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Hamiltonian derivation of a gyrofluid model for collisionless magnetic reconnection

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Abstract. We consider a simple electromagnetic gyrokinetic model for collisionless plasmas and show that it possesses a Hamiltonian structure. Subsequently, from this model we derive a two-moment gyrofluid model by means of a procedure which guarantees that the resulting gyrofluid model is also Hamiltonian. The first step in the derivation consists of imposing a generic fluid closure in the Poisson bracket of the gyrokinetic model, after expressing such bracket in terms of the gyrofluid moments. The constraint of the Jacobi identity, which every Poisson bracket has to satisfy, selects then what closures can lead to a Hamiltonian gyrofluid system. For the case at hand, it turns out that the only closures (not involving integro/differential operators or an explicit dependence on the spatial coordinates) that lead to a valid Poisson bracket are those for which the second order parallel moment, independently for each species, is proportional to the zero order moment. In particular, if one chooses an isothermal closure based on the equilibrium temperatures and derives accordingly the Hamiltonian of the system from the Hamiltonian of the parent gyrokinetic model, one recovers a known Hamiltonian gyrofluid model for collisionless reconnection. The proposed procedure, in addition to yield a gyrofluid model which automatically conserves the total energy, provides also, through the resulting Poisson bracket, a way to derive further conservation laws of the gyrofluid model, associated with the so called Casimir invariants. We show that a relation exists between Casimir invariants of the gyrofluid model and those of the gyrokinetic parent model. The application of such Hamiltonian derivation procedure to this two-moment gyrofluid model is a first step toward its application to more realistic, higher-order fluid or gyrofluid models for tokamaks. It also extends to the electromagnetic gyrokinetic case, recent applications of the same procedure to Vlasov and drift-kinetic systems.

1. Introduction

Fluid models represent a widespread and effective tool for investigating important phenomena in fusion plasmas such as instabilities, turbulence and reconnection events. Indeed, fluid models offer a considerable advantage, in terms of required computational resources, with respect to kinetic models. On the other hand, compared to these, they obviously suffer from limitations in the range of scales and frequencies of the phenomena that they can describe. The derivation of sophisticated fluid models aiming at partially remedying such limitations is an active line of research since a long time. An essential part of the problem is related to the closure adopted to truncate the fluid hierarchy of equations obtained by taking moments of a parent kinetic model. A considerable effort, directed also toward applications to space plasma turbulence, has been carried out to derive fluid and gyrofluid models (see, e.g. Refs. [1–8]) by taking moments of

kinetic or gyrokinetic models and imposing closures designed to satisfy desired properties, such as for instance consistency with kinetic linear theory or energy conservation. Relatively little attention has been paid, however, to ensure that the adopted closures satisfy a further criterion, which is the preservation of a Hamiltonian structure in the derivation of the non-dissipative part of the model. In other words, the parent kinetic model, in its non-dissipative limit, is supposed to possess a Hamiltonian structure, and, unless forcing or dissipative terms are voluntarily added when applying the fluid closure, the resulting fluid model is also supposed to possess a Hamiltonian structure. Apart from being an important feature from a fundamental point of view, the preservation of a Hamiltonian structure provides also some practical advantages. On one hand, it provides an unambiguously defined conserved total energy (the Hamiltonian functional). Moreover, given that fluid models for plasmas are often expressed in Eulerian form, their Hamiltonian structure is typically noncanonical (see, e.g. Ref. [9]), which implies the existence of additional conserved quantities, denoted as Casimirs. Their knowledge can provide useful information on the nonlinear dynamics of the fluid model. In the presence of dissipation, the decay rate and the cascade of such invariants can provide a further way to characterize plasma turbulence. Moreover, they can be used as additional invariants to test the conservation properties of numerical codes. Namely because with Hamiltonian plasma models one has to deal with noncanonical Poisson brackets, a delicate point in preserving the Hamiltonian structure throughout the derivation, is that of not violating the Jacobi identity. The latter is one of the properties defining a Poisson bracket. Although it is always satisfied for canonical Poisson brackets, showing its validity for noncanonical Poisson brackets is often far from obvious.

