Numerical performances of low rank stap based on different heterogeneous clutter subspace estimators

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NUMERICAL PERFORMANCES OF LOW RANK STAP BASED ON DIFFERENT HETEROGENEOUS CLUTTER SUBSPACE ESTIMATORS

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ABSTRACT

Space time Adaptive Processing (STAP) for airborne RADAR fits the context of a disturbance composed of a Low Rank (LR) clutter, here modeled by a Compound Gaussian (CG) process, plus a white Gaussian noise (WGN). In such context, the corresponding LR adaptive filters used to detect a target require less training vectors than classical methods to reach equivalent performance. Unlike the classical filter which is based on the Covariance Matrix (CM) of the noise, the LR filter is based on the clutter subspace projector, which is usually derived from a Singular Value Decomposition (SVD) of a noise CM estimate. Regarding to the considered model of LR-CG plus WGN, recent results are providing both direct estimators of the clutter subspace [1][2] and an exact MLE of the noise CM [3]. To promote the use of these new estimation methods, this paper proposes to apply them to realistic STAP simulations.

Index Terms— Covariance Matrix and Projector estimation, Maximum Likelihood Estimator, Low-Rank clutter, SIRV, STAP.

I. INTRODUCTION

In Space Time Adaptive Processing (STAP) [4], the additive disturbance can be modeled by a sum of two noises: a White Gaussian Noise (WGN), due to electronics, and the so-called ground clutter, the response of the environment to the emitted signal. In side looking STAP, the clutter is known to be Low Rank (LR) structured [5] i.e. to have a singular Covariance Matric (CM) of rank $R \ll M$ (with M the dimension of the data). The rank R can be evaluated thanks to the Brennan Rule [5].

In practice, this LR structure of the disturbance can be exploited to build LR adaptive filters. The approach consists of canceling the clutter instead of performing a classical whitening the noise. Thus LR filters are not based on the estimated noise CM but on the estimated clutter subspace projector only (usually derived from an SVD of a CM estimate). LR adaptive techniques present two main advantages. Firstly, estimating the clutter subspace projector requires only $K \geq 2R$ secondary data to ensure a classical 3dB loss of the output SNR compared to optimal filtering [6], while classical filter requires $K \geq 2M$ secondary data to reach equivalent performance. Secondly, LR adaptive filters and detectors are robust to secondary data contamination by outliers [7].

Classically the LR clutter has been modeled by a correlated Gaussian noise with eigenvalues that largely exceeds the power of the WGN, leading to the Sample Covariance Matrix (SCM) as estimator of the CM and the clutter subspace. Nevertheless, the SCM is not well adapted in highly heterogeneous or impulsive clutter environment. Therefore, developing filters/detectors based on it may lead to poor performance.

To describe an heterogeneous clutter, one of the most general model is provided by the Complex Elliptically Symmetric distribution (CES) [8]. Among the general CES class, this paper will focus on the Compound-Gaussian (CG) distributions[1] that covers a large panel of well known distributions, notably Weibull and K-distribution. Eventually, the disturbance will be modeled in this paper as a LR-CG clutter plus WGN. This general model as already been used in [10][11][1][12].

The noise CM and the clutter subspace may then be estimated with Tyler's estimator [13][2]. Nevertheless, this approach presents two drawbacks. Firstly, this estimator is not the MLE of the CM in our context, and ignoring the LR property of the heterogeneous noise, as well as ignoring the additive WGN, may lead to loss of performance [15]. Secondly, the FPE requires $K > M$ secondary data to be computed, which is a problem for high-dimensional data. Moreover, this requirement does not allows to take full advantage of the LR assumption in the cases where 2R $\ll$ M.

Regarding to the considered model of LR-CG plus WGN, recent results are providing both direct estimators of the clutter subspace [1][2] and an exact MLE of the noise CM [3]. In [1] is proposed an estimator of the clutter subspace under specific hypotheses: the CM of the low-

[1]Also referred to as Spherically Invariant Random Vectors (SIRV) in the literature [9]

[2]Also known as the Fixed-Point estimator (FPE) in the complex case [14].

rank CG clutter is assumed to have identical eigenvalues, and the Probability Density Function (PDF) of the texture is assumed to be known. The assumption of known texture PDF has been relaxed in [2] by considering texture as an unknown deterministic parameter. [3] relaxes the hypothesis of identical eigenvalues of the clutter CM and introduces a parametric MLE of the clutter CM (where the parameters are the eigenvectors corresponding eigenvalues). To promote the use of these new estimation methods, this paper presents an application of them to realistic STAP simulations.

