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# Broadcast Channel Feedback in Multiple-Antenna Transmitter Cooperation Networks: Accuracy or Consistency?

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**Abstract**—We consider the use of broadcast feedback of channel state information (CSI) in multi-cell cooperative networks. The salient feature of this form of feedback lies in the exploitation of the broadcast nature of the wireless medium in order to convey CSI to all transmitters in contrast with other forms of feedback exchange mechanisms relying on inter-transmitter signaling. We show that this particular design approach for feedback naturally leads to a scalable partially distributed precoding framework for multi-cell MIMO networks. A model based on channel quantization is provided to assess the performance of such a system. The analysis exhibits a novel trade-off between feedback consistency and accuracy which is specific to the multi-transmitter cooperation scenario. The optimal trade-off is explored through analysis and numerical simulations.

## I. INTRODUCTION

The use of transmitter-based cooperation in interference-limited wireless networks promises substantial gains. In particular the joint design of MIMO precoders at the various transmitters offers a powerful approach to interference avoidance, in the context of multicell/network MIMO [1], or interference coordination, e.g. using alignment strategies [2] [3]. However it is well known that the benefits of multiple antenna transmitter cooperation go at the expense of requiring channel state information (CSI) at *all* the transmitters. In practice current standardized systems operating in frequency division duplex FDD bands allow for a limited rate feedback channel to convey channel information from a given receiver back to its serving transmitter (Serving eNodeB in the 3GPP terminology) alone. In a second step, CSI is exchanged among the various transmitters over a backhaul signaling link [4]. Unfortunately, this approach has three drawbacks. First it is suitable for networks having a pre-existing backhaul signaling infrastructure, less so for ad-hoc deployments (e.g. HetNets etc.). Second, inter-transmitter information exchange is not easily scalable as the network grows dense. Third, this approach fails to exploit the fundamental broadcast nature of wireless propagation. The proposed broadcast feedback of CSI

has the capability to overcome the drawbacks of CSI exchange over the backhaul.

In this paper, we study the performance of downlink transmitter MIMO cooperation (with shared data messages) in the context of a broadcast feedback design, i.e. where a receiver feeds back quantized CSI [5] [6] [7] to *whichever* transmitter that can hear the feedback message over the uplink. Since not all transmitters will be able to successfully decode the feedback data, this scenario naturally leads to a *distributed* precoder optimization problem, i.e. one where individual transmitters must make precoding decisions on the basis of individual (local) channel state information. As a first step to solve this very challenging problem, we form distinct cooperation clusters. As a nice feature of our design, the cooperation clusters need not be computed but instead are a direct by-product of the broadcast feedback scheme. We present a simple algorithm for partially distributed precoding over the formed clusters and evaluate its performance analytically and over monte-carlo simulations. Our results exhibit an interesting trade-off between *consistency* and *accuracy* in the channel quantization design. The intuition behind this trade-off is that a finely quantized channel feedback message will provide accurate CSI to a small subset of transmitters nearby the terminal but may not be decoded at other transmitters. In contrast, a coarsely quantized channel vector will provide useful (albeit not very precise) CSI at *most* of the transmitters, allowing them to (i) form a bigger cooperation cluster, and (ii) reach a consistent joint precoding decision.

## II. SIGNAL AND SYSTEM MODEL

We consider an FDD wireless communication network consisting of  $M$  base stations and a total of  $K$  active users. Each base station is equipped with  $J$  antennas and the user terminals are assumed to have a single antenna. We assume that all or a subgroup of base stations serve the users in a Network MIMO fashion, also referred to as Joint Processing CoMP in the 3GPP terminology.

