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# Credit imperfections, labor market frictions and unemployment: a DSGE approach

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## Abstract

We construct and estimate a new-Keynesian DSGE model, integrating sticky prices in goods market and frictions in labor and credit markets. A search and matching labor market and a costly-state verification framework in the credit market are introduced. Capital spending, vacancy posting costs and wage bill require to be paid in advance of production and thus require external financing in a frictional credit market. Search and matching in labor market is uncertain and costly. We find that the procyclicality of the risk premium (the cost of external over internal funds) impacts the vacancy posting decision and thus the level of unemployment in the economy. Credit markets frictions may be the source of lower posting vacancies, lower bargained wages and higher unemployment level. Then, the transmission channel of shocks among both markets is investigated. We also analyze to what extent frictions alter main variables response to monetary and credit shocks. An empirical evidence is presented by regressing principle variables in both markets using a Bayesian VAR method. The shocks identification is based on sign restrictions and penalty function strategies. Moreover, the theoretical model is log-linearized around the steady-state and estimated using a Bayesian approach. The calibration is based on post-war US data and observed variables cover the period 1960-2007. We find that asymmetric information in the credit market pushes up the marginal cost of labor by adding a risk premium to the negotiated wage, to compensate for uncertainty related to the recruitment process efficiency. Although the wage is a result of a right-to-manage bargaining process, credit market imperfections affect labor supply through the final good mark-up.

*JEL classification:* E24, E32, E44, E52

*Keywords:* New-Keynesian model, labor and credit markets frictions, sticky prices, intensive and extensive margins, monetary and credit shocks

## 1 Introduction

Standard new-Keynesian DSGE models have been successful in terms of empirical performances, associated with a micro-founded structure. However, these models are missing a strong reference to unemployment. Indeed, a neoclassical labor market is assumed, in which individuals modify only hours worked (intensive margin) but the number of unemployed (extensive margin) does not change. This may seem annoying to the extent that unemployment is an important indicator of performances of the economy in its use of resources and it is a major policy issue. Thus, our model dispenses with a neoclassical model

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of labor market, by introducing frictions on the labor market through a Mortensen-Pissarides search and matching model of unemployment (Mortensen and Pissarides (1994)).

At the same time, this search-matching model is not performing well to explain high volatility of unemployment, vacancies and market tightness (Shimer (2004)). In order to tackle this issue, credit market frictions are integrated in the new-Keynesian model, through a costly-state verification framework similar to the one of Carlstrom and Fuerst (2001) and associated to a right-to-manage Nash bargaining wage as in Trigari (2009).

The model obtained enables to better understand cyclical fluctuations in key labor market variables (unemployment, hours worked and wages) and in credit market central variables (risk premium and default rate). Indeed, since capital spending, wage bill and posting vacancies (recruitment costs) are assumed to be paid partially by external funds, the interaction between frictions in both markets are key to better analyze the propagation and amplification of shocks (monetary and credit) on principal variables of both markets.

Credit frictions generate a cyclical component: the risk premium (the cost of external over internal funds) increases during downturns, increases the cost of a new employee and thus generates less vacancy posting and a higher equilibrium unemployment. In period of upturns, the risk premium decreases and net worth of firm increases (decrease of their dependence on external funds), making job posting less expensive.

Using a costly state verification framework similar to Carlstrom and Fuerst (1998) and Paustian (2004), asymmetric information is introduced between entrepreneurs and financial intermediaries on the credit market. In order to launch the production process, entrepreneurs need external funds to post vacancies (and recruit workers), to pay workers' wages and to finance capital spending in advance. Entrepreneurial production is then subject to a periodical idiosyncratic shock, privately observed by entrepreneurs, while banks need to pay a monitoring cost to check the real output produced. This agency problem will alter recruitment costs and the marginal cost of production for wholesale firms.

Further, according to Mortensen and Pissarides (1994), the job search and the matching process are costly and time-consuming for workers as well as for wholesale firms. Job creation occurs when an entrepreneur and an unemployed worker match and agree on a Nash bargaining wage. The match keeps going on until it is exogenously destroyed<sup>1</sup>.

Following Bernanke et al. (1999), a layered production sector is considered in order to separate asymmetric information and financial contract complications from price rigidity problem. A homogeneous wholesale good is produced by a continuum of competitive firms using borrowed funds. This produced good is then used by a continuum of monopolistic firms, that differentiate it costlessly into an intermediate good. Finally, a competitive bundler aggregates the different varieties into final output goods.

At the beginning of each period, after the occurrence of aggregate shocks, wholesale firms decide on their total inputs demand including hiring new workers. External funds are obtained from the financial intermediary via the establishment of an optimal financial contract. Households decide how to allocate their wealth between consumption and deposits, supply labor and capital and receive benefits for its unemployed members. Then, wholesale firms produce homogeneous goods and sell them to the retail sector. If revenues are sufficient, they repay their debt. Otherwise, they default and the bank monitors the output and seizes it. Intermediate-goods firms produce differentiated goods by using homogeneous goods as inputs. Due to differentiation, these firms are able to have a market power and are price makers. But all intermediate-goods firms are not allowed to modify their prices after the occurrence of aggregate shocks, because prices are subject to rigidity in the spirit of Calvo (1983). At the end, final goods are purchased by households and entrepreneurs for their own consumption.

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<sup>1</sup>No endogenous destruction is considered in our framework. See Trigari (2009), Christiano et al. (2011) for models with endogenous separation.

## 2 Related literature

The purpose of our work is at the intersection of different lines of research. In the relatively recent literature, a number of research papers introduce search and matching frictions on labor markets in real business cycle (RBC) models or in new-Keynesian (NK) models. Other articles highlight the role of financial frictions for macroeconomic dynamics, without taking into account search and matching frictions. Finally, more recent studies embody simultaneously frictions in labor and credit markets in partial equilibrium models or in dynamic stochastic general equilibrium (DSGE) models, to study interactions and implications of these two types of frictions.

The assumption of Walrasian labor markets is considered as a weakness of standard RBC and NK models. So, these models do not take into account variations in the number of unemployed, the extensive margin (that never changes). They allow only to study variations in hours worked per employee, the intensive margin. Furthermore, this kind of models is ineffective to explain many stylized facts, such as the observed inertia of inflation together with the large and persistent response of output after a monetary policy shock or the propagation mechanism of output after a technological shock. They are not able also to study the effect of aggregate shocks on unemployment dynamics. As a consequence, many articles have introduced search and matching frictions in labor markets, based on [Mortensen and Pissarides \(1994\)](#) framework, in RBC models or in NK models ([Merz \(1995\)](#), [Andolfatto \(1996\)](#), [Walsh \(2005\)](#), [Gertler and Trigari \(2009\)](#), [Krause et al. \(2008\)](#), [Thomas and Zanetti \(2009\)](#), [Trigari \(2009\)](#), [Christoffel et al. \(2009\)](#), [Lechthaler et al. \(2010\)](#), [Blanchard and Galí \(2010\)](#), [Galí et al. \(2011\)](#), [Campolmi and Faia \(2011\)](#), [Christiano et al. \(2013\)](#)).

Papers, as those of [Merz \(1995\)](#) and [Andolfatto \(1996\)](#), study implications of search and matching frictions for economic fluctuations in a standard RBC model. Both model show that labor market frictions are a mechanism of amplification and persistence for technology shocks. These frictions improve the empirical performance of RBC models, compared to a standard one (even if they do not predict enough cyclical movements in vacancies and output compared to data). For example, they succeed to generate for United-States (US) data the negative correlation between unemployment and vacancies and to replicate a high degree of persistence in unemployment and output. [Andolfatto \(1996\)](#), by introducing extensive and intensive margins, finds also that most of the variability of total hours worked is due to changes in unemployment level rather than hours worked per employee.

Then, several papers in the same spirit ([Walsh \(2005\)](#), [Trigari \(2009\)](#), [Thomas and Zanetti \(2009\)](#), [Lechthaler et al. \(2010\)](#) and [Campolmi and Faia \(2011\)](#)), study the role of matching frictions in new-Keynesian models. [Walsh \(2005\)](#) develops a new-Keynesian DSGE model with labor market frictions and with different potential sources of persistence (habit persistence, price stickiness and policy inertia) to see if it generates persistence in output and inflation after a monetary policy shock as observed in data. He finds through calibration that it amplifies for US data the output response, and decreases the inflation response to a monetary policy shock, as well as it generates persistence in output and inflation as observed in data and as standard NK models do not succeed to generate. As [Andolfatto \(1996\)](#), he shows that fluctuations in total hours worked occur by cyclical adjustments in the number of worker employed rather than in the number of hours worked per worker.

[Trigari \(2009\)](#) studies cyclical fluctuations of output, inflation and labor market variables following a monetary shock. She introduces the possibility of endogenous separation between firms and workers (idiosyncratic preference shock specific to each worker), as well as extensive and intensive margins. Her estimated model is able to replicate well for US data the observed responses of output, inflation and labor market data to a monetary policy shock. Using a VAR, she finds as observed in data that in a model with labor market frictions, the response of inflation is less volatile and response of output more persistent after a monetary policy shock than in a standard NK model. She finds also that the extensive margins fluctuate more than the intensive margin with a monetary shock, as observed in data and in previous articles.

[Thomas and Zanetti \(2009\)](#) focus on the impact of reductions in firing and hiring costs and unemployment

benefits (so on the impact of labor market reforms) on inflation in the Euro area. They find that it alters volatility of inflation, through a higher volatility of components of marginal costs. But quantitatively, neither shocks seem to have an important impact on inflation volatility, as these components have only a small contribution to inflation dynamics.

Finally, [Campolmi and Faia \(2011\)](#) study also the role of search and matching frictions in new-Keynesian model, but by adopting the framework of a small open economy. They find that real wages, marginal costs and profits are more sensitives to productivity and monetary policy shocks for countries with lower replacement rates.