II. STAP MODEL AND LR APPROXIMATION

Typically, the radar receiver consists in an array of Q antenna elements processing P pulses in a coherent processing interval ($M = PQ$). In this framework, we assume that the received signal $\mathbf{z}$ is a complex known signal $\mathbf{d}$ corrupted by an additive disturbance $\mathbf{n}$. One also have K secondary data $\mathbf{z}_k$ which are signal free realizations of the disturbance:

$$\mathbf{z} = \mathbf{d} + \mathbf{n} \tag{1}$$

$$\mathbf{z}_k = \mathbf{n}_k \quad \text{for} \quad k \in [\![1, K]\!] \tag{2}$$

The additive disturbance is the sum of the ground clutter $\mathbf{c}$ and a thermal noise $\mathbf{g}$, model commonly used in Radar community [11][1][12][2].

$$\mathbf{n} = \mathbf{c} + \mathbf{g} \tag{3}$$

The thermal noise is modeled by a WGN of known power σ^2 i.e. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_m)$. The hypothesis of known σ^2 is made for describing a valid theoretical framework. In practice, presented results could be applied with a prior estimate of σ^2 used as its actual value. The ground clutter is an heterogeneous noise that has a different power in each cell k. The randomness of its power is induced by spatial variation in the radar backscattering. In such a situation, it is common to model this kind of clutter by a CG process [9][8]. A realization of a CG process corresponds to a Gaussian random vector multiplied the square root of a random power factor called the texture τ of Probability Density Function (PDF) f_τ. Moreover, in side looking STAP, the rank R of the clutter CM can be evaluated [5] and is verifying $R \ll M$. One has then $\mathbf{c} \sim \mathcal{CG}(\mathbf{0}, \mathbf{\Sigma}_c, f_\tau)$. With the rank R clutter CM defined by its eigendecomposition:

$$\mathbf{\Sigma}_c = \sum_{r=1}^{R} c_r \mathbf{v}_r \mathbf{v}_r^H \tag{4}$$

The whole noise covariance matrix is then defined as

$$\mathbf{\Sigma} = \sigma^2 \mathbf{I}_M + E(\tau)\mathbf{\Sigma}_c \tag{5}$$

However, in a realistic STAP application, no prior information is available on the PDF f_τ. In that case, each secondary data may be described conditionally to the texture as $(\mathbf{z}_k|\tau_k) \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_k)$, with

$$\mathbf{\Sigma}_k = \sigma^2 \mathbf{I}_M + \tau_k \mathbf{\Sigma}_c \ , \tag{6}$$

where the textures of each realizations τ_k are considered as unknown deterministic parameters.

Usual adaptive processes (filtering, detection, estimation of signal parameters) require an estimate of the noise CM $\mathbf{\Sigma}$, obtained with the set of secondary data $\{\mathbf{z}_k\}$. However, considering the previous framework, one can exploit the LR structure of the noise and cancel the clutter instead of whitening it. This approach presents an interest, notably when there are few secondary data or when they may be corrupted by outliers. Adaptive LR processes are therefore based on the following LR approximation:

$$\mathbf{\Sigma}^{-1} \sim \frac{1}{\sigma^2}\mathbf{\Pi}_c^\perp \propto \mathbf{\Pi}_c^\perp \tag{7}$$

$\mathbf{\Pi}_c^\perp$ is the projector onto the clutter subspace complementary, namely

$$\mathbf{\Pi}_c^\perp = \mathbf{I}_M - \mathbf{\Pi}_c \ , \tag{8}$$

where $\mathbf{\Pi}_c$ is the rank R projector onto the clutter subspace is constructed with the eigenvectors of the clutter CM:

$$\mathbf{\Pi}_c = \sum_{r=1}^{R} \mathbf{v}_r \mathbf{v}_r^H \tag{9}$$

Classically, an estimator of the clutter subspace projector $\hat{\mathbf{\Pi}}_c$ is derived from the SVD of an estimate of the noise CM $\hat{\mathbf{\Sigma}}$. The following section will recall the classical CM estimators and newly introduced estimators, directly derived from the considered model.