All cooperating base stations are required to have the knowledge of downlink channels of users to be served for

The present work was carried out within the framework of Celtic-Plus Self-organized Heterogeneous Advanced Radlo Networks Generation (SHARING) project

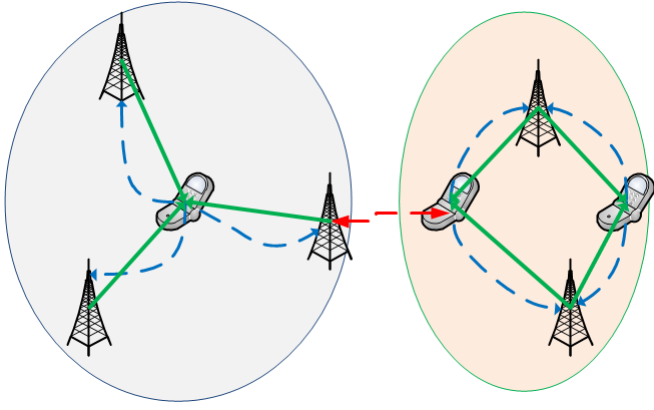


Fig. 1: Example of joint CSIT and cooperation cluster. The dashed arrows represent successfully decoded broadcast feedback on the uplink. The solid (green) arrows represent BS serving users in the downlink, belonging to one of two joint CSIT clusters.

implementing CoMP. Since we consider an FDD network, channel reciprocity is not valid except in terms of average SNR. We assume that each user in the network is able to estimate its downlink channel from all the  $M$  base stations through a training phase. This channel is quantized and then fed back to the base stations using non-interfering feedback links. The channel estimated by the  $k^{\text{th}}$  user is denoted as  $\mathbf{h}_k \in \mathbb{C}^{1 \times MJ}$ .

$$\mathbf{h}_k = [\mathbf{h}_{k1}, \mathbf{h}_{k2} \dots \mathbf{h}_{kM}] \quad (1)$$

where,  $\mathbf{h}_{km} \in \mathbb{C}^{1 \times J}$  is a row vector corresponding to the channel from  $m^{\text{th}}$  base station to the  $k^{\text{th}}$  user. The collection of channels experienced by all the users in the network is represented by the channel matrix  $\mathbf{H} \in \mathbb{C}^{K \times MJ}$ .

$$\mathbf{H} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \dots, \mathbf{h}_k^T, \dots, \mathbf{h}_K^T]^T \quad (2)$$

#### A. Clustered Network MIMO

We consider Network MIMO applied through possibly disjoint clusters where the number of clusters may vary from the ideal single cluster case (whole network cooperation) to the extreme case of  $M$  clusters (ie, no cooperation between base stations). The number of clusters, denoted by  $N$ , depends on the channel state information being successfully decoded at the base station side. Let  $\mathcal{S}_n^{bs}$  be the  $n^{\text{th}}$  ( $n \in 1 : N$ ) disjoint cluster of base stations and  $\mathcal{S}_n^u$  be the set of scheduled (active) users in the same cluster.  $\mathcal{S}_n$  refers to the whole set of users and base stations in the  $n^{\text{th}}$  cluster. We assume that a user cannot be served simultaneously by two base stations located in different BS clusters.

The stacked vector of received signals for the users in the  $n^{\text{th}}$  cluster  $\mathcal{S}_n$  is given by the expression :

$$\mathbf{y}(\mathcal{S}_n^u) = \mathbf{H}(\mathcal{S}_n^u, \mathcal{S}_n^{bs})\mathbf{u}(\mathcal{S}_n^{bs}) + \sum_{t \neq n} \mathbf{H}(\mathcal{S}_n^u, \mathcal{S}_t^{bs})\mathbf{u}(\mathcal{S}_t^{bs}) + \boldsymbol{\eta}(\mathcal{S}_n^u) \quad (3)$$

where  $\mathbf{H}(\mathcal{S}_n^u, \mathcal{S}_n^{bs})$  is the channel sub-matrix corresponding to the users and base stations in the cluster of interest,  $\mathbf{u}(\mathcal{S}_n^{bs})$  is the vector of transmit precoded symbols at each base station antennas in the same cluster.  $\sum_{t \neq n} \mathbf{H}(\mathcal{S}_n^u, \mathcal{S}_t^{bs})\mathbf{u}(\mathcal{S}_t^{bs})$  represents the inter-cluster interference and  $\boldsymbol{\eta}$  is the thermal noise at the receivers modeled as iid normalized gaussian RV  $\mathcal{CN}(0, \sigma_n^2)$ .