However, the Mortensen-Pissarides search and matching model of unemployment remains unable to match important stylized facts observed in data. In particular, these types of models are not performing well to explain high volatility and persistence of unemployment, vacancies and market tightness ([Shimer \(2004\)](#) and [Hall \(2005\)](#)) and the relative smooth behavior of real wages found in data. The framework of wage Nash bargaining leads to an exaggerated procyclical movements in wages after a positive productivity shock, that dampens the firm's incentives to hire. Wages absorb much of the change in the expected benefit to a new worker induced by fluctuations in labor productivity. As a consequence, several papers try to tackle this issue by introducing wage rigidity mechanism ([Shimer \(2004\)](#), [Hall, 2005](#), [Gertler and Trigari \(2009\)](#) and [Christiano et al. \(2013\)](#)), hiring and firing costs. First, for productivity shock, [Blanchard and Galí \(2010\)](#) found that search and matching frictions modify the level of unemployment but the unemployment rate is invariant to productivity shocks. Thus, they study alternative wage-setting (Nash bargaining wage and more rigid real wages) and show that rigid wages enable to have inefficient fluctuations in unemployment after a productivity shock.

[Lechthaler et al. \(2010\)](#) introduce in a new-Keynesian model labor market frictions, through hiring and firing costs (and no wage rigidity). They find through a calibration on a given European country, more persistence in output and unemployment in response to real and monetary shock and in inflation in response to real shocks, as well as a strong amplification effect of these shocks on unemployment and on the job finding rate. The negative correlation between job creation and job destruction is founded, as well as between unemployment and job-finding rate.

[Gertler and Trigari \(2009\)](#) introduce in a standard Mortensen-Pissarides search and matching framework, a staggered multiperiod Nash wage contracting to reproduce, by calibration, the relatively volatile behavior of unemployment and the relative smooth behavior of real wages over the business cycle observed in data. [Christiano et al. \(2013\)](#) introduce in a New-Keynesian model with capital, an endogenous wage inertia (and no an exogenous wage nominal rigidity), coming from how firms and workers bargain the wage (period-by-period bargaining or present discounted value bargaining).

Another focus of some papers is optimal monetary policies in new-Keynesian models with matching frictions, as the ones of [Thomas and Zanetti \(2009\)](#), [Blanchard and Galí \(2010\)](#). For example, with flexible wages, [Blanchard and Galí \(2010\)](#) show that unemployment can be stabilized by targeting inflation. And with rigid wages, it is possible to stabilize unemployment in response to productivity shock by allowing transitory movements in inflation. Otherwise, stabilizing inflation would induce large and inefficient variations in unemployment.

On the other hand, frictions have been also studied on the credit market side by a certain number of papers, as those of [Bernanke and Gertler \(1989\)](#), [Bernanke and Gertler \(1995\)](#), [Carlstrom and Fuerst \(1998\)](#), [Kiyotaki and Moore \(1997\)](#), [Bernanke et al. \(1999\)](#), [Carlstrom and Fuerst \(2001\)](#), [Paustian \(2004\)](#) and [Fiore and Tristani \(2013\)](#). They have been devoted to understand the relationship between financial markets and overall macroeconomic performances. Financial factors are indeed suspected to amplify and increase persistence of macroeconomic variables responses to aggregate shocks. The idea behind is that deteriorating credit conditions could be the source of poor economic activity and not the consequence of a declining real economy.

Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1999) develop the concept of a financial accelerator in DSGE models, with money and price stickiness. Without credit frictions, an entrepreneur can resort to external financing to raise capital at a risk-free interest rate. With credit market frictions, information asymmetry appears in the form of moral hazard between the lender and the borrower (here, called the entrepreneur). Borrower will indeed be induced to report to the lender a lower real output produced than their true level. As a consequence, this type of asymmetric information leads either to borrowing restrictions for borrowers on the amount of external financing, based on the existence of collateral constraints to cover their potential inability to reimburse loans under a financial contract, as in Kiyotaki and Moore (1997). Or it leads to a second type of financial frictions, namely a higher cost of external financing compared to opportunity costs of internal financing (the deposit risk-free rate), that to say to an external finance premium or a risk premium, pays by entrepreneurs. The lender have indeed to pay for a state verification cost to monitor the borrower's realized return on capital, when he defaults, as introduced by Townsend (1979) and Gale and Hellwig (1985). The external finance premium can vary a lot and depends on many characteristics, as company net worth or risk. As the latter are pro-cyclical, with asset prices for example, the external finance premium is counter-cyclical, amplifying as a consequence the fluctuations in borrowing, investment, employment, and output, through the financial accelerator mechanism. For example, if the economy is hit by an adverse shock, firm's net worth decreases and the external finance premium increases as a consequence. It reduces the possibility to resort to external financing and worsens the financial position of borrowers ; which increases again the external finance premium charged by banks, and reduces the demand for external funds. Less investment are thus financed, less output is produced and less profits are triggered. It makes then the external financing all the more crucial and makes the net worth decreases... which leads to another increase in the external finance premium. Thus, endogenous development in imperfect credit markets, due to the inverse relationship between firm's net worth and external finance premium, helps to amplify and propagate shocks to the economy. Credit market frictions amplify and propagate more precisely real and nominal shocks and could give an insight of the financial accelerator mechanism by which small shocks may have large real consequences on the economy. In a very similar spirit and few years before, ? introduce in a canonical real business cycle model the same kind of informational asymmetry between lenders and borrowers and show that it leads the economy to return more slowly to the steady-state after being hit by a shock (propagation mechanism) and leads to less amplification because agency costs create an endogenous mark-up in an output model. The markup distorts factor markets, so wages and capital rental rates are below their corresponding productivities. Thus, increases in net worth lower agency costs and hence the mark-up. Debt arises as the optimal financial contract between firms and banks and firms must borrow at a premium over the risk-free rate. The financial contract is designed to minimize the expected agency costs. It specifies the returns when bankruptcy or success occurs and a monitoring threshold (for reported profits below the threshold, the lender pays the state verification costs and above the threshold, the lender does not pay to audit the project result). The threshold is a decreasing function of borrower's capital and an increasing function of the deposit risk free-rate (opportunity cost).

Paustian (2004) compare monetary policy rules in a utility based welfare metric, through a new-Keynesian model with staggered prices and wages, and financial frictions. He finds that firm's net worth affects the availability of external finance, and that financial frictions reduce the economy's response to technology and monetary shocks, and make them more persistent.

Fiore and Tristani (2013) show, by adopting the costly state verification set-up, that financial market conditions are important to explain macroeconomic outcomes because its affects firms' marginal costs. Higher credit spreads increase lending rates and marginal cost of credit for firms, which lead to increase prices and as a consequence, it affects output.

All these papers assume standard Walrasien labor markets. Only a relative small number of papers try to introduce credit and labor markets frictions, as Thomas and Zanetti (2009), Christiano et al. (2011), Petrosky-Nadeau (2014) and Zanetti and Mumtaz (2013). As seen before, labor market frictions imply

that it is costly to hire new workers. The functioning of labor markets prevent the competitive allocation of labor resources, and thus it will interact with financial frictions to impact production, unemployment, investment and capital accumulation. Those models enhance the [Bernanke et al. \(1999\)](#) framework with a more realistic labor market. [Christiano et al. \(2011\)](#) show in a new-Keynesian model that financial and employment frictions are able to change the model dynamics in an open economy setting, and improve the forecasting properties of the model for Swedish data, in particular for inflation. [Thomas and Zanetti \(2009\)](#) makes out that financial shocks are important to explain business cycles fluctuations, because they impact firm’s ability to raise funds and amplify/dampen the response of macroeconomic variables (such as unemployment, wages and vacancy posting) to shocks. [Petrosky-Nadeau \(2014\)](#) considers that firms finance only their vacancy costs with external financing on frictional credit markets. He finds that the easing of financing constraints during an expansion (a productivity shock) reduces the opportunity cost for resources allocated to job creation (cost channel), because firms are able to accumulate net worth. Credit market frictions generate persistence in the dynamics of labor-market tightness and have a moderate effect on amplification. [Zanetti and Mumtaz \(2013\)](#) demonstrate that labor and financial frictions are supported by the data and that they play together to amplify or reduce the variables’ reaction to shocks.

Note that our framework is different from the one of [Acemoglu \(2001\)](#) or [Wasmer and Weil \(2004\)](#), who assumed search frictions on both labor and credit markets. Search frictions and agency costs are both credit markets imperfections but we focus on agency costs by a costly-state verification framework. [Wasmer and Weil \(2004\)](#) find also that labor and credit market frictions work together to amplify macroeconomic volatility. Note also that our work is included in the spirit of researches about the impact of credit markets imperfections on investment flows but we will focus on their impact on employment flows, hours worked and wages, and not on capital investment as in [Bernanke et al. \(1999\)](#) or [Kiyotaki and Moore \(1997\)](#).

### 3 Empirical evidence

A Bayesian VAR method is used to present empirical evidence for the United States (US) between 1960Q1-2007Q4, by regressing principle variables in labor and credit markets. Monetary and credit shocks are identified. The literature is followed for the identification of shocks and a mixture of sign restrictions and penalty function strategies is applied.

Bayesian VAR (BVAR) uses Bayesian methods to estimate a VAR ([Stock and Watson \(2001\)](#)). As a consequence, the difference with VAR models is that model parameters are treated as random variables, and priors are assigned to them. Indeed, with VAR models, the parameter space increases with the number of dependent variables and the number of lags. But our quarterly database is only of moderate size. Thus, empirical Bayes estimators enable to overcome the over-fitting problem of standard VAR models.

#### 3.1 Shocks identification

A VAR is an  $n$ -equations,  $n$ -variables model in which each variable is explained by its lagged values, current and past values of the others variables. The structural VAR considered is given by

$$X_t = \alpha_{(0)} + A_{(1)}X_{t-1} + A_{(2)}X_{t-2} + \dots + A_{(L)}X_{t-L} + e_t, t = 1, \dots, T \quad (1)$$

where  $X_t$  is an  $p \times 1$  vector of variables at date  $t$ ,  $A_i$  is a  $p \times p$  coefficient matrix for each lag of the variable vector with  $\alpha_{(0)}$  the constant term.  $e_t$  is the structural shocks vector associated to the variable vector with variance-covariance matrix  $\Sigma$ . In our estimation, the lag length,  $L$  is kept at and structural shocks are supposed to be independent. If they were correlated, this would suggest some remaining unexplained causal relationship between them. The vector  $X_t$  corresponds to:

$$X_t = \begin{bmatrix} \ln(Y_t) \\ R_t \\ \pi_t \\ U_t \\ V_t \\ \ln(H_t) \\ \ln(W_t) \\ Jcrea_t \\ Jdestr_t \\ D_t \\ GZspread_t \end{bmatrix}$$

The variables included are real output ( $Y_t$ ), inflation ( $\pi_t$ ) and nominal interest rate ( $R_t$ ) to which we add six labor market variables and two credit market variables. The labor market variables are unemployment ( $U_t$ ), vacancies ( $V_t$ ), number of hours worked per worker ( $H_t$ ), real wages ( $W_t$ ) and job creation and destruction rate ( $Jcrea$  and  $Jdestr$ ). The credit market variables are GZ spread ( $GZspread_t$ ) and default rate ( $D_t$ ).