Recently, derivation procedures that preserve the Hamiltonian structure of the parent model have been applied to kinetic and drift-kinetic models [10–12]. A key element in these procedures consists of inserting a generic fluid closure relation in the bracket obtained from the parent kinetic model, and then imposing the constraint that the Jacobi identity be satisfied. Such constraint selects fluid closures that automatically lead to Hamiltonian fluid models. The purpose of the present paper is two-fold. In the first place, we show in Sec. 2 the existence of a Hamiltonian structure for a simple gyrokinetic electromagnetic model in slab geometry. In the second place, in Sec. 3 we apply the procedure of Ref. [11] to this model in order to derive a two-moment Hamiltonian gyrofluid model for magnetic reconnection driven by electron inertia. This type of reconnection is believed to be relevant for sawtooth oscillations in tokamaks (see, e.g. Ref. [13]). Because of the relative simplicity of the models under consideration, and of the limited number of fluid moments involved, the present results cannot be directly relevant for a realistic modelling of tokamak plasmas. They provide, however, an explicit example of a structure-preserving derivation which is a first step toward applications to more realistic higher-order fluid models. They also provide new knowledge about the Hamiltonian structures of the more basic gyrokinetic and gyrofluid models involved in the paper. The results are summarized in Sec. 4, where also limitations of the present analysis and possible future directions of investigation are discussed.

2. Parent gyrokinetic model and its Hamiltonian structure

In a Cartesian coordinate system (x, y, z) we consider a gyrokinetic model in slab geometry in the δf approximation. The model consists of the following evolution equations

$$\frac{\partial g_i}{\partial t} + \frac{c}{B}\left[J_0\left(\phi - \frac{v}{c}A\right), g_i\right] + v\frac{\partial}{\partial z}\left(g_i + \frac{\mathcal{F}_i}{T_i}eJ_0\left(\phi - \frac{v}{c}A\right)\right) = 0, \tag{1}$$

$$\frac{\partial g_e}{\partial t} + \frac{c}{B}\left[\phi - \frac{v}{c}A, g_e\right] + v\frac{\partial}{\partial z}\left(g_e - \frac{\mathcal{F}_e}{T_e}e\left(\phi - \frac{v}{c}A\right)\right) = 0, \tag{2}$$

of the quasi-neutrality relation

$$\frac{e^2 n_0}{T_i}(\Gamma_0 - 1)\phi + e\int d\mathcal{W}(J_0 g_i - g_e) = 0, \tag{3}$$

and of Ampère's law

$$\nabla_\perp^2 A - 4\pi\frac{e^2}{c^2}n_0\left(\frac{\Gamma_0}{M_i}+\frac{1}{M_e}\right)A + 4\pi\frac{e}{c}\int d\mathcal{W}v(J_0 g_i - g_e) = 0. \tag{4}$$

Eqs. (1) and (2) govern the evolution of $g_i = f_i + \mathcal{F}_i(e/T_i)(v/c)J_0 A$ and $g_e = f_e - \mathcal{F}_e(e/T_e)(v/c)A$, where $f_{i,e}(x,y,z,v,\mu,t)$ indicate the perturbations of the ion and electron equilibrium distribution functions $\mathcal{F}_{i,e}$, respectively, which are defined as

$$\mathcal{F}_i(v,\mu) = n_0\left(\frac{M_i}{2\pi T_i}\right)^{3/2} \mathrm{e}^{-\frac{M_i v^2 + 2\mu B}{2T_i}}, \qquad \mathcal{F}_e(v,\mu) = n_0\left(\frac{M_e}{2\pi T_e}\right)^{3/2} \mathrm{e}^{-\frac{M_e v^2 + 2(M_e/M_i)\mu B}{2T_e}}. \tag{5}$$