#### B. Zero Forcing Precoder

Zero-forcing (ZF) beamforming [8] is a convenient precoding solution for tractability and simplicity of implementation. It will be assumed in this paper. It performs very well in the high signal-to-noise-ratio (SNR) regime or when the number of users is sufficiently large (taking advantage of the multi-user diversity), and is known to provide full degrees of freedom [9].

If all the cooperating base stations have perfect CSIT from the users, ZF beamforming can perfectly nullify the intra-cluster interference. As obtaining perfect CSIT is not feasible in most practical cases, we will focus on the quantized CSIT obtained through limited (finite rate) feedback channel [5]. The limited feedback will introduce imperfection at the precoding scheme and results in intra-cluster interference.

### III. DISTRIBUTED BROADCAST FEEDBACK

#### A. Quantized Broadcast Channel Feedback

In current wireless standards, a feedback mechanism is designed which allows for a terminal to convey downlink channel information back to the serving "home" base station only. Yet, if a neighboring BS interferes with the terminal on the downlink, it is likely that the same terminal can also be heard at the interfering base station on the uplink as well due to path loss reciprocity. In this paper we are interested in exploiting the broadcast nature of the wireless channel in designing the feedback, in allowing every BS to overhear the feedback sent by any terminal over the uplink. We further assume that some orthogonality is maintained between the feedback messages sent by the multiple terminals, hence no interference is considered on the feedback channel.

We consider that each UE estimates and quantizes the aggregated DL channels and feeds back this quantized version to all the base stations. The channel vector of any user is quantized over a  $B$  bit codebook before being transmitted over  $\beta_{fb}$  channel uses, where  $\beta_{fb}$  takes an arbitrary value.

A direct consequence of the broadcast feedback design is that not all base stations may be able to decode the uplink feedback message as a result of the possibly larger base stations-terminal distance or the fading events. However, we point out that the success (failure) of decoding the uplink broadcast feedback messages serve as a good indication of how strong (weak) the interference is from a given BS. In this paper the success of feedback message detection is modeled by the following comparison with the uplink Shannon capacity  $C_{mk}$  from user  $k \in [1, K]$  to base station  $m \in [1, M]$ .

$$C_{mk} = \beta_{fb} \cdot \log\left(1 + \frac{|\mathbf{h}_{mk}^U|^2 P}{\sigma_n^2}\right) \quad (4)$$

where  $\beta_{fb}$  is the number of uplink channel uses for feeding back  $B$  bits,  $\mathbf{h}_{mk}^U \in \mathbb{C}^{1 \times J}$  is the uplink channel from  $k^{th}$  UE to  $m^{th}$  BS,  $P$  is the transmit power from UE.  $\sigma_n^2$  is the thermal noise power at the base station, assume to be equal to that at the UE for ease of notation.

Our feedback model is simplified into a simple comparison with the offered capacity on the uplink channel and goes as follows:

- if  $C_{mk} < B$  then BS  $m$  is not able to decode the feedback sent by terminal  $k$  (no feedback) and will not attempt to serve it.
- if  $C_{mk} \geq B$  then BS  $m$  decodes the feedback from terminal  $k$  without error.

We are interested in finding the best possible disjoint cooperation clusters naturally arising out of the CSI feedback pattern in order to perform JP-CoMP.

### B. A Novel Dynamic Clustering Algorithm

The knowledge of a given user channel at one particular base station clearly conditions the ability of the base station to serve (jointly with other cooperating bases) that user. We consider a clustering approach where cluster formation is dynamic and based on the availability of channel knowledge, hence the notion of CoMP cluster and joint CSIT cluster will coincide. We define a joint CSIT cluster as follows:

**Definition:** A *Joint CSIT Cluster*  $\mathcal{S}_n$  is defined as a disjoint subgroup of users and base stations such that feedback from all users in the cluster has been successfully decoded by all the base stations in the cluster.