The series for the real output are the natural logarithm of quarterly real gross domestic product (GDP); the nominal interest rate is the quarterly federal funds rate, annualized and averaged over the quarter and inflation is measured as the change in natural logarithm of the GDP deflator between two consecutive quarters. Unemployment is given by the ratio of civilian unemployed persons to the civilian labor force and vacancies are. The series for the average number of hours per worker come from the non-farm business sector weekly average hours. Finally, job-creation and job-destruction are obtained from.

The default rate is the default rate for Moody's rated U.S speculative-grade corporate bonds, as used by [Helbling et al. \(2011\)](#) and [Meeks \(2012\)](#). The GZ spread is a credit spread index constructed by [Gilchrist and Zakrajšek \(2012\)](#). A credit spread measures the difference in yields between a private debt instrument and government securities of comparable maturity. Formally, the authors have constructed the index as follow:

$$S_t^{GZ} = \frac{1}{N_t} \sum_i \sum_k S_{it}(k) \quad (2)$$

where  $N_t$  is the number of bond observations in quarter  $t$  and  $S_{it}(k)$  is the credit spread, expressed as the difference between the yield of the corporate bond  $k$  and a hypothetical Treasury security with exactly the same cash-flows as the underlying corporate bond. More precisely, for each individual bond issue in the sample, they construct a theoretical risk-free security that replicates exactly the promised cash-flows of the corresponding corporate debt instrument. So they compare the interest rate actually paid by the firm with what the US government would have paid on a loan with a similar maturity<sup>2</sup>.

The model is used to study the impact of monetary and credit shocks on labor and credit market variables. Identification assumptions are required in order to interpret correlation as causality among variables. Economic theory is needed to sort out links among variables. Monetary shocks are identified using a set of sign restrictions imposed on impulse responses following [Uhlig \(2005\)](#). Credit shocks are identified in the same spirit of [Meeks \(2012\)](#), who use a bayesian VAR methodology in a CSV framework.

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<sup>2</sup>The GZ spread is an arithmetic average of the credit spreads on bonds in any given quarter. According to the authors, it has a huge predictive power for the economic activity over the 1973-2010 period. It is based on micro level data set of secondary market prices of outstanding senior unsecured bonds issued by a large panel of US non financial corporations, which allows to have a spread not subject to the duration mismatch problem of standard credit spread indexes.



### 3.1.1 Monetary shocks identification

Following Uhlig (2005), sign restrictions are imposed on the impulse responses of prices and on the federal funds rate in response to a monetary policy shock but no restrictions are imposed on the response of real output and labor market variables (to have a so called "agnostic" identification procedure) to answer the key question we are focusing on. Monetary shocks are identified by assuming that they do not affect output within the same quarters. It does not lead to an increase in prices and a decrease in federal funds rate for a certain period.

The horizon  $K$  of sign restrictions has to be precised. Indeed, we have to investigate how much sensitive are the results to changes in the horizon.

### 3.1.2 Credit shocks identification

Credit shocks are frictions that prevent firms from funding all desired recruitment or investment. This incapacity is assumed to be due to credit constraints or inability to borrow. Credit shock does not mean financial distress or default risk or economic distress, although these things are for sure correlated with credit shocks.

Similar to Helbling et al. (2011), we want to see if exogenous shocks originating in credit markets are important source of labor market variables fluctuations because linkages between the real economy and credit markets are worth to focus on.

A negative credit shock is identified by assuming that it leads to an increase in the price of credit (risk premium), that productivity does not fall and default rate do not rise. An adverse credit shock has a clear natural impact on the price of credit. Sign restrictions on productivity and default rate are made in order to be sure to identify a credit supply shock originating from financial sector and not an endogenous credit response to expected fluctuations in future macroeconomic activity by lenders, as explained by Meeks (2012). Thus, it is not necessary that an increase of credit spread leads to an expected decline in productivity or an increase in default rates. These sign restrictions are imposed for four quarters after the initial shock, as Helbling et al. (2011) and Meeks (2012).

## 3.2 Impulse responses

Impulse-response functions represent a one-unit increase in the current value of one of the VAR errors, assuming that this error returns to zero in future periods, and that all other errors are equal to zero (errors have to be uncorrelated across equations as a consequence). It enables to see what are the effects of structural shocks on labor market variables.

## 3.3 Forecast error variance decomposition

Now, the importance of the effects of structural shocks for the volatility of labor market variables at business cycles frequencies are studied.

# 4 The model

## 4.1 Timing Summary

The economy consists of seven types of agents: households, who are either workers or unemployed, wholesale-goods firms managed by entrepreneurs, intermediate-goods and final-goods producing firms, banks and a government.

The agents are interacting in 7 different markets (labor market, capital market, credit market, liquidity market, wholesale-goods market, intermediate-goods market and final-goods market), where the timing of

events is given by:

0- An exogenous amount is given to entrepreneurs who went bankrupt last period; a given fraction of entrepreneurs born so as to ensure a constant fraction of entrepreneurs in the economy.

1- Liquidity market opens.

Monetary policy and aggregate shocks are realized.

2- Credit market opens: banks accumulate the deposited amounts by households at the end of period  $t - 1$  in order to grant them as loans at the beginning of the current period  $t$ . The credit market clears when the amount of deposits equals the amount of granted loans.

Entrepreneurs own and manage the wholesale production sector. They enter period  $t$  with a net worth accumulated at the end of period  $t - 1$  for the last period solvent entrepreneurs or with a net worth given by the exogenous endowment for the entrepreneurs who went bankrupt last period. They borrow from banks using a financial contract, in order to cover their expected production bill (labour and capital costs) and vacancies posting costs. Capital, labor costs and vacancies posting costs are supposed to be paid in advance.

3- Capital market opens: households rent the effective capital to entrepreneurs at a perfectly competitive price. Households own capital; they have accumulated it at the end of the period  $t - 1$  to lent it during the current period  $t$  to entrepreneurs. The capital market clears when the amount of effective capital supplied by households is equal to the amount demanded by entrepreneurs. Entrepreneurs have to pay immediately their capital costs to households (using their loans).

4- Labor market opens: Entrepreneurs post vacancies at a real unit cost and recruit a number of workers. Then, the wage is established after a Nash bargaining process and new hired employees start working immediately. The Nash bargained wage and the vacancy posting costs have to be paid also immediately by entrepreneurs (using their loans).

Then, each entrepreneur maximizes its expected return in order to decide the total number of hours worked.

5- Wholesale-good market: Wholesale goods are produced by wholesale-goods firms, thanks to capital and labour. They have used their entire loan.

After wholesale production occurs, entrepreneurs sell it to retailers and declare either being solvent or bankrupt, after having observed privately their own idiosyncratic shocks. Solvent entrepreneurs, characterized by a sufficiently high idiosyncratic shock, pay back their loan and keep the remaining amount to use it at the end of the period  $t$ . The bank spends a monitoring cost, proportional to the credit amount, in order to check the output produced by bankrupt entrepreneurs and confiscates the proceeds of production left by entrepreneurs, so that these latter cannot neither consume nor carry over net worth to the coming period.

Banks reimburse households deposits. The role of the financial intermediary is well defined : it allows to mitigate the monitoring cost and avoid its duplication.

6- Intermediate-good market: Retailers are a set of monopolistically competitive firms owned by households. They buy the wholesale good from the wholesale firms and differentiate it costlessly into intermediate goods varieties. Only a given fraction of retailers are able to fully re-optimize their prices. Then, the intermediate good is sold to final-good production firms. All the realized profits are transferred to households at the end of the period.

7- Final-good market: Final-goods firms are also owned by households. They proceed in a perfectly

competitive market. They aggregate intermediate goods bought from retailers to produce final goods. Given all prices and revenues in the economy, households decide on their level of consumption, deposits, money, of investment and on the capital rate of utilization.

Solvent entrepreneurs decide on their consumption and their net worth, depending on their probability of death in the current period  $t$ : those existing the economy at the end of the period  $t$  can consume all their net worth just before death, and those keeping in the economy will accumulate totally their net worth thanks to money.

8- All markets clear.

## 4.2 Households

### 4.2.1 Timing

Households consume, save, invest and supply passively labor to entrepreneurs.

First, households supply labor passively and without any cost to entrepreneurs, when the labor market opens. A fraction of them is matched with entrepreneurs and begins to work immediately. The other fraction is not matched and becomes unemployed.

The newly matched workers and the workers who have been matched without destruction in the period  $t - 1$  receive their wages immediately after the matching. The unemployed have to wait the end of the period to receive their unemployment benefits.

After, at the end of the period  $t$ , when the final-good production takes place, households will decide how much they want to consume, save through deposits and invest to accumulate capital (and they decide also the capital utilization rate for the next period). To make their decision, households have as resources, wages earned by workers, the unemployment benefits earned by the unemployed (at the end of period  $t$ ). They earn also the profits from retailers and final-goods firms, as they are the owners of these firms. They receive also from banks after the reimbursement of wholesale-goods firms the last period interest rate, multiplied by the amount of deposits they have decided last period. Finally, they have to pay also at this precise moment lump-sum taxes to the government, that will finance unemployment benefits.

To summarize, they decide how much they want to consume final goods. They decide how much they want to save through deposits to banks. These deposits will be then used by banks to lend to wholesale-goods firms. And these deposits will be remunerated by banks to households after the reimbursement of wholesale-goods firms in period  $t + 1$ , at the risk-free interest of the current period  $t$ . This revenue is sure because we assume that the financial intermediation is realized through a large number of atomistic risk-neutral bank, holding enough large and diversified portfolios to ensure perfect risk pooling for their main creditors, the households.