We indicate with v the velocity coordinate along the magnetic guide field direction (which we take to be the z direction) and with μ the ion magnetic moment. Ion and electron masses are denoted as M_i and M_e, respectively, whereas $T_{i,e}$ indicate constant temperatures characterizing the ion and electron equilibrium distribution functions. The constants c and e indicate the speed of light and the proton charge, respectively. The flux function $A(x,y,z,t)$ is related to the magnetic field by $\mathbf{B} = \nabla A \times \hat{z} + B\hat{z}$, with B indicating the uniform and constant amplitude of the guide field component. The constant n_0 is the equilibrium density of both electron and ions, whereas $\phi(x,y,z,t)$ denotes the electrostatic potential. The symbol $\nabla_\perp^2$ in Eq. (4) indicates the Laplacian operator restricted to the $xy-$plane. The integrations in Eqs. (3) and (4) are carried out over the element $d\mathcal{W} = 2\pi B d\mu/M_i$. We assume that all fields are periodic along the x, y and z directions and that $f_{i,e}$ decay to zero sufficiently fast as $v \to \infty$ and $\mu \to +\infty$. The operators J_0 and Γ_0 are the standard operators appearing in gyrokinetic theory and correspond to the multiplication, in Fourier space, times the zeroth order Bessel function $J_0(k_\perp\sqrt{2\mu B/M_i\omega_{ci}^2})$ and times the function $I_0(k_\perp^2\rho_i^2)\mathrm{e}^{-k_\perp^2\rho_i^2}$, respectively, where $k_\perp^2 = k_x^2 + k_y^2$ and where we indicated with $\omega_{ci} = eB/M_i c$ the ion cyclotron frequency, and with $\rho_i = \sqrt{T_i/M_i}/\omega_{ci}$ the ion thermal gyroradius. Finally, the canonical bracket appearing in Eqs. (1)-(2) is defined by $[f,g] = \partial_x f \partial_y g - \partial_y f \partial_x g$, for two functions f and g.

In the model (1)-(4) we neglected, as is customary by virtue of the small electron/ion mass ratio, the effects of the gyration radius for the electrons. However, the results present in the paper, can easily be extended to account for them as well.

Showing that the model (1)-(4) possesses a Hamiltonian structure amounts to show that it can be cast in the form

$$\frac{\partial \chi}{\partial t} = \{\chi, H\}, \tag{6}$$

where χ indicates a vector of dynamical variables, corresponding, in our case, to g_i and g_e. $H = H(\chi)$ is a functional, denoted as Hamiltonian, and $\{,\}$ is a Poisson bracket, that is an antisymmetric bilinear operation satisfying the Leibniz identity $\{FG, H\} = F\{G,H\} + G\{F,H\}$, $\forall F, G, H$ and the Jacobi identity

$$\{\{F,G\},H\} + \{\{G,H\},F\} + \{\{H,F\},G\} = 0, \qquad \forall F, G, H. \tag{7}$$

Because the Hamiltonian physically represents the conserved total energy of the system, a good candidate for H for our model is the conserved quantity [8]

$$\begin{aligned} H(g_i, g_e) = \frac{1}{2}\int d^3x d\mathcal{W}\Bigg[&\frac{T_i}{\mathcal{F}_i}g_i^2 + \frac{T_e}{\mathcal{F}_e}g_e^2 + e(J_0 g_i - g_e)L_0^{-1}\int d\mathcal{W}'(J_0 g_i - g_e) \\ &-e\frac{v}{c}(J_0 g_i - g_e)L_1^{-1}\int d\mathcal{W}'(J_0 g_i - g_e)\Bigg]. \end{aligned} \tag{8}$$

The expression (8) assumes the validity of the relations

$$\phi = L_0^{-1} \int d\mathcal{W}(J_0 g_i - g_e), \qquad A = L_1^{-1} \int d\mathcal{W} v (J_0 g_i - g_e), \tag{9}$$

that is the existence of invertible operators $L_{0,1}$ that permit, from Eqs. (3) and (4), to express ϕ and A in terms of g_i and g_e. In Fourier space it is not difficult to see that this inversion can be carried out for each $\mathbf{k}$, apart from the case concerning Eq. (3) and modes with $k_x = k_y = 0$, for which $\Gamma_0 - 1 = 0$. We restrict then to electrostatic potentials such that $\phi_{0,0,k_z} = 0$.