From now on, the MIMO cooperation clusters will be identical to the obtained joint CSIT clusters. Furthermore, we assume a basic signaling mechanism by which the clusters' identities are exchanged among the base stations, hence our scheme is *partially distributed*.

We define a feedback detection index  $v_{mk}$  corresponding to the success of feedback decoding in the uplink channel from the  $i^{th}$  UE to the  $m^{th}$  BS.

$$v_{mk} = \begin{cases} 1, & \text{if } C_{mk} \geq B, \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

Hence  $v_{mk} = 1$  implies that  $m^{th}$  BS knows the channel of  $k^{th}$  UE.

The feedback index vector for a user  $k$ ,  $\mathbf{v}_k \in \{0, 1\}^{[M \times 1]}$  shows which base stations could decode its CSI. Feedback index matrix  $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_K]^T$  is set of feedback vectors of all  $K$  users in the network.  $\mathbf{V}$  carry implicit information about the downlink interference pattern due to reciprocity of slow-fading coefficients, hence can be used for MIMO clustering. Although the quantized broadcast feedback leads to a distributed framework, in this work, we focus on a partially distributed scheme. We use the following algorithm to find the largest sets of BS and users forming joint CSIT clusters.

- 1) Find permutations  $\pi^u$  and  $\pi^{bs}$  such that,  $\pi^u \mathbf{V} \pi^{bs}$  has a '1-block' diagonal structure.

$$\pi^u \mathbf{V} \pi^{bs} = \begin{pmatrix} \mathbf{V}_1 & & \cdots & \\ & \mathbf{V}_2 & & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ & & \cdots & \mathbf{V}_N \end{pmatrix}$$

where a '1-block'  $\mathbf{V}_n$  is a  $M_n \times K_n$  matrix containing only ones. We select this matrices so that

$$M_n \cdot J \geq K_n \quad \forall n \in [1, N]$$

and where  $K_n$  and  $M_n$  denote the number of rows and columns in  $\mathbf{V}_n$ . Note that  $\sum M_n = M$  and  $\sum K_n = K$ .

- 2) Extract the blocks to strictly block diagonal matrix

$$\mathbf{V}_{cluster} = \text{diag}(\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n, \dots, \mathbf{V}_N)$$

Assign the user indices and BS indices corresponding to  $n^{th}$  1-block  $\mathbf{V}_n$  to  $\mathcal{S}_n \forall n \in [1, N]$ .

### C. Partially Distributed Precoding Algorithm

Once the Joint CSIT clusters have been identified, we apply network MIMO using ZF beamformer in each of the clusters. ZF precoder for the  $n^{th}$  cluster  $\mathcal{S}_n$  is found through:

$$\mathbf{W}_n = \mathbf{H}(\mathcal{S}_n)^H (\mathbf{H}(\mathcal{S}_n) \mathbf{H}(\mathcal{S}_n)^H)^{-1} / \mu_n \quad (6)$$

where,  $\mathbf{H}(\mathcal{S}_n) \in \mathbb{C}^{M_n \times K_n}$  is the submatrix containing the downlink channels of users and base stations assigned to the joint CSIT cluster  $\mathcal{S}_n$ .  $\mu_n$  depends on the power constraint for the base stations in  $\mathcal{S}_n$  and is defined as,

$$\mu_n = \|\mathbf{H}(\mathcal{S}_n)^H (\mathbf{H}(\mathcal{S}_n) \mathbf{H}(\mathcal{S}_n)^H)^{-1}\| \cdot \frac{1}{\sqrt{M_n \cdot P}} \quad (7)$$

where,  $P$  is the average power constraint at each BS.