Finally, households decide how much they want to invest final-goods to accumulate capital, which will be rented at the next period  $t + 1$  to entrepreneurs on a perfectly competitive capital market (so at a perfectly competitive price). They decide also at the same time the capital utilization rate for the next period, which will determine the effective capital submitted for rent.

Households are seen as a large representative family, consisting of a continuum of members, represented by the unit interval and composed of workers who are either employed or unemployed. The family pools its income before choosing the per capita consumption and deposits such that a perfect consumption is fully insured for all members. As in [Merz \(1995\)](#) and [Andolfatto \(1996\)](#), there is a full risk sharing of consumption and savings in order to avoid distributional issues due to heterogeneity in incomes. So that, the same notation is used for the consumption of the representative household and for the consumption

of each member<sup>3</sup>.

After a costly and time-consuming search and matching process à la [Mortensen and Pissarides \(1994\)](#) on the labor market, the number of employed household members is  $n_t \in (0, 1)$ , whereas the remaining members,  $1 - n_t$ , are unemployed. Each employed worker supplies hours of labor at the real hourly wage  $W_t$ . The real wage is determined through a decentralized Nash bargaining between matched couples of entrepreneurs and workers. But, wholesale firms set unilaterally effective hours of work,  $H_t$ , at the time of the financial contract establishment. Therefore, the total number of hours worked by a representative household,  $N_t$ , is given by:

$$N_t = n_t H_t$$

In addition to the real wage income,  $W_t N_t$ , earned by employed workers and real unemployment benefits,  $b^4$ , received by unemployed members, the representative family has a diversified ownership stake in monopolistic retailers, paying out a nominal lump-sum profit,  $\Pi_t$ . Conditional on  $n_t$  and  $H_t$ , the household makes also a nominal lump-sum transfer of taxes for an amount of  $T_t$  to the government and rent effective capital,  $K_t$ , to firms at a given real rate of  $r_t^K$ . Finally, the remaining total income is used to consume an amount  $C_t$  of final goods, to save a nominal amount,  $D_t$ , of bank deposits remunerated at a risk free rate,  $R_t$ , and to invest  $I_t$ . Thus, the representative household budget constraint is given by:

$$C_t + I_t + \frac{D_t}{P_t} \leq n_t W_t H_t + (1 - n_t)b + \frac{R_{t-1} D_{t-1}}{P_t} + r_t^K \nu_t K_{t-1}^p - \Upsilon(\nu_t) K_{t-1}^p + \frac{\Pi_t}{P_t} + \frac{T_t}{P_t} \quad (3)$$

where  $P_t$  is the nominal aggregate price level.

Households own the economy's stock of physical capital and choose the capital utilization rate,  $\nu_t$ , which transforms physical capital,  $K_t^p$ , into effective capital,  $K_t$ . So the amount of effective capital that households can rent to wholesale firms is:

$$K_t = \nu_t K_{t-1}^p \quad (4)$$

$\Upsilon(\nu_t)$  is the real cost of capital utilization per unit of physical capital, specified to match the following conditions at the steady state:  $\nu_t = 1$ ,  $\Upsilon(1) = 0$  and  $\frac{\Upsilon'(1)}{\Upsilon''(1)} = \eta_\nu$ .

By investing  $I_t$  units of consumption goods during period  $t$ , the representative household increases the capital stock,  $K_t^p$ , available during period  $t + 1$  according to:

$$K_t^p = (1 - \delta_K) K_{t-1}^p + \epsilon_t^I \left[ 1 - \Lambda\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \quad (5)$$

where  $\delta_K$  is the capital depreciation rate and  $\Lambda$  is the adjustment cost function, with  $\Lambda(\gamma_\omega) = \Lambda'(\gamma_\omega) = 0$  and  $\Lambda''(\gamma_\omega) = \nu_K > 0$ , with  $\gamma_\omega$  the economy's steady state growth rate.  $\epsilon_t^I$  is an investment-specific technological shock affecting the efficiency with which consumption goods are transformed into capital, following an auto-regressive process:

$$\log(\epsilon_t^I) = \rho_I \log(\epsilon_{t-1}^I) + u_t^I, \quad \rho_I \in [0, 1) \text{ where } u_t^I \stackrel{iid}{\sim} N(0, \sigma_I^2)$$

Conditional on  $\{H_t, n_t\}_{t=0}^\infty$  and taking as given the set of prices  $\{P_t, W_t, R_t, r_t^K\}_{t=0}^\infty$ , the household chooses the streams of optimal consumption of final good  $\{C_t\}_{t=0}^\infty$ , deposits amount  $\{D_t\}_{t=0}^\infty$ , investment

<sup>3</sup>The family optimally allocates the same consumption for each member, regardless their individual income.

<sup>4</sup> $b$  can be interpreted as home production or as unemployment benefits, as we do, provided by the government (financed by lump-sum taxes).

$\{I_t\}_{t=0}^{\infty}$ , capital utilization rate  $\{\nu_t\}_{t=0}^{\infty}$  and the stream of physical capital  $\{K_t^P\}_{t=0}^{\infty}$ , maximizing the following discounted utility function<sup>5</sup> subject to the budget constraint (3) and the capital accumulation constraint (5):

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \epsilon_t^C \log(C_t - hC_{t-1}) - \epsilon_t^H \frac{H_t^{1+\tau} n_t}{1+\tau} \right] \quad (6)$$

where  $0 < \beta < 1$  is the intertemporal discount factor,  $h$  is a habit persistence parameter,  $\epsilon_t^C$  is a consumption preference shock,  $\epsilon_t^H$  is a shock to supplied hours and  $\tau$  denotes the inverse Frisch intertemporal elasticity of labor supply to the hourly real wage. When  $h > 0$ , the model allows for habit persistence in consumption preferences, in order to take into account the necessary empirical persistence in the consumption process. There is here no money in the utility function, since the monetary policy is modelled by a Taylor rule and not by a money growth rule.

Preference and hours supply shocks obey to the following stochastic processes:

$$\begin{aligned} \log(\epsilon_t^C) &= \rho_C \log(\epsilon_{t-1}^C) + u_t^C, & \rho_C &\in [0, 1) \text{ where } u_t^C \stackrel{iid}{\sim} N(0, \sigma_C^2) \\ \log(\epsilon_t^H) &= (1 - \rho_H) \log(\epsilon_t^H) + \rho_H \log(\epsilon_{t-1}^H) + u_t^H, & \rho_H &\in [0, 1) \text{ where } u_t^H \stackrel{iid}{\sim} N(0, \sigma_H^2) \end{aligned}$$

where  $\epsilon^H$  is the steady state value of  $\epsilon_t^H$ .

The first-order conditions of the representative household's problem are given by:

$$(C_t) \quad \lambda_t = \frac{\epsilon_t^C}{C_t - hC_{t-1}} - \beta h E_t \frac{\epsilon_{t+1}^C}{C_{t+1} - hC_t} \quad (7)$$

$$(D_t) \quad 1 = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \right] \quad (8)$$

$$(\nu_t) \quad r_t^K = \Upsilon'(\nu_t) \quad (9)$$

$$(I_t) \quad \epsilon_t^I Q_t \left[ 1 - \Lambda\left(\frac{I_t}{I_{t-1}}\right) \right] = 1 + \epsilon_t^I Q_t \frac{I_t}{I_{t-1}} \Lambda'\left(\frac{I_t}{I_{t-1}}\right) - \beta E_t \epsilon_{t+1}^I \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 \Lambda'\left(\frac{I_{t+1}}{I_t}\right) \quad (10)$$

$$(K_t^P) \quad Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \delta_K) Q_{t+1} + r_{t+1}^K \nu_{t+1} - \Upsilon(\nu_{t+1}) \right] \quad (11)$$

where  $\lambda_t$  is the Lagrange multiplier associated to the household's budget constraint and  $Q_t$  is the marginal value of installed capital in consumption units and  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is the inflation rate.

### 4.3 Workers, searching dynamics and matching

At time  $t$ , a wholesale firm  $i$  posts actively  $V_{it}$  job vacancies at a real unit cost  $\gamma$  to attract unemployed workers, who are searching passively for a job.

**Assumption 1.** Only unemployed workers can search passively for a job and can be hired. Current employed workers are not allowed to look for another job. Job-to-job transition is not considered.

Job creation occurs when an entrepreneur and an unemployed worker meet on the labor market after a search and matching process à la [Mortensen and Pissarides \(1994\)](#) and agree on a Nash bargaining wage.

<sup>5</sup>The form of the utility function is based on the one used by [Gertler et al. \(2008\)](#), [Blanchard and Galí \(2010\)](#) and [Christiano et al. \(2011\)](#).

At the end of this process, a wholesale firm  $i$  employs  $n_{it}$  workers. As there exists a continuum of wholesale firms represented by the unit interval, the total number of vacancies and employed workers at time  $t$  are:

$$\begin{aligned} V_t &= \int_0^1 V_{it} di \\ n_t &= \int_0^1 n_{it} di \end{aligned}$$

**Assumption 2.** There is a full participation of workers. At each period  $t$ , they are either employed or unemployed workers looking for a job. The transition between in and out the labor force is ignored.

Formally, total vacancies,  $V_t$ , will be filled by unemployed workers,  $U_t$ , via an aggregate constant return to scale matching function,  $M(U_t, V_t)$ , specified by<sup>6</sup>:

$$M(U_t, V_t) = \epsilon_t^M U_t^\rho V_t^{1-\rho} \quad (12)$$

where  $\rho \in (0, 1)$  is the elasticity of matches to unemployment and  $\epsilon_t^M$  is the matching process efficiency shock. It follows a first-order auto-regressive process:

$$\epsilon_t^M = (1 - \rho_M)\epsilon^M + \rho_M \log \epsilon_{t-1}^M + u_t^M, \quad \rho_M \in [0, 1) \text{ where } u_t^M \stackrel{iid}{\sim} N(0, \sigma_M^2)$$

$\epsilon^M$  is the steady-state value of the matching process efficiency,  $\epsilon_t^M$ . Moreover, as standard in the literature, the matching technology is assumed to be concave and increasing in both arguments.