The operators L_0^{-1} and L_1^{-1} also turn out to satisfy the symmetry property $\int d^3x f(\mathbf{x}) L_{0,1}^{-1} g(\mathbf{x}) = \int d^3x g(\mathbf{x}) L_{0,1}^{-1} f(\mathbf{x})$ and the operator J_0 is known (see, e.g. Ref. [5]) to satisfy the relation $\int d^3x d\mathcal{W} f(\mathbf{x}, v, \mu) J_0 g(\mathbf{x}, v, \mu) = \int d^3x d\mathcal{W} g(\mathbf{x}, v, \mu) J_0 f(\mathbf{x}, v, \mu)$. By virtue of these properties, we obtain that the functional derivatives of H with respect to $g_{i,e}$ can be written as

$$\frac{\delta H}{\delta g_i} = \frac{T_i}{\mathcal{F}_i} g_i + e J_0 \left(\phi - \frac{v}{c} A \right), \qquad \frac{\delta H}{\delta g_e} = \frac{T_e}{\mathcal{F}_e} g_e - e \left(\phi - \frac{v}{c} A \right). \tag{10}$$

Direct calculations show that, when using (8) as Hamiltonian, with the help of Eqs. (10) and of integration by parts, the Poisson bracket that yields Eqs. (1)-(2), accounting for Eqs. (3) and (4), is given by

$$\{F, G\} = \sum_{s=i,e} \int d^3x d\mathcal{W} \left(\mp \frac{c}{eB} g_s [F_{g_s}, G_{g_s}] - v \frac{\mathcal{F}_s}{T_s} F_{g_s} \frac{\partial}{\partial z} G_{g_s} \right), \tag{11}$$

where the upper (lower) sign should be taken for ions (electrons). The subscripts on functionals, on the other hand, indicate functional derivatives, so that, for instance $F_{g_s} = \delta F / \delta g_s$.

The operation (11) is the direct sum of two independent Poisson brackets and consequently satisfies the properties of a Poisson bracket. We conclude then that the gyrokinetic model (1)-(4) possesses a Hamiltonian structure. A similar Hamiltonian structure was presented in Ref. [15] for the case of a drift-kinetic model.

3. Derivation of the Hamiltonian gyrofluid model

We define moments of $g_{i,e}$ as $P_{i,e_{jk}} = \int d\mathcal{W} v^j \mu^k g_{i,e}$, where j and k are non-negative integers. Functionals $F(g_i, g_e)$ can be expressed as functionals of the infinite moments $P_{i,e_{jk}}$ by using scalar invariance, so that $F(g_i, g_e) = \bar{F}(\{P_{i,e_{jk}}\}_{j,k \in \mathbb{N}})$ and the functional derivatives consequently transform as $F_{g_{i,e}} = \sum_{j,k \in \mathbb{N}} v^j \mu^k \bar{F}_{i,e_{jk}}$, where we denoted $\bar{F}_{i,e_{jk}} = \delta \bar{F} / \delta P_{i,e_{jk}}$. Inserting this expression into (11) one formally obtains the Poisson bracket in terms of all the moments, which reads

$$\{F, G\} = \sum_{s=i,e} \sum_{m,n,j,k \in \mathbb{N}} \int d^3x \left(\mp \frac{c}{eB} P_{s_{m+j\, n+k}} [F_{s_{mn}}, G_{s_{jk}}] - F_{s_{mn}} \frac{\partial}{\partial z} G_{s_{jk}} \int d\mathcal{W} v^{m+j+1} \mu^{n+k} \frac{\mathcal{F}_s}{T_s} \right), \tag{12}$$

In practice, however, one is interested in deriving a Hamiltonian fluid model, describing the evolution of a finite number of moments, for instance the first $M(N)$ moments for the parallel (perpendicular) direction, for both the ions and the electrons, where M and N are two positive integers. This means that the functionals of interest will be of the kind $F(P_{i_{00}}, P_{i_{10}}, P_{i_{01}} \cdots P_{i_{MN}}, P_{e_{00}}, P_{e_{10}}, P_{e_{01}} \cdots P_{e_{MN}})$ and will no longer depend on the infinite set of moments. The operation (12) between two such functionals, however, is not closed, in the sense that $\{F, G\}$, in general, will not be again a functional of the same kind, but it will depend also on moments of order higher than $M(N)$ in the parallel (perpendicular) direction. In order

to obtain a closed system, the way we follow is that of modifying the Poisson bracket, imposing that such extra higher order moments be functions of the first $M(N)$ moments in the parallel (perpendicular) direction. Imposing such constraint in the bracket, however, can break the properties defining a Poisson bracket. In particular, the Jacobi identity can be violated by this operation. However, in some cases, which we refer to as to Hamiltonian closures, imposing the closure relation in the Poisson bracket, preserves the properties of the Poisson bracket. Hamiltonian fluid models can then be derived adopting Hamiltonian closures.

