done by the tensile machine. We computed the work done by the tensile machine by numerically integrating the product of the load and elongation with respect to time. In Fig. 8, the total temperature rise is plotted as a function of the tensile machine’s work for both brass and aluminum. In this plot, the density and specific heat of the brass and aluminum are taken into account. The relationship between the tensile machine’s work and the temperature rise is reasonably linear, supporting the above interpretation.

Conclusion

The dynamics of plastic deformation was formulated in terms of the energy conservative and dissipative forces possessed by materials. Experiments were conducted to measure the temporal change in the temperature and load of thin-plate brass and aluminum samples under tensile load at a constant. Simultaneously, the in-plane displacement field of the sample was observed with optical interferometry. Temperature rise was observed at the fracturing stage. Interferometric fringe images showed charge-like patterns that drifted as the crack developed from the notch. They also showed high optical intensity patterns that were believed to indicate stress concentration. These observations were explained based on the energy conservative and dissipative forces.

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