**Assumption 3.** As soon as the matching happens, new hired workers start working immediately<sup>7</sup>. Employed workers for whom the matching ends exogenously during period  $t$  (as explained below) are allowed to search for a new job in the same period<sup>8</sup>. But a new matching is only possible after financial contracts and loans are decided and obtained, so only at the beginning of next period.

The ratio of total vacancies to unemployed workers at time  $t$  (that to say at the beginning of the matching process of period  $t$ ) defines the labor market tightness from the firm point of view,  $\theta_t$ , so that:

$$\theta_t \equiv \frac{V_t}{U_t}$$

The probability at time  $t$  for a firm to fill a vacancy job,  $p_t$ , is given by:

$$p_t = \frac{M(U_t, V_t)}{V_t} = M(\theta_t^{-1}, 1) \quad (13)$$

And the probability at time  $t$  for an unemployed worker to find a job,  $q_t$ , is given by:

$$q_t = \frac{M(U_t, V_t)}{U_t} = M(1, \theta_t) \quad (14)$$

Therefore, the number of hired workers at time  $t$ ,  $\psi_t$ , is given by:

$$\psi_t = p_t V_t = M(U_t, V_t) \quad (15)$$

<sup>6</sup>Gertler et al. (2008) use the same specification. The Cobb-Douglas matching function is used in almost all macroeconomic models with search and matching frictions. Furthermore, the constant returns to scale assumption (homogeneity of degree one) seems to be supported empirically (Petrongolo and Pissarides (2001)).

<sup>7</sup>Following Krause and Lubik (2007), Gertler et al. (2008), Thomas and Zanetti (2009) and Blanchard and Galí (2010), workers are assumed to be immediately productive after being hired.

<sup>8</sup>Many papers on searching and matching literature are considering the same assumption. See Blanchard and Galí (2010) for example.

Note that  $q_t = \theta_t p_t$  and  $\partial p_t / \partial \theta_t < 0$ ,  $\partial q_t / \partial \theta_t > 0$ . The higher vacancy posts on unemployment (or the higher the labor market tightness from the firm point of view), the higher the probability for an unemployed worker to find a job and the lower the probability for a firm to fill a job. Both workers and firms take  $q_t$  and  $p_t$  as given.

**Assumption 4.** A job is assumed to be destroyed at an exogenous rate,  $\delta_t$ , which evolves according to a first-order auto-regressive process:

$$\log(\delta_t) = (1 - \rho_\delta) \log(\delta) + \rho_\delta \log(\delta_{t-1}) + u_t^\delta, \quad \rho_\delta \in [0, 1) \quad \text{where } u_t^\delta \stackrel{iid}{\sim} N(0, \sigma_\delta^2)$$

$\delta$  is the steady state value of  $\delta_t$ .

Finally, the total employment just after the matching process of the period  $t$ ,  $n_t$ , is defined as the sum of the surviving workers from the exogenous separation at the end of period  $t - 1$ ,  $(1 - \delta_{t-1})n_{t-1}$ , and the new hires from the matching of the period  $t$ ,  $p_t V_t$ :

$$n_t = (1 - \delta_{t-1})n_{t-1} + \psi_t \tag{16}$$

As the labor force is normalized to one, the number of unemployed members after the matching process of the period  $t$  is:

$$U_t = 1 - n_t \tag{17}$$

Since workers who discontinue their match during the period  $t$  are assumed to be allowed to search for a new job in the same period, the number of searching workers at the end of the period  $t$  is  $1 - (1 - \delta_t)n_t$ .

## 4.4 Production and financial sectors

The production sector has three different layers as in [Bernanke et al. \(1999\)](#). At the first layer, where agency problem and search and matching frictions occur, a continuum of perfectly competitive wholesale firms produce a homogeneous good using external financing. At the second layer, where prices stickiness arises, the wholesale good is differentiated costlessly by a continuum of monopolistic firms to produce intermediate goods indexed by  $j \in [0, 1]$ . Finally, a competitive bundler aggregates these varieties to produce the final good, which can be used for consumption, investment, posting vacancies and government spending.

### 4.4.1 Wholesale-good production: hiring decision and financial contract

Each wholesale firm, indexed by  $i \in [0, 1]$ , is managed by an infinitely lived risk-neutral entrepreneur, who may die at each period with a probability  $(1 - \varsigma)$ . This assumption is made to be sure that entrepreneurs will not accumulate net worth and will borrow from banks at each period<sup>9</sup>. Intra-period entry into and exit out of wholesale firms at each period are ruled out.

At the beginning of each period, first, wholesale-goods firms make their hiring decision. They determine the number of new workers,  $\psi_{it}$ , they have to take on. Second, they post vacancies on the labor market at the real unit cost  $\gamma$ , partially financed externally on a frictional credit market. Posting vacancies costs are assumed to be not paid immediately but they are settled after the loan contract negotiation. The negotiated loan with the financial intermediary is supposed to cover also the bill of inputs used to produce the wholesale good, namely wages and capital expenditures, and to be reimbursed after production occurs.

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<sup>9</sup>The same assumption is made by [Bernanke et al. \(1999\)](#) and [Paustian \(2004\)](#). [Carlstrom and Fuerst \(1998\)](#) make the different assumption, that consumers and entrepreneurs have different time-discount factors, with entrepreneurs less impatient than consumers.

Hence each new hire costs the firm  $Z_t$  that can be seen as the total cost of hiring or equivalently, as the total replacement cost of a unit of labor, that includes financial costs.

### Hiring dynamics

The hiring decision of a wholesale firm  $i$  is the optimal solution to the following bellman equation:

$$\begin{aligned} J_{it} = \max_{\psi_{it}} \quad & \frac{P_t^{ws}}{P_t} Y_{it}^{ws} - r_t^K K_{it} - W_t N_{it} - \frac{Z_t}{P_t} \psi_{it} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{it+1} \\ \text{subject to} \quad & n_{it} = (1 - \delta_{t-1}) n_{i,t-1} + \psi_{it} \\ & N_{it} = n_{it} H_{it} \end{aligned} \quad (18)$$

where  $P_t^{ws}$  is the wholesale-good price,  $\beta \frac{\lambda_{t+1}}{\lambda_t}$  is the firm's discount factor<sup>10</sup> and  $Y_{it}^{ws}$  is the quantity of wholesale goods produced by the firm using  $N_{it}$  hours of labor and  $K_{it}$  units of capital, according to a constant return to scale production function given by:

$$Y_{it}^{ws} = A_t K_{it}^\alpha N_{it}^{1-\alpha} \quad (19)$$

where  $A_t$  is the aggregate technology shock, source of systematic risk, supposed to be stationary and evolves according to:

$$\log A_t = \rho_A \log A_{t-1} + u_t^A, \quad \rho_A \in [0, 1) \quad \text{where } u_t^A \stackrel{iid}{\sim} N(0, \sigma_A^2)$$

Normally, by taking as given the wage schedule,  $W_t$ , the hours of work per employee,  $H_{it}$ , the consumer price index,  $P_t$ , the wholesale-good price,  $P_t^{ws}$ , as well as the used capital,  $K_{it}$ , and its unit price,  $r_t^K$ , a wholesale firm chooses first the number of hirings,  $\psi_{it}$ , and consequently the number of employees,  $n_{it}$ , so as to maximize its discounted value of future profits. However, an univariate optimization problem can be obtained in  $n_{it}$  by embedding both constraints into the problem, since by choosing  $n_{it}$ , the firm determines implicitly  $\psi_{it}$ . So that the following first-order condition can be derived:

$$\frac{Z_t}{P_t} = \frac{P_t^{ws}}{P_t} \frac{(1 - \alpha) Y_{it}^{ws}}{n_{it}} - W_t H_{it} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{n_t, it+1} \quad (20)$$

Using the envelop theorem, one obtains:

$$J_{n_{t-1}, it} = (1 - \delta_{t-1}) \frac{Z_t}{P_t} \quad (21)$$

By taking equation (21) one period forward and plugging it in equation (20), the following Euler equation is derived:

$$\frac{Z_t}{P_t} = \frac{P_t^{ws}}{P_t} \frac{(1 - \alpha) Y_{it}^{ws}}{n_{it}} - W_t H_{it} + (1 - \delta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{Z_{t+1}}{P_{t+1}} \quad (22)$$

The expected cost of hiring a new worker is equal to the expected value of a match. So the real marginal value of a new hire is the sum of the net real return of a worker, which corresponds to its marginal productivity net of the marginal labor cost that is defined as the hourly wage, plus the real value of the continued relationship with the same worker, realized with the probability  $(1 - \delta_t)$ . Therefore, the number of people hired,  $\psi_{it}$ , is such that the expected profit from maintaining a match with an existing worker is equal to

<sup>10</sup>Since firms are owned by households, profits are evaluated in terms of utility brought to them.



the expected costs of posting a new vacancy.

### *Optimal financial contact*

The financial intermediation is realized through a large number of atomistic risk-neutral banks. Banks are assumed to hold enough large and diversified portfolios to ensure perfect risk pooling for their main creditors, the households.<sup>11</sup> Under a costly state verification framework, the perfectly competitive financial intermediaries' setting ensures that each firm-bank pair will write the borrowing contract that maximizes the expected return of the borrower under the constraint that the expected return to the lender exceeds its opportunity cost, namely the risk free interest rate,  $R_t$ . So the optimal incentive-compatible financial arrangement is just a standard risky debt contract, whose terms are the optimal solution to a standard principle-agent problem between entrepreneurs and banks<sup>12</sup>. Unlike the costly state verification framework similar to [Carlstrom and Fuerst \(1998\)](#), credit contracts are here nominal. They stipulate one-period loans, established after all aggregate shocks have occurred and repaid at the end of the same period. Then, each entrepreneur draws an idiosyncratic shock,  $\omega_{it}$ , reflecting its management skills, recruitment efficiency, hires' quality and input utilization skills, which is the source of wholesale firms' heterogeneity.  $\omega_{it}$  is i.i.d. with a continuous distribution function  $\Phi(\cdot)$  and a density function  $\phi(\cdot)$ , defined over a non-negative support, has a mean of unity and  $\Phi(0) = 0$ . Moreover, its variance, reflecting the shock's volatility and its entrepreneurs' riskiness, is time-varying and its standard deviation,  $\sigma_t^\omega$ , follows a first-order auto-regressive process given by:

$$\log \sigma_t^\omega = (1 - \rho_\sigma) \log(\sigma^\omega) + \rho_\sigma \log \sigma_{t-1}^\omega + u_t^\sigma, \quad \rho_\sigma \in [0, 1) \quad \text{where } u_t^\sigma \stackrel{iid}{\sim} N(0, \sigma_\sigma^2)$$

$\sigma^\omega$  is the steady-state value of the standard deviation,  $\sigma_t^\omega$ .