In particular, we consider here the case $M = 1$ and $N = 0$, that is we restrict to functionals of the first two moments in the parallel direction. The bracket (12) then becomes

$$\{F, G\} = \sum_{s=i,e} \int d^3x \left(\mp \frac{c}{eB} P_{s_0}[F_{s_0}, G_{s_0}] \mp \frac{c}{eB} P_{s_1}([F_{s_1}, G_{s_0}] + [F_{s_0}, G_{s_1}]) \mp \frac{c}{eB} P_{s_2}[F_{s_1}, G_{s_1}] \right.$$
$$\left. - \frac{n_0}{M_s} \left(F_{s_1} \frac{\partial}{\partial z} G_{s_0} + F_{s_0} \frac{\partial}{\partial z} G_{s_1} \right) \right), \tag{13}$$

where, in order to simplify the notation, given that the index for the powers of μ has been fixed equal to zero, we have defined $P_{s_j} = P_{s_{j0}}$, for $j = 0, 1, 2$. It is evident that, as above anticipated, the result of the operation $\{F, G\}$ in (13) depends also on P_{s_2} and consequently the resulting equations of motion cannot lead to a closed fluid model, for any Hamiltonian depending on P_{s_0} and P_{s_1} only. One remedy to this is to look for two functions $\mathcal{P}_s(P_{s_0}, P_{s_1})$, if they exist, such that, when inserting $P_{s_2} = \mathcal{P}_s$ into Eq. (13), the resulting operation (which will now be closed) still satisfies the Jacobi identity and can consequently still provide a Poisson bracket. It turns out that, for brackets of the form (13), it has been shown [11] that the constraint of the Jacobi identity (7) selects, among all the possible closures (not considering, however, those involving integro/differential operators or an explicit dependence on the spatial coordinates), the following one:

$$P_{s_2} = a_s P_{s_0}, \qquad \text{for } s = i, e, \tag{14}$$

where a_s are two arbitrary constants, which dimensionally must be homogeneous to a velocity squared. We remark that the Hamiltonian closure (14) does not depend on first order parallel moments, which is a consequence of having chosen a Maxwellian with zero mean flow as equilibrium distribution function [11].

Once that a valid Poisson bracket, inherited from the original gyrokinetic bracket (11), has been found, in principle, Hamiltonian fluid models can be constructed by choosing the total energy functional (the Hamiltonian) $H(P_{s_0}, P_{s_1})$ that one would like to be conserved, and then using the definition of Hamiltonian system (6) to get the four dynamical model equations, by replacing χ with the four moments P_{s_0} and P_{s_1}.

A relevant example of Hamiltonian closure, which is also consistent with the Hamiltonian of the original gyrokinetic model, is obtained by choosing $a_s = T_s/M_s$. This corresponds to an isothermal closure, based on the equilibrium temperature, for the linearized pressure. The Poisson bracket then reads

$$\{F, G\} = \sum_{s=i,e} \int d^3x \left(\mp \frac{c}{eB} P_{s_0}[F_{s_0}, G_{s_0}] \mp \frac{c}{eB} P_{s_1}([F_{s_1}, G_{s_0}] + [F_{s_0}, G_{s_1}]) \mp \frac{c}{eB} \frac{T_s}{M_s} P_{s_0}[F_{s_1}, G_{s_1}] \right.$$
$$\left. - \frac{n_0}{M_s} \left(F_{s_1} \frac{\partial}{\partial z} G_{s_0} + F_{s_0} \frac{\partial}{\partial z} G_{s_1} \right) \right). \tag{15}$$