### D. Ergodic Achievable Rate

For the purpose of identifying a suitable operational regime for the number of feedback bits  $B$ , we derive an approximation of the ergodic sum rate below. If  $\gamma_i$  is the SINR experienced by  $i^{th}$  UE belonging to an arbitrary cluster  $\mathcal{S}_n$ , the average rate achieved per user with the proposed scheme can be written as,

$$R_i = \mathbb{E}_h [\log_2(1 + \gamma_i)] \quad (8)$$

where,

$$\gamma_i = \frac{|\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{ij}|^2}{\sum_{k \neq i} \left( |\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}| + \sum_{j \notin \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}|^2 \right) + \sigma_n^2} \quad (9)$$

$\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}$  correspond to the intra-cluster interference (ICI) and  $\sum_{j \notin \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}$  correspond to the other-cluster interference (OCI).

### E. Rate Lower Bound

Since the derivation of a closed form expression for the rate is challenging in general, we limit ourselves to an analysis within the context of a dense network where the path loss coefficients are roughly the same from any user to any BS. Furthermore, we place ourselves in the case of single antenna BS and hence, clustering algorithm selects an equal number of base stations and users in a cluster. Finally, we assume that the average cluster size as a function of  $B$  can be modeled through repeated experiments. Clearly, in the simulations section, the equal path loss assumption is relaxed.

**Proposition** : The *Ergodic achievable rate* of a user, for a given feedback bits  $B$  and signal to noise ratio  $P$  can be lower-bounded by,

$$R_i \geq \log_2(P \cdot \mathcal{L}^2) - \log_2 \left( 1 + \left( 2^{\frac{-B}{M-1}} + (M - \mathbb{E}(M_n))^2 \right) \cdot P \cdot \mathcal{L}^2 \right) \quad (10)$$

where,  $\mathcal{L}$  is the pathloss,  $\mathbb{E}(M_n)$  is the average cluster size.

*Proof*: See Appendix.

### F. The Accuracy vs Consistency trade-off

The distributed broadcast feedback model exhibits an interesting novel trade-off between feedback *accuracy* and *consistency*. From (10), the trade-off is interpreted as follows. If the channels are quantized finely ( $B$  large), this will favor the accuracy of feedback at the base station which are closest to the terminal, but will result in some base stations not being able to obtain CSIT and hence the cluster size will reduce. This will increase other-cluster interference(OCI) but reduce intra-cluster interference(ICI). On the other hand a coarser quantization ( $B$  small) will lead to a consistent (yet less accurate) estimation of the channel state at *most* base stations. This will increase ICI due to imperfection in the precoder design but reduce OCI. This points to an existence of an optimum number of bits,  $B^{opt}$  by which if we quantize the CSI will yield higher sum rate, which is confirmed from our simulations below.

## IV. NUMERICAL RESULTS

In this section, the performance of the proposed broadcast feedback scheme along with the dynamic clustering is evaluated via monte carlo simulations. We first validate the rate lower bound expression (10) for simplified network and proceed to a general network layout.

### A. Validation of the Rate Approximations

Here we assess the tightness of the ergodic rate lower bound in (10). We assume that users are affected by pathloss which is same for all the users, and fast fading. The variation of average cluster size as a function of number of feedback bits  $B$  is shown in Fig. 2 for different values of average SNR. The cluster size decreases, although not linearly, wrt  $B$ . We use the result from this simulation to validate the rate lower bound.

Fig. 3 compares the analytical results for the rate lower bound derived in (10) with the simulated performance for various SNR values. There is an optimum feedback bits  $B$

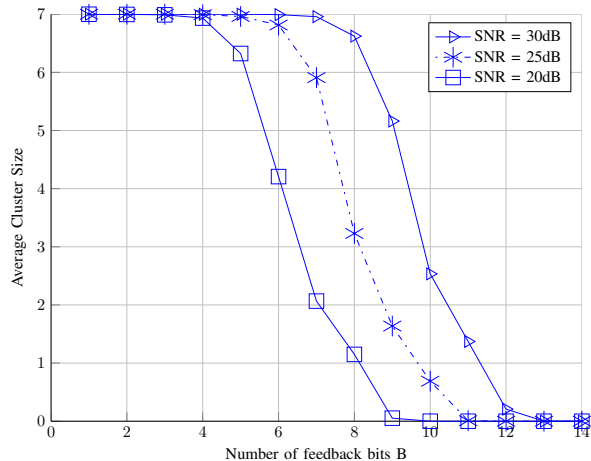


Fig. 2: Cluster size as a function of  $B$  in a 7 cell equal pathloss network.

which depending on the network settings as seen from the figure.