In order to eliminate aggregate uncertainty from the lender-borrower relationship, the aggregate technology shock,  $A_t$ , is assumed to be observed by all agents in the economy and it is realized before any loan contract is established. On the other hand, the idiosyncratic shock,  $\omega_{it}$ , is privately observed by the entrepreneur  $i$  after the production takes place and it can be verified by the lender only at a monitoring cost,  $\mu_t$ , proportional to the realized value of the firm. Thus, this private information creates a moral hazard problem as the entrepreneur may be encouraged to under-report the true value of his or her production.

Then, to justify the recourse to the external funding, entrepreneurs are supposed to pay their real production bill,  $r_t^K K_{it} + W_t N_{it}$ , as well as the real vacancies posting costs,  $\gamma V_{it}$ , before production occurs. But due to idiosyncratic shocks, firms face default risk on their debt. For low values of  $\omega_{it}$ , some firms may not be able to reimburse the credit. Let  $B_{it}$  be the total real amount of the production bill for a firm  $i$ , so that:

$$B_{it} = W_t N_{it} + r_t^K K_{it} + \gamma V_{it}$$

The wholesale firm borrows an amount of  $P_t(B_{it} - X_{it})$  from the bank at a implicit interest rate  $R_t^l$ , where  $X_{it}$  is his or her real net worth. Entrepreneurs and banks agree on a financial contract specifying a break-even entrepreneur-specific productivity level  $\bar{\omega}_{it}$  satisfying  $R_t^l P_t(B_{it} - X_{it}) = \bar{\omega}_{it}(P_t^{ws} Y_t^{ws} + Z_t \psi_{it})$ . If  $\omega_{it} < \bar{\omega}_{it}$ , the firm is insolvent and the bank confiscates the total output produced. The bank can observe this state of nature at a monitoring cost  $\mu_t \in (0, 1)$ , a fraction of the realized value of the firm. After the

<sup>11</sup>Infinitely-lived households are risk averse, but they become risk neutral for the financial contract. [Carlstrom and Fuerst \(1998\)](#) explain this fact by the absence of uncertainty about the term of the one-period contract since the aggregate uncertainty is realized before the contract establishment. Furthermore, by the law of large numbers as banks are financing a continuum of different entrepreneurs, households know they will receive the expected return of the idiosyncratic shock.

<sup>12</sup>See [Townsend \(1979\)](#), [Gale and Hellwig \(1985\)](#)

realization of shocks and production occurrence, this firm's value is given by  $\omega_{it}(P_t^{ws}Y_t^{ws} + Z_t\psi_{it})$ . The monitoring is non-stochastic and the lender actions are pre-committed. If  $\omega_{it} \geq \bar{\omega}_{it}$ , the entrepreneur pays back the value  $\bar{\omega}_{it}(P_t^{ws}Y_t^{ws} + Z_t\psi_{it})$ , the loan amount augmented with interest. So, this framework breaks down the Modigliani-Miller theorem and makes the firms' external borrowing costs higher than internal funds opportunity costs. Indeed, firms must borrow at a premium over the risk-free rate as seen below.

The expected return earned by the firm  $i$ ,  $E_{it}^f$ , is given by:

$$E_{it}^f = (P_t^{ws}Y_{it}^{ws} + Z_t\psi_{it}) \int_{\omega > \bar{\omega}_{it}} (\omega - \bar{\omega}_{it})\phi(\omega)d\omega$$

Using the statistic properties of the random idiosyncratic shock, equation (23) can be developed to see that the expected return of the firm is a fraction of its total realized value:

$$E_{it}^f = (P_t^{ws}Y_{it}^{ws} + Z_t\psi_{it})f(\bar{\omega}_{it}) \quad (23)$$

where  $f(\bar{\omega}_{it}) = \int_{\omega > \bar{\omega}_{it}} \omega\phi(\omega)d\omega - \bar{\omega}_{it}[1 - \Phi(\bar{\omega}_{it})]$ . Note that  $f(\bar{\omega}_{it}) \in (0, 1)$ <sup>13</sup> and  $f'(\bar{\omega}_{it}) = \Phi(\bar{\omega}_{it}) - 1 \leq 0$ . The firm's expected return is a decreasing function of  $\bar{\omega}_{it}$ . This result is quite intuitive since an increase of the default rate  $\Phi(\bar{\omega}_{it})$  reduces the gross share of return going to the firm.

Similarly, the expected return earned by the bank,  $E_{it}^b$ , is given by:

$$E_{it}^b = (P_t^{ws}Y_{it}^{ws} + Z_t\psi_{it}) \left[ \int_{\omega > \bar{\omega}_{it}} \bar{\omega}_{it}\phi(\omega)d\omega + (1 - \mu_t) \int_{\omega < \bar{\omega}_{it}} \omega\phi(\omega)d\omega \right]$$

It is straightforward to show that lender's expected return is also a fraction  $g(\bar{\omega}_{it}) \in (0, 1)$ , of the total return of the wholesale firm<sup>14</sup>. Then,

$$E_{it}^b = (P_t^{ws}Y_{it}^{ws} + Z_t\psi_{it})g(\bar{\omega}_{it}). \quad (24)$$

where,

$$g(\bar{\omega}_{it}) = 1 - f(\bar{\omega}_{it}) - \mu_t\Gamma(\bar{\omega}_{it})$$

with  $\Gamma(\bar{\omega}_{it}) = \int_{\omega < \bar{\omega}_{it}} \omega\phi(\omega)d\omega$ . An amount of the realized firm's value,  $\mu_t\Gamma(\bar{\omega}_{it})$ , is lost due to monitoring in cases of declaring bankruptcy by the borrower. In this context, the monitoring cost spending is a synonym of bankruptcy and it is spent in terms of currency so that bankruptcy has no impact on the real output. As a consequence, the proportion of the total amount recovered by the bank in case of bankruptcy is  $(1 - \mu_t)\Gamma(\bar{\omega}_{it})$ .  $(1 - \mu_t)$  is interpreted as the recovery rate. Following Livdan et al. (2009) and Petrosky-Nadeau (2014), this recovery rate is assumed to be time-varying and to have the following specification:

$$1 - \mu_t = s_{0,t} \exp s_1(\omega_{it} - 1) \quad (25)$$

$s_1$  is the elasticity of the recovery rate to the entrepreneurial productivity level and  $s_{0,t}$  is interpreted as a credit shock, following a first-order auto-regressive process:

$$\log s_{0,t} = (1 - \rho_{s0}) \log s_0 + \rho_{s0} \log s_{0,t-1} + u_t^{s0}, \quad \rho_{s0} \in [0, 1) \quad \text{where } u_t^{s0} \stackrel{iid}{\sim} N(0, \sigma_{s0}^2)$$

<sup>13</sup>  $f'(\bar{\omega}) \leq 0, \forall \bar{\omega} \in [0, \infty)$ . In addition,  $\lim_{\bar{\omega} \rightarrow 0} f(\bar{\omega}) = 1$  and  $\lim_{\bar{\omega} \rightarrow \infty} f(\bar{\omega}) = 0$ . Then,  $f(\bar{\omega}) \in (0, 1)$ .

<sup>14</sup> Since  $1 - f(\bar{\omega}) \in (0, 1)$  and by definition, we have  $\int_{\omega < \bar{\omega}_{it}} \omega\phi(\omega)d\omega \in (0, 1)$  and  $\int_{\omega > \bar{\omega}_{it}} \bar{\omega}_{it}\phi(\omega)d\omega + \int_{\omega < \bar{\omega}_{it}} \omega\phi(\omega)d\omega \in (0, 1)$ , then  $g(\bar{\omega}) \in (0, 1)$ ,  $\lim_{\bar{\omega} \rightarrow 0} g(\bar{\omega}) = 0$  and  $\lim_{\bar{\omega} \rightarrow \infty} g(\bar{\omega}) = 1 - \mu$ .

where  $s_0$  is the steady-state value of the credit shock,  $s_{0,t}$ .

The optimal contract is a solution to a maximization problem, where the entrepreneur maximizes its expected return subject to the bank's participation constraint (Paustian (2004)). The bank is willing to lend funds only and only if the contract yields an expected return greater or equal to the riskless rate of return,  $R_t$ :

$$\begin{aligned} \max_{K_{it}, H_{it}, V_{it}, \bar{\omega}_{it}} & [P_t^{ws} Y_{it} + Z_t \psi_{it}] f(\bar{\omega}_{it}) \\ \text{subject to} & [P_t^{ws} Y_{it} + Z_t \psi_{it}] g(\bar{\omega}_{it}) \geq R_t P_t (W_t N_{it} + \gamma V_{it} + r_t^K K_{it} - X_{it}) \\ & \psi_{it} = p_t V_{it} \\ & N_{it} = n_{it} H_{it} \end{aligned} \quad (26)$$

The first-order conditions for the firm's problem are summarized by the three following equations:

$$Y_K(K_{it}, H_{it} | n_{it}) = \frac{P_t}{P_t^{ws}} r_t^K R_t S_{it} \quad (27)$$

$$Y_H(K_{it}, H_{it} | n_{it}) = \frac{P_t}{P_t^{ws}} n_{it} W_t R_t S_{it} \quad (28)$$

$$\frac{Z_t}{P_t} = \frac{\gamma}{p_t} R_t S_{it} \quad (29)$$

where  $S_{it} = \{1 - \mu_t [\Gamma(\bar{\omega}_{it}) + \bar{\omega}_{it} h(\bar{\omega}_{it}) f(\bar{\omega}_{it})]\}^{-1}$ , with  $h(\bar{\omega}_{it})$  the hazard rate<sup>15</sup> defined by  $h(\bar{\omega}_{it}) = \frac{\phi(\bar{\omega}_{it})}{1 - \Phi(\bar{\omega}_{it})}$ .