III. CM AND CLUTTER SUBSPACE ESTIMATORS

This section simply recalls the expression of the estimators that are going to be tested. For a more detailed review of their related model / properties / computation methods, we refer the reader to the associated references.

- The classical Sample Covariance Matrix (SCM), which is the MLE of the CM in a Gaussian context:

$$\hat{\mathbf{\Sigma}}_{\text{SCM}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{z}_k \mathbf{z}_k^H \tag{10}$$

  The projector estimate derived from the SVD of the SCM will be denoted $\hat{\mathbf{\Pi}}_{\text{SCM}}$.
- The Fixed Point Estimator (FPE), or Tyler's estimator[14][13], is defined only for $K > M$ as the unique solution of the fixed point equation:

$$\hat{\mathbf{\Sigma}}_{\text{FPE}} = \frac{M}{K}\sum_{k=1}^{K} \frac{\mathbf{z}_k \mathbf{z}_k^H}{\mathbf{z}_k^H \hat{\mathbf{\Sigma}}_{\text{FPE}}^{-1} \mathbf{z}_k} \tag{11}$$

  The projector estimate derived from the SVD of the FPE will be denoted $\hat{\mathbf{\Pi}}_{\text{FPE}}$.

- The Shrinkage-FPE (SFPE), also known as Diagonnaly-Loaded FPE [16][17][18], defined for $\beta \in]0,1]$ by the fixed point equation:

$$\hat{\mathbf{\Sigma}}_{\text{S-FPE}}(\beta) = (1-\beta)\frac{M}{K}\sum_{k=1}^{K}\frac{\mathbf{z}_k\mathbf{z}_k^H}{\mathbf{z}_k^H\hat{\mathbf{\Sigma}}_{\text{S-FPE}}^{-1}(\beta)\mathbf{z}_k} + \beta\mathbf{I}_M \quad (12)$$

The projector estimate derived from the SVD of the S-FPE will be denoted $\hat{\mathbf{\Pi}}_{\text{S-FPE}}$.
- Under the assumption of equals eigenvalues of the clutter CM, the approached MLE of the clutter subspace (A-MLE) [2], denoted $\hat{\mathbf{\Pi}}_{\text{A-MLE}}$, is the projector onto the subspace defined by the R strongest eigenvectors of the matrix:

$$\hat{\mathbf{R}} = \sum_{k=1}^{K}\frac{\hat{\tau}_k}{\sigma^2+\hat{\tau}_k}\mathbf{z}_k\mathbf{z}_k^H \ , \quad (13)$$

with the estimated textures:

$$\hat{\tau}_k = \begin{cases} ||\hat{\mathbf{\Pi}}_c\mathbf{z}_k||^2/R - \sigma^2 & \text{if } ||\hat{\mathbf{\Pi}}_c\mathbf{z}_k||^2 > R\sigma^2 \\ 0 & \text{else} \end{cases} \quad (14)$$

This estimator's expression stands when there is no prior information on the texture PDF, which is more realistic for a STAP application. However, the case of known texture PDF is treated in [1].
- The exact MLE of the clutter subspace [3], denoted $\hat{\mathbf{\Pi}}_{\text{MLE}}$, is the projector onto the subspace defined as the subspace satisfying:

$$\hat{\mathbf{\Pi}}_{\text{MLE}}\hat{\mathbf{M}} = \hat{\mathbf{M}}^H\hat{\mathbf{\Pi}}_{\text{MLE}} \quad (15)$$

with the definition of the rank 1 projectors $\hat{\mathbf{\Pi}}_r = \hat{\mathbf{v}}_r\hat{\mathbf{v}}_r^H$, and with:

$$\hat{\mathbf{M}} = \sum_{r=1}^{R}\hat{\mathbf{M}}_r\hat{\mathbf{\Pi}}_r \ , \quad (16)$$

$$\hat{\mathbf{M}}_r = \sum_{k=1}^{K}\frac{\hat{\tau}_k\hat{c}_r}{\sigma^2+\hat{\tau}_k\hat{c}_r}\mathbf{z}_k\mathbf{z}_k^H \ , \quad (17)$$