Analogously to Ref. [8], one can then make use of the approximate decomposition

$$g_s = \mathcal{F}_s \left(\frac{P_{s_0}}{n_0} + \frac{M_s}{T_s} v \frac{P_{s_1}}{n_0} \right), \qquad \text{for } s = i, e, \tag{16}$$

which respects the moments definition as well as the isothermal closure relation. Inserting the expression (16) into Eq. (8) yields

$$H(P_{i_0}, P_{i_1}, P_{e_0}, P_{e_1}) = \frac{1}{2}\sum_{s=i,e}\int d^3x\left(\frac{T_s}{n_0}P_{s_0}^2 + M_s\frac{P_{s_1}^2}{n_0}\right) + \frac{e}{2}\int d^3x(\Gamma_1 P_{i_0} - P_{e_0})L_0^{-1}(\Gamma_1 P_{i_0} - P_{e_0})$$
$$- \frac{e}{2c}\int d^3x(\Gamma_1 P_{i_1} - P_{e_1})L_1^{-1}(\Gamma_1 P_{i_1} - P_{e_1}), \tag{17}$$

where we made use of the common closure approximations (see, e.g. Ref. [8]) $\Gamma_1 = \Gamma_0^{1/2}$, $\int d\mathcal{W} J_0 g_i = \Gamma_1 P_{i_0}$ and $\int d\mathcal{W} J_0 v g_i = \Gamma_1 P_{i_1}$. From such closure approximations it follows also that, using Eqs. (3) and (4), the following well-known relations hold:

$$\nabla_\perp^2 A = -\frac{4\pi e}{c}n_0(\Gamma_1 u_i - u_e), \qquad n_0 e\frac{\Gamma_0 - 1}{T_i}\phi = -(\Gamma_1 n_i - n_e), \tag{18}$$

where we introduced the ion and electron density fluctuations $n_{i,e} = \int d\mathcal{W} f_{i,e}$ as well as the parallel fluid velocities $u_{i,e} = \int d\mathcal{W} v f_{i,e}/n_0$. Making use of Eqs. (18), the fluid Hamiltonian can be rewritten, in a physically more transparent way, as

$$H = \frac{1}{2}\sum_{s=i,e}\int d^3x\left(\frac{T_s}{n_0}n_s^2 + M_s n_0 u_s^2\right) - \frac{e^2 n_0}{2T_i}\int d^3x\phi(\Gamma_0 - 1)\phi + \frac{1}{8\pi}\int d^3x|\nabla_\perp A|^2. \tag{19}$$

The first and the second term in Eq. (19) account for the thermal and kinetic energy, whereas the third and the fourth term represent the electrostatic energy due to the polarization term and the magnetic energy, respectively.

Due to the symmetry of the operators $L_{0,1}^{-1}$ and Γ_1 involved in Eq. (17), one obtains the relations

$$\frac{\delta H}{\delta P_{i_0}} = \frac{T_i}{n_0}P_{i_0} + e\Gamma_1\phi, \qquad \frac{\delta H}{\delta P_{i_1}} = \frac{M_i}{n_0}P_{i_1} - \frac{e}{c}\Gamma_1 A, \tag{20}$$

$$\frac{\delta H}{\delta P_{e_0}} = \frac{T_e}{n_0}P_{e_0} - e\phi, \qquad \frac{\delta H}{\delta P_{e_1}} = \frac{M_e}{n_0}P_{e_1} + \frac{e}{c}A. \tag{21}$$

From Eq. (6), using the Poisson bracket (15), the Hamiltonian (17) and with the help of the relations (20)-(21), one can finally obtain the Hamiltonian gyrofluid model

$$\frac{\partial n_i}{\partial t} + \frac{c}{B}[\Gamma_1\phi, n_i] - \frac{n_0}{B}[\Gamma_1 A, u_i] + n_0\frac{\partial u_i}{\partial z} = 0, \tag{22}$$

$$\frac{\partial D}{\partial t} + \frac{c}{B}[\Gamma_1\phi, D] - \frac{T_i}{M_i B}[\Gamma_1 A, n_i] + \frac{T_i}{M_i}\frac{\partial n_i}{\partial z} + \frac{e n_0}{M_i}\frac{\partial \Gamma_1\phi}{\partial z} = 0, \tag{23}$$

$$\frac{\partial n_e}{\partial t} + \frac{c}{B}[\phi, n_e] - \frac{n_0}{B}[A, u_e] + n_0\frac{\partial u_e}{\partial z} = 0, \tag{24}$$

$$\frac{\partial F}{\partial t} + \frac{c}{B}[\phi, F] - \frac{T_e}{M_e B}[A, n_e] + \frac{T_e}{M_e}\frac{\partial n_e}{\partial z} - \frac{e n_0}{M_e}\frac{\partial \phi}{\partial z} = 0, \tag{25}$$

where we introduced the quantities $D = P_{i_1} = n_0(u_i + e\Gamma_1 A/M_i c)$ and $F = P_{e_1} = n_0(u_e - eA/M_e c)$, corresponding to the first order fluid momenta and which represent the parallel canonical momenta of the ion and electron fluid, respectively.