Equations (27) and (28) show that the ratio of marginal products of inputs is equal for all firms, so that the capital-labor ratio is constant across firms because of the linearity assumption on the monitoring technology and the homogeneity of the Cobb-Douglas production function. Therefore,  $S_{it}$  does not depend on  $i$  and the threshold value of the entrepreneurial productivity,  $\bar{\omega}_{it}$ , is identical for all firms. Consequently, the subscript  $i$  can be dropped in what follows. So, the assets' distribution among entrepreneurs does not matter for the equilibrium<sup>16</sup>.

Further, asymmetric information in the credit market generates inefficiencies in both markets: the wholesale-goods market and the labor market. On the one hand, the respective marginal productivity of capital and labor are higher than their corresponding real marginal costs. The final real price of the wholesale good is augmented by a financial mark-up,  $S_t > 1$ <sup>17</sup>, used to overcome the agency problem between entrepreneurs and banks. As a consequence, credit market conditions matter because they affect firms' marginal costs and are transmitted to the rest of the economy through the selling price's mark-up. Aggregating over entrepreneurs and embedding equations (27)-(29) in the lender's break even constraint, the firms' leverage ratio,  $\mathcal{L}_t$ , is given by:

$$\mathcal{L}_t \equiv \frac{B_t}{X_t} = \frac{1}{1 - S_t(\bar{\omega}_t) g(\bar{\omega}_t)} \quad (30)$$

with,  $\mathcal{L}_{\bar{\omega}} > 0$ . For a given level of net worth  $X_t$ , a higher leverage ratio est associated with a high default rate. The probability of default increases as the loan amount raises (see Stiglitz and Weiss (1981)).

<sup>15</sup>We assume that  $\bar{\omega}h(\bar{\omega})$  is increasing in  $\bar{\omega}$  in order to ensure the concavity of the lender's net share of return,  $g(\bar{\omega})$ , and avoid any credit rationing at the equilibrium. This regularity condition is without loss of generality and it is satisfied by most of the continuous probability distributions. See Bernanke et al. (1999) for details.

<sup>16</sup>The ex-post heterogeneity among entrepreneurs is introduced by the level of their idiosyncratic productivity at each period, which makes their wealth fluctuating over time. Thanks to the linearity assumptions, the aggregation over entrepreneurs is possible and complications stemming from heterogeneity are ruled out.

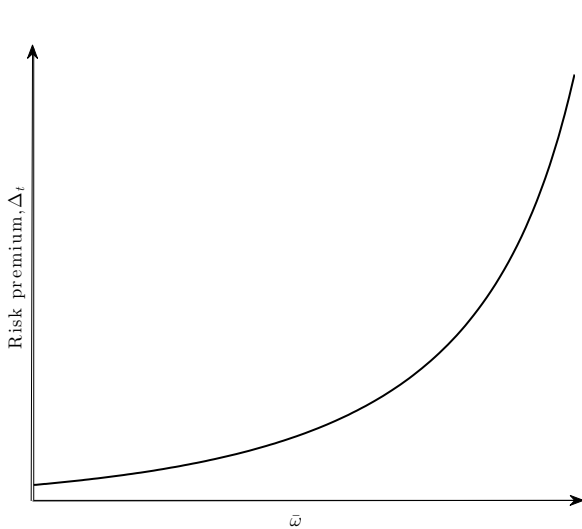
<sup>17</sup>See appendix A for the proof.

It is straightforward to show that, at the optimum, equation (26) is binding. Using this result with the condition on  $\bar{\omega}_t$ , the risk premium,  $\Delta_t$ , defined as the ratio of the lending rate to the risk free rate,  $R_t^l/R_t$ , is given by:

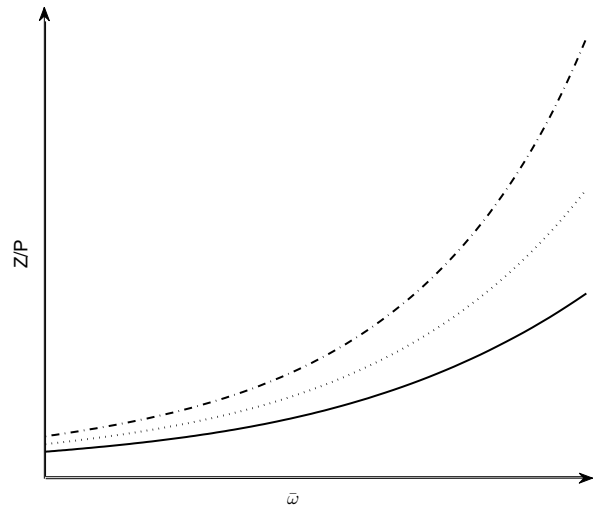
$$\Delta_t = \frac{\bar{\omega}_t}{g(\bar{\omega}_t)}$$

Figure 1. shows the evolution of the risk premium as a convex increasing function of the entrepreneurial productivity threshold,  $\bar{\omega}$ . This result is quite familiar in the financial accelerator literature. A higher default probability of firms induces a higher cost of lending for banks and consequently, a higher loan spread. In that case, an expansionary monetary policy for example affects the loan rate by decreasing the opportunity cost of lending funds for banks. The loan rate decreases more than one-to-one with respect to the risk-free rate: firms are able to repay their debts more easily and the default probability must decrease. As a result, in equilibrium, credit spreads, average financial distortion and the mark-up must fall.

On the other hand, the real value of a new hire depends also on the financial contract conditions. Besides the unit cost stemming from hiring,  $\gamma$ , and the average duration of vacancies,  $1/p_t$ , the total cost of vacancy posting is augmented by the same financial mark-up. Financial contract conditions affect the labor market efficiency through the total vacancy posting cost, that becomes an endogenous variable. This relation is presented by figure 2., where the real posting cost is also an increasing and convex function of  $\bar{\omega}$  and its slope raises with monitoring costs,  $\mu$ . For a higher default likelihood (higher  $\bar{\omega}$ ), banks charge a higher risk premium,  $\Delta$ , so that entrepreneurs obtain their credit at a higher lending rate,  $R^l$ , which makes their external funds more expensive and reduces their willingness to open vacancies.



**Figure 1:** The risk premium as a function of  $\bar{\omega}$



**Figure 2:** Real vacancy posting cost as a function of  $\bar{\omega}$  for different values of monitoring costs:  $\mu = 0.15$  (solid line),  $\mu = 0.2$  (dotted line) and  $\mu = 0.25$  (dashed line)

An increase of  $\mu$  shifts the real vacancy posting cost upward. For a fixed level of  $\bar{\omega}$ , the real cost of a new hire raises with monitoring costs. As these costs are expressed in terms of currency, and not in terms of physical goods, they do not generate a loss of resources through a destruction of goods, which could have been used for consumption, but they generate an additional cost taken into account by banks when agreeing on an appropriate interest rate on loans. Fluctuations in monitoring costs and bankruptcy rates will have an impact on welfare only indirectly, through their implications on the mark-up or on the amount of loans.

Now, by making use of the optimal financial contract conditions derived below and the hiring condition

given by equation (22), the job creation condition under financial frictions in the credit market is obtained:

$$\frac{\gamma}{p_t} R_t S_t = W_t H_t [R_t S_t - 1] + (1 - \delta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\gamma}{p_{t+1}} R_{t+1} S_{t+1} \quad (31)$$

For any positive cost of monitoring, financial frictions increase as expected the average cost of filling a vacancy.

#### 4.4.2 Intermediate-good production

A continuum of retailers of measure one buy the homogeneous good at the price  $P_t^{ws}$  produced by wholesale firms and differentiate it costlessly into intermediate goods varieties indexed by  $j$ . The intermediate-goods sector is thus made of monopolistically competitive firms. These firms are assumed to have no other inputs or costs than the homogeneous good. Hence, the price index of the composite good is given by:

$$P_t = \left[ \int_0^1 P_t(j)^{1-\epsilon_t} dj \right]^{\frac{1}{1-\epsilon_t}} \quad (32)$$

where  $\epsilon_t > 1$  is a time-varying parameter governing the degree of monopolistic competition (or the time-varying elasticity of substitution between intermediate goods in final-goods production). It follows an exogenous first-order auto-regressive process:

$$\log \epsilon_t = (1 - \rho_\epsilon) \log(\epsilon) + \rho_\epsilon \log \epsilon_{t-1} + u_t^\epsilon, \quad \rho_\epsilon \in [0, 1) \quad \text{where } u_t^\epsilon \stackrel{iid}{\sim} N(0, \sigma_\epsilon^2) \quad (33)$$

where  $\epsilon$  is the steady-state value for the degree of monopolistic competition,  $\epsilon_t$ .

Following Yun (1996) and Christiano et al. (2005), a simple price stickiness is formulated in the spirit of Calvo (1983): nominal prices are not subject to continuous revisions. Every period, only a random fraction  $\varrho$  of firms, where  $\varrho \in [0, 1)$ , is able to fully reoptimize their nominal prices knowing the aggregate shock,  $A_t$ . This fraction  $\varrho$  of firms, the so-called flexible-price firms, sets its price,  $P_t^{fl}$ , after the realization of period  $t$  uncertainty and before the realization of monetary policy shocks. The remaining fraction of firms, the so-called fixed-price firms, sets its price,  $P_t^{fx}$ , before any shock's realization and indexes it to the most recent aggregate rate of inflation, namely the lagged inflation. Thus, all firms adjust their prices each period.