At this point, there is no direct way to obtain closed form for the projector MLE. However, [3] proposes an Ad-Hoc iterative procedure that increases the likelihood at each step. This algorithm requires to alternatively compute the parameters $\{\tau_k, c_r\}$ and the clutter subspace projector $\hat{\mathbf{\Pi}}_c$. The estimated textures and clutter CM eigenvalues (that are not true MLE) are given by:

$$\hat{\tau}_k = \max\left(\frac{||\hat{\mathbf{\Pi}}_c\mathbf{z}_k||^2}{R} - 1 \ , \ 0 \ \right) \quad (18)$$

$$\hat{c}_r = R\frac{\sum_{k=1}^{K}\max(||\hat{\mathbf{\Pi}}_r\mathbf{z}_k||^2 - 1 \ , \ 0)}{\sum_{k=1}^{K}\max(||\hat{\mathbf{\Pi}}_c\mathbf{z}_k||^2 - R \ , \ 0)} \quad (19)$$

And the projector can be evaluated iteratively using the SVD of:

$$\hat{\mathbf{M}}^{(n)} = \sum_{r=1}^{R}\hat{\mathbf{M}}_r\hat{\mathbf{\Pi}}_r^{(n)} = \mathbf{U}^{(n)}\mathbf{D}^{(n)}\mathbf{V}^{H(n)} \ , \quad (20)$$

and the update:

$$\hat{\mathbf{\Pi}}_c^{(n+1)} = \mathbf{U}^{(n)}\mathbf{U}^{H(n)} \quad (21)$$

The next section will recall LR adaptive STAP filtering principle and the application of the different presented estimators to it.

IV. LR ADAPTIVE FILTERING

The optimal STAP filter is [4]

$$\mathbf{w}_{opt} = \mathbf{\Sigma}^{-1}\mathbf{d} \quad (22)$$

In the context of a LR clutter, it is well known that a correct sub-optimal filter is [19][20][21]:

$$\mathbf{w}_{lr} = \mathbf{\Pi}_c^{\perp}\mathbf{d} = (\mathbf{I}_m - \mathbf{\Pi}_c)\mathbf{d} \ , \quad (23)$$

where $\mathbf{\Pi}_c$ is the projector onto the clutter subspace, described in (9). In practice, this projector is unknown and have to be estimated using the secondary data $\{\mathbf{z}_k\}$. The adaptive filter is then:

$$\hat{\mathbf{w}}_{lr} = \hat{\mathbf{\Pi}}_c^{\perp}\mathbf{d} = \left(\mathbf{I}_m - \hat{\mathbf{\Pi}}_c\right)\mathbf{d}, \quad (24)$$

with $\hat{\mathbf{\Pi}}_c$, a given estimate of the clutter subspace projector. Of course, the performance of the LR filters will directly rely on the accuracy of the estimation of $\mathbf{\Pi}_c$.

To evaluate the performance of a sub-optimal filter, the SINR-Loss [4], denoted ρ is currenlty used: it is the mean ratio between the $SINR_{out}$, computed for $\hat{\mathbf{w}}_{lr}$, and $SINR_{max}$ computed for the optimal filter $\mathbf{w} = \mathbf{\Sigma}^{-1}\mathbf{d}$. For an estimate of the clutter subspace $\hat{\mathbf{\Pi}}_c$, the SINR-Loss expression is given by :

$$\rho_{\hat{\mathbf{\Pi}}_c} = \frac{SINR_{out}}{SINR_{max}} = \frac{(\mathbf{d}^H\hat{\mathbf{\Pi}}_c^{\perp}\mathbf{d})^2}{\mathbf{d}^H\hat{\mathbf{\Pi}}_c^{\perp}\mathbf{\Sigma}\hat{\mathbf{\Pi}}_c^{\perp}\mathbf{d}} \quad (25)$$

We consider the following STAP configuration. The number Q of sensors is 8 and the number P of coherent pulses is also 8. The center frequency and the bandwidth are respectively equal to $f_0 = 450$ MHz and $B = 4$ MHz. The radar velocity is 100 m/s. The inter-element spacing is $d = \frac{c}{2f_0}$ (c is the celerity of light) and the pulse repetition frequency is $f_r = 600$ Hz. The clutter rank is computed from Brennan rule [5] and is equal to $R = 15 \ll 64$, therefore, the low rank assumption is valid. The texture PDF is a Gamma law of shape parameter $\nu = 0.1$ and scale parameter $\frac{1}{\nu}$. Mean SINR-Loss are computed from 10^4 Monte Carlo realizations.