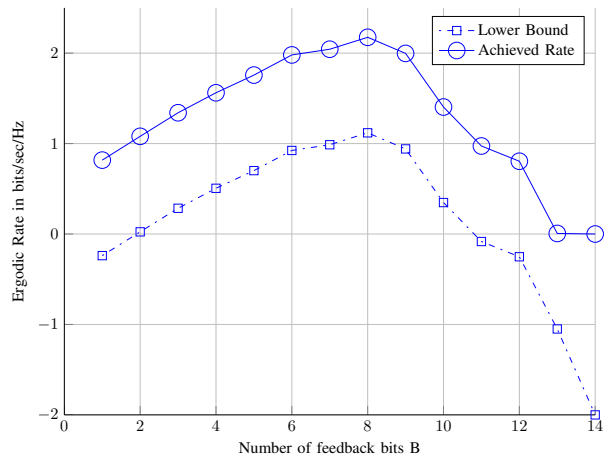


Fig. 3: Ergodic rate lower-bound validation for  $SNR = 30dB$  for a 7 cell equal pathloss network.

### B. Simulation Result for General Network Topology

In this section, we show that the behaviour of the broadcast feedback can be generalized to non-equal pathloss case for 19 cell network (two-tier). Here we consider the BS-s are located in a hexagonal lattice, with users randomly located around each BS. In such a scenario, the users are affected by distant-dependent pathloss and fast fading.

Fig. 4 shows that there is an optimum number of bits  $B^{opt}$  for the network.  $B^{opt}$  increases with average SNR of users. When users in the cells are experiencing better channel (higher average SNR), the number of feedback bits can be increased to get better downlink transmission rates, as confirmed from this simulation.

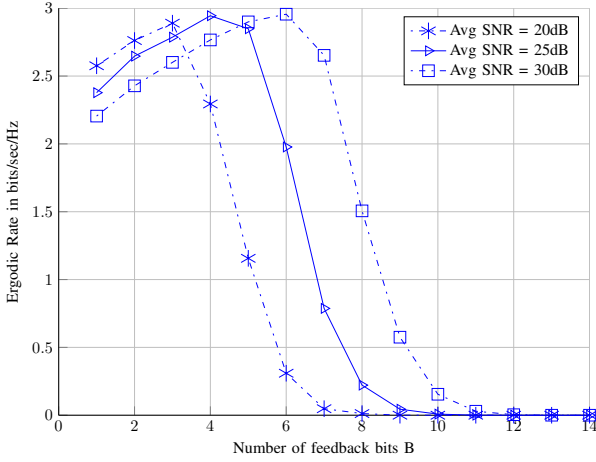


Fig. 4: Ergodic rate as a function of B, showing an optimum feedback bits behaviour for a 19 cell network

## V. CONCLUSIONS

We have proposed the concept of broadcast feedback for use in multi-cell CoMP systems. We have shown that this particular design approach naturally leads to a distributed precoding framework for multi-cell MIMO networks. Quantized broadcast channel state feedback leads to a novel trade-off between feedback *consistency* and *accuracy* which is specific to the multi-transmitter cooperation scenario. This broadcast feedback strategy leads to a scalable implementation for dense multi-cell CoMP scenarios.