By multiplying Eq. (25) times M_e one sees that, because of terms proportional to the electron inertia, the magnetic flux A is not frozen into any fluid. Indeed, in spite of the Hamiltonian

character of the model, the presence of electron inertia violates the frozen-in condition, allowing for magnetic reconnection.

The model (22)-(25) and its Hamiltonian structure were presented in Ref. [14], but in that work the Hamiltonian structure was "guessed" a posteriori, after deriving the model from the more general gyrofluid model of Ref. [6]. The present derivation, on the other hand, originates from the gyrokinetic description, and guarantees, by construction, the Hamiltonian structure of the resulting model.

The above gyrofluid model is characterized by the presence of four Casimirs, that is functionals C, such that $\{C, F\} = 0$, for every functional F. From this definition and from Eq. (6) it follows that $dC/dt = 0$, so that Casimir are invariants of motion and consequently provide information about constraints on the dynamics, also in the nonlinear phase. The four Casimirs for the gyrofluid model can be written as $\int d^3x C_{s_\pm}$ for $s = i, e$, where

$$C_{s_\pm} = \sqrt{\frac{M_s}{T_s}} P_{s_1} \pm P_{s_0}. \tag{26}$$

The parent gyrokinetic bracket (11), on the other hand, possesses the two Casimirs $\int d^3x d\mathcal{W} g_s$, for $s = i, e$. Note that the Casimirs of the gyrofluid model are "inherited" from those of the gyrokinetic model, in the sense that they can be obtained from those by using the decomposition (16) and replacing v with plus or minus the equilibrium thermal speed $\sqrt{T_{i,e}/M_{i,e}}$.

In the two-dimensional limit $\partial/\partial_z = 0$ the Casimirs of both the gyrokinetic and the gyrofluid model extend to infinite families, given by $\int d^3x d\mathcal{W} \mathcal{G}_s(g_s)$ and $\int d^3x \mathcal{C}_{s_\pm}(C_{s_\pm})$, respectively, for $s = i, e$, where $\mathcal{G}_s$ and $\mathcal{C}_{s_\pm}$ are arbitrary functions. The relevance of the Casimirs of the gyrofluid model for two-dimensional reconnection has been shown in Ref. [16].

4. Conclusions

In this paper we have shown that the electromagnetic gyrokinetic model (1)-(4) possesses a noncanonical Hamiltonian structure. From such model we have then derived a gyrofluid model, which, by construction, also possesses a Hamiltonian structure. The resulting model corresponds to a known model [14] for magnetic reconnection. The derivation, however, is new and shows, by taking advantage of the results of Ref. [11], that the constraint of the Jacobi identity selects this model as a Hamiltonian model derived consistently from the Poisson bracket and the Hamiltonian of the parent gyrokinetic model. The Casimir invariants of the gyrokinetic parent model have been shown to be related to those of the final gyrofluid model, which were known to play a role in the cascade toward small scale characterizing collisionless reconnection. Clearly the present results possess evident limitations and they only represent a first step in a long term project aiming at deriving Hamiltonian closures for more realistic models. An obvious direction for improvement is that of including elements of tokamak geometry and physics in the Hamiltonian model. Obvious examples in this context would be toroidal geometry , magnetic, temperature and density equilibrium gradients (although the latter have already been partially treated in Ref. [12]). Perhaps even more fundamental is the problem of considering Hamiltonian closures at higher-order moments. Advanced fluid and gyrofluid models such as those of Refs. [1–8] reduced the gap between kinetic and fluid descriptions by evolving moments up to the heat fluxes, in both the parallel and perpendicular directions. A Hamiltonian derivation of three or four-moment models from kinetic theories is a natural step for the future, although the difficulty in identifying Hamiltonian closures might rapidly and nonlinearly increase when moving to higher order moments.

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