Figure 1 and 2 show the SINR-Loss evolution with respect to K for LR-STAP filters based on different clutter subspace estimators. We notice that the LR-STAP filter built from MLE and A-MLE outperforms the others ones. Estimation is especially improved for low CNR and highly heterogeneous clutter, as shown in figure 2. This result could have been inferred since MLE and A-MLE are both defined as the

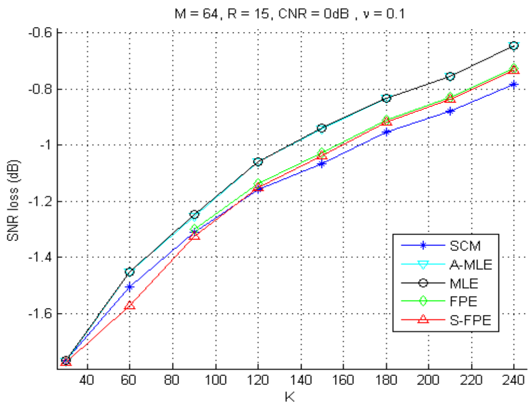

Fig. 1. SINR-Loss versus K. $M = 64$, $R = 15$, $\nu = 0.1$, CNR= 0dB.

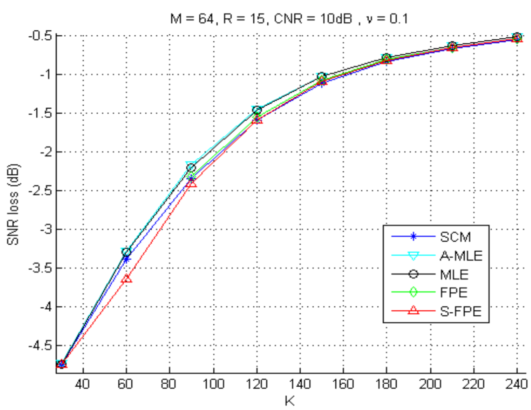

Fig. 2. SINR-Loss versus K. $M = 64$, $R = 15$, $\nu = 0.1$, CNR= 10dB.

R dominant eigenvectors of SCMs of the data $\mathbf{z}_k$ scaled by factors that are increasing with the textures τ_k, i.e. the power of the clutter of the realization k. With these scaling factors, realizations that contain more power in the subspace of interest are given more significance in the estimation process, which may be useful if the clutter subspace is not well represented over the data set (for example with a low CNR or for very impulsive CG's).

Figure 3 and 4 show the mean SINR-Loss versus V, the speed of the target. The same conclusions as the previous simulations can be drawn: MLE and A-MLE are slightly improving the resolution of the LR STAP.

Figure 5 and 6 show the mean SINR-Loss versus ν, the parameter of the texture's Gamma law. This figure allows to study the comportment of estimators facing different level of heterogeneity of the clutter. The closer to 0 ν is, the more heterogeneous the clutter is. On the contrary, if ν increases, the clutter will have a more Gaussian behavior. One can notice that the MLE and A-MLE outperform the state of art when $\nu \sim 0$ and tend to have performance equivalent as the SCM (theoretical MLE) when the noise is Gaussian.

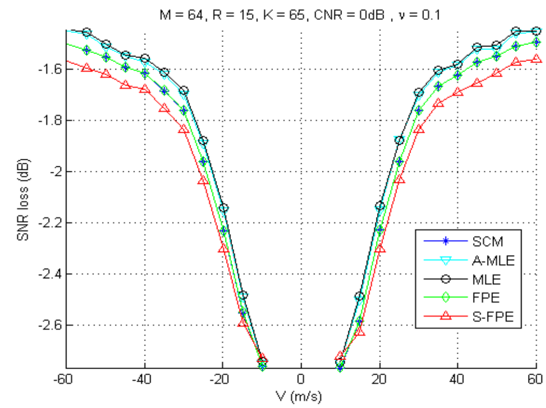

Fig. 3. SINR-Loss versus V. $M = 64$, $R = 15$, $K = M + 1$, $\nu = 0.1$, CNR= 0dB.