## APPENDIX

### Proof of Proposition :

From (8) and (9), we can approximate  $1 + \gamma_i$  to  $\gamma_i$  at high SINR region. As our clustering approach reduces the interference, this is a good assumption at high SNR. We can rewrite the rate expression as,

$$R_i \approx \mathbb{E}_h[\log_2(\gamma_i)] \quad (11)$$

where,

$$\begin{aligned} \gamma_i &= \frac{|\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{ij}|^2}{1 + \sum_{k \neq i} \left( |\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}| + |\sum_{j \notin \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}|^2 \right)} \\ &\stackrel{(a)}{\approx} \frac{|\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{ij}|^2}{1 + \sum_{k \neq i} \left( |\sum_{j \in \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}|^2 + |\sum_{j \notin \mathcal{S}_n} \mathbf{h}_{ij} \mathbf{w}_{kj}|^2 \right)} \end{aligned}$$

where, (a) assumes that intra-cluster interference term and inter-cluster interference are uncorrelated as they are composed of channels and precoding vectors two disjoint clusters.

Using Jensen's inequality for logarithmic (concave) function, the ergodic rate expression can be lowerbounded by,

$$\begin{aligned} R_i &\geq \log_2(P \cdot \mathcal{L}^2) - \log_2 \left[ 1 + \sum_{k \neq i} \mathbb{E}_h \left( \sum_{j \in \mathcal{S}_n} |\mathbf{h}_{ij} \mathbf{w}_{kj}|^2 \right) \right. \\ &\quad \left. + \sum_{k \neq i} \mathbb{E}_h \left( \sum_{j \notin \mathcal{S}_n} |\mathbf{h}_{ij} \mathbf{w}_{kj}|^2 \right) \right] \\ &= \log_2(P \cdot \mathcal{L}^2) - \log_2 [1 + \overline{ICI} + \overline{OCI}] \end{aligned}$$

where,  $\overline{ICI}$  and  $\overline{OCI}$  denote the average intra-cluster interference and other-cluster interference powers respectively. We first derive the expression for ICI using the similar approach followed in [6], where a beta random variable approximation is used to model it.

$$\overline{ICI} = \frac{\mathbb{E}\{|\mathbf{h}_k|^2\}}{M-1} \cdot 2^B \cdot \beta \left( 2^B, \frac{M}{M-1} \right) \quad (12)$$

$$\stackrel{(b)}{\leq} 2^{\frac{-B}{M-1}} \cdot P \cdot \mathcal{L}^2 \quad (13)$$

where (b) results from the upperbound on beta function.

The average other-cluster interference  $\overline{OCI}$  is due the interference from base stations (transmitting to different set of users) another cluster other than the cluster of interest. Since there are  $M - \mathbb{E}(M_n)$  users and base stations outside  $\mathcal{S}_n$ , we remove the two summations in (12) corresponding to  $\overline{OCI}$  and express it as,

$$\overline{OCI} = (M - \mathbb{E}(M_n))^2 \mathbb{E}_h(|\mathbf{h}_{ij} \mathbf{w}_{kj}|^2) \quad (14)$$

where,

$$\mathbb{E}_h(|\mathbf{h}_{ij} \mathbf{w}_{kj}|^2) = \mathbb{E}_h\{(\mathbf{h}_{ij} \mathbf{w}_{kj})^H (\mathbf{h}_{ij} \mathbf{w}_{kj})\} \quad (15)$$

$$= \mathbb{E}_h\{\mathbf{w}_{kj}^H \mathbf{h}_{ij}^H \mathbf{h}_{ij} \mathbf{w}_{kj}\} \quad (16)$$

$$\stackrel{(c)}{=} \mathbf{w}_{kj}^H \mathbb{E}_h\{\mathbf{h}_{ij}^H \mathbf{h}_{ij}\} \mathbf{w}_{kj} \quad (17)$$

$$\stackrel{(d)}{=} \mathbf{w}_{kj}^H \mathbf{w}_{kj} \cdot \mathcal{L}^2 \stackrel{(e)}{\leq} P \cdot \mathcal{L}^2 \quad (18)$$

where (c) follows from the fact that the precoders in other clusters are computed independent of the channel experienced by the user in the current cluster, (d) is substituting the value of norm square of the channel and (e) upperbounds the norm of the precoders from other clusters.

The rate lower bound (10) in the proposition is obtained by direct substitution, hence completing the proof.

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