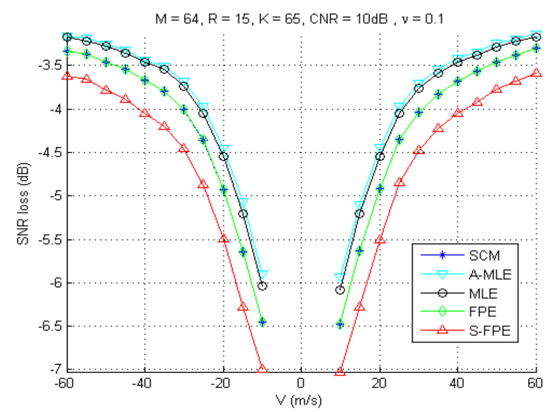

Fig. 4. SINR-Loss versus V. $M = 64$, $R = 15$, $K = M + 1$, $\nu = 0.1$, CNR= 10dB.

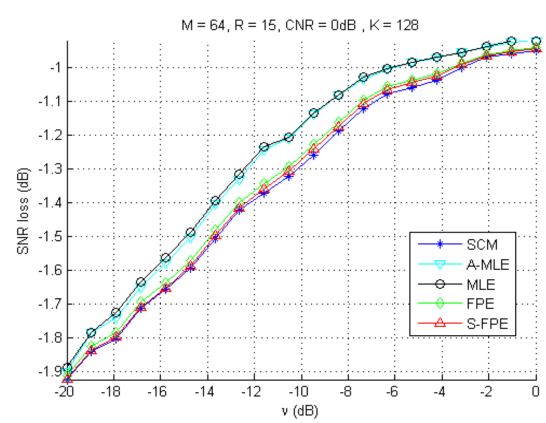

Fig. 5. SINR-Loss versus ν. $M = 64$, $R = 15$, $K = 2M$, CNR= 0dB.

This property illustrates robustness to heterogeneity of these estimation methods.

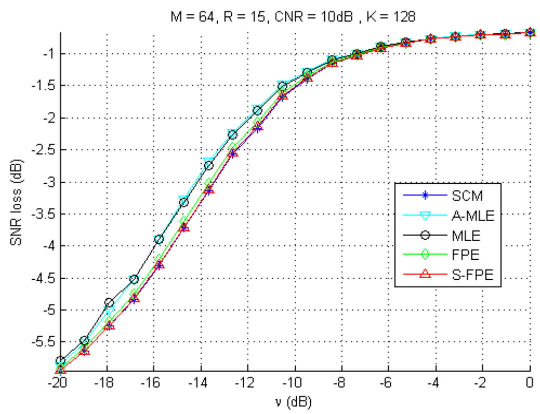

Fig. 6. SINR-Loss versus ν. $M = 64$, $R = 15$, $K = 2M$, CNR= 10dB.

V. CONCLUSION

Classically, the clutter subspace estimator can be derived from the SVD of an estimator of the CM. However, [1][2] show that, in the considered framework of LR-Compound Gaussian plus white Gaussian noise, the clutter subspace projector MLE is derived from a matrix that is not an estimate of the CM. This intermediary matrix is the SCM of the data scaled by a factor that gives more significance to samples that have strong power into the subspace of interest.

In this paper, these new estimations methods [2][3] have been applied to LR STAP simulations. Results illustrates that they could improve LR adaptive processes. Indeed, in standard cases (high CNR and weak heterogeneity), they will preform equivalently as the SCM. However, in some specific scenarios, involving data dropouts, weak clutter samples or limited sample support (modeled by low CNR and/or very impulsive clutter) the performances are improved.

It is also interesting to point out that clutter subspace estimates derived from robust estimation methods (FPE,S-FPE) are not necessarily performing a better estimation than the simple SCM for the considered model, even it involves non Gaussian noise. Indeed, these robust estimates of the CM and not necessarily robust from the point of view of estimating the clutter subspace. Nevertheless, FPE and S-FPE could bring robustness to data corruption by outliers, which have not been treated in this paper, but is investigated in [23].

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