The consumer-price index or the average price level is a CES aggregate of all prices in the economy at  $t$ . A costless price regulation mechanism is assumed, which guarantees that a consumer pays the same price whatever the firm at which he or she realizes his or her purchases<sup>18</sup>. So, the uniform consumer-price index corresponds to a weighted average price of the fraction  $\varrho$  of firms who can adjust their prices after the aggregate shock,  $A_t$ , and the fraction  $(1 - \varrho)$  of firms who can not. The sum in equation (32) can be transformed into a convex combination of two prices because flexible-price firms will choose the same optimal reset price and fixed-price firms will do also the same, since they are symmetric. So that equation (32) becomes:

$$P_t = \left[ \varrho (P_t^{fl})^{1-\epsilon_t} + (1 - \varrho) (P_t^{fx})^{1-\epsilon_t} \right]^{\frac{1}{1-\epsilon_t}} \quad (34)$$

Fixed-price firms, which can modify their prices only before the realization of period  $t$  uncertainty, maximize their profit,  $\Pi_t^{fx}$ , at time  $t$ :

$$\Pi_t^{fx} = E_{t-1} \left[ \frac{P_t^{fx} - P_t^{ws}}{P_t} Y_t^{fx} \right]$$

<sup>18</sup>The matching of consumers and firms is ignored. See xx for more details.

where  $Y_t^{fx}(Y_t^{fl})$  is the quantity of output sold by the the fixed(flexible)-price retailer. Then, the price  $P_t^{fx}$  is set to satisfy:

$$E_{t-1} \left[ P_t^{\epsilon-1} Y_t^{fx} ((\epsilon_t - 1) P_t^{fx} - \epsilon_t P_t^{ws}) \right] = 0 \quad (35)$$

Flexible-price firms set their price such that  $P_t^{fl}$  is equal to a mark-up over the marginal cost,  $P_t^{ws}$ , so the monopoly standard mark-up formula is obtained:

$$P_t^{fl} = \frac{\epsilon_t}{\epsilon_t - 1} P_t^{ws}, \quad \text{where } \frac{\epsilon_t}{\epsilon_t - 1} > 1 \quad (36)$$

Finally, the wholesale-goods market clearing condition suggests that the total output, after realization of the entrepreneurial idiosyncratic shock,  $\omega_t$ , has to be absorbed by fixed-price and flexible-price firms' input demands, respectively:

$$\omega_t Y_t^{ws} = \varrho Y_t^{fl} + (1 - \varrho) Y_t^{fx} \quad (37)$$

#### 4.4.3 Final-goods production

Final-goods firms proceed in a perfectly competitive market. They purchase a continuum of differentiated intermediate goods,  $Y_t(j)$ , at the nominal price  $P_t(j)$  and aggregate these varieties to produce  $Y_t^f$  units of the final good. The total quantity of final goods produced is given by the CES aggregator of the different quantities of intermediate goods. Given that fixed-price firms and flexible-price firms are symmetric, final goods are produced using a standard constant return to scale technology given by:

$$Y_t^f = \left[ \int_0^1 Y_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}} dj \right]^{\frac{\epsilon_t}{\epsilon_t-1}} = \left[ \varrho \left[ Y_t^{fl} \right]^{\frac{\epsilon_t-1}{\epsilon_t}} + (1 - \varrho) \left[ Y_t^{fx} \right]^{\frac{\epsilon_t-1}{\epsilon_t}} \right]^{\frac{\epsilon_t}{\epsilon_t-1}} \quad (38)$$

Output of the final good is equal to a weighted average sum of the intermediate goods output. Each competitive final-good firm chooses its demand function for each variety of intermediate goods,  $Y_t(j)$ , so as to maximize its nominal profit,  $\Pi_t^f$ :

$$\Pi_t^f = P_t Y_t^f - \int_0^1 P_t(j) Y_t(j) dj$$

where  $P_t$  is the bundler's technology price-index that corresponds to the consumer's price index and  $P_t(j)$  is the input price. Both prices are taken as given. The solution to the maximization problem<sup>19</sup> yields the following demand function for the intermediate good of variety  $j$ :

$$Y_t(j) = \left[ \frac{P_t(j)}{P_t} \right]^{-\epsilon_t} Y_t^f \quad (39)$$

So, the demand for each variety of intermediate good is a downward sloping demand curve, which gives to the intermediate-goods flexible-price firms some pricing power. According to equation (36), the mark-up depends negatively on the time-varying elasticity of substitution,  $\epsilon_t$ . The higher the elasticity, the lower the mark-up is and the higher is the demanded quantity.

In a competitive setting, the zero-profit condition applies at the equilibrium and it yields:

$$P_t Y_t^f = \int_{j=0}^1 P_t(j) Y_t(j) dj$$

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<sup>19</sup>Final-goods firms maximize their expected stream of profits, which is equivalent to maximizing their profit period by period since they purchase intermediate goods at the same frequency.

From this condition, the output price can be easily derived:  $P_t$  is the minimum cost of producing one unit of final goods.

The final-goods market clearing condition implies that final goods,  $Y_t^f$ , may be consumed by households,  $C_t$ , and entrepreneurs,  $C_t^e$ , or be used as investment,  $I_t$ , government expenditures,  $G_t$ , as capital utilization costs,  $\Upsilon(\nu_t)K_{t-1}^p$ , or as job creation costs,  $\gamma R_t S_t V_t$ , (augmented by the costs of credit market imperfections). The following aggregate resource constraint is thus obtained:

$$Y_t^f = C_t + C_t^e + I_t + G_t + \Upsilon(\nu_t)K_{t-1}^p + \gamma R_t S_t V_t \quad (40)$$

#### 4.4.4 Entrepreneurs

Entrepreneurs live infinitely and their time-discount factor is the same as households. The birth of rate of entrepreneurs is supposed to ensure that the fraction of agents who are entrepreneurs is constant. They aim to maximize their stream of consumption by resolving:

$$\max_{C_t^e} E_t \sum_{t=0}^{\infty} \beta^t C_t^e \quad (41)$$

In order to ensure that entrepreneurs do not accumulate net worth and to avoid any eventual self-financing, each entrepreneur is assumed to have a constant probability  $(1 - \varsigma)$  to die next period. It will limit the size of aggregate net worth in an infinite horizon set up. Indeed, since the rate of return on internal funds is higher than the one of external funds, due to asymmetric information on credit markets, risk neutral entrepreneurs may be willing to postpone consumption and would only accumulate funds. Therefore, solvent entrepreneurs are assumed to consume all their net worth if they exit the economy at the end of the period. Thus, the aggregate entrepreneurial consumption,  $C_t^e$ , is given by:

$$C_t^e = (1 - \varsigma) \frac{P_t^{ws}}{P_t} Y_t^{ws} f(\bar{\omega}_t) \quad (42)$$

On the other hand, solvent entrepreneurs that are not exiting the economy keep accumulating net worth using their realized return. Consequently, the evolution of the aggregate entrepreneurial real net worth is given by:

$$X_{t+1} = \varsigma \frac{P_t^{ws}}{P_t} Y_t^{ws} f(\bar{\omega}_t) \quad (43)$$

## 4.5 Labor market and wage bargaining

### 4.5.1 Nash bargaining and wage setting

#### *Bellman equations*

The real hourly wage is assumed to be determined on a period-by-period basis and through a Nash bargaining between a representative firm and a representative worker. The Nash real hourly wage splits the joint surplus of the employment relationship between them, depending on their respective bargaining power.

Given that the worker's welfare is independent of the number of employed persons by the firm at which he or she is working, all workers have the same welfare (and thus the same surplus). So that, the

discounted value of employment for a worker in terms of current consumption at time  $t$  is denoted by  $\mathcal{W}_t^N$  and given by:

$$\mathcal{W}_t^N = W_t H_t - \frac{\varepsilon_t^H H_t^{1+\tau}}{(1+\tau)\lambda_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \delta_t(1 - q_{t+1}))\mathcal{W}_{t+1}^N + \delta_t(1 - q_{t+1})\mathcal{W}_{t+1}^U \right] \quad (44)$$

The discounted value of a job for a worker in terms of current consumption is the sum of the total real wage earned, reduced for the marginal disutility of working and the expected discounted gain from being either employed or unemployed during the subsequent period. A worker will be again employed at the period  $t + 1$  if the match has not been destroyed, with a probability  $(1 - \delta_t)$ , after the matching of the period  $t$ ; or if the match has been destroyed with the probability  $\delta_t$  after the matching of the period  $t$ , but that another matching occurs at the period  $t + 1$  with the probability  $q_{t+1}$ . And a worker will become unemployed at the period  $t + 1$  if the match is destroyed after the matching of the period  $t$  and if he or she does not find a job at the period  $t + 1$ , that to say with the probability  $\delta_t(1 - q_{t+1})$ .

The discounted value of unemployment for a worker in terms of current consumption at time  $t$  is denoted by  $\mathcal{W}_t^U$  and given by:

$$\mathcal{W}_t^U = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1}\mathcal{W}_{t+1}^N + (1 - q_{t+1})\mathcal{W}_{t+1}^U \right] \quad (45)$$

In the same spirit, the discounted value to be unemployed for a worker in terms of current consumption is the sum of real unemployment benefits and the expected discounted gain from either being employed (with probability  $q_{t+1}$ ) or unemployed (with probability  $1 - q_{t+1}$ ) after the matching of the period  $t + 1$ . Therefore, the worker's surplus of an employment relationship is given by:

$$\mathcal{W}_t^N - \mathcal{W}_t^U = W_t H_t - \frac{\varepsilon_t^H H_t^{1+\tau}}{(1+\tau)\lambda_t} - b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \delta_t)(1 - q_{t+1})(\mathcal{W}_{t+1}^N - \mathcal{W}_{t+1}^U) \right] \quad (46)$$

For a firm, the discounted value of an employed worker at time  $t$  in terms of current consumption is defined as:

$$J_{n,t} = \frac{P_t^{ws}}{P_t} \frac{(1 - \alpha)Y_t^{ws}}{n_t} - W_t H_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - \delta_t) J_{n,t+1} \quad (47)$$

This discounted value is equal to the current profits from an employed worker, plus the expected discounted continuation value. The job is still provided at the period  $t + 1$  if the match has not been destroyed with a probability  $(1 - \delta_t)$  after the matching of the period  $t$ . And if the job is not provided at the period  $t + 1$ , the continuation value will be equal to zero.

Then, the discounted value of an open vacancy for a firm in terms of current consumption at time  $t$  is given by:

$$V_t = -\frac{Z_t}{P_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ p_{t+1} J_{n,t+1} + (1 - p_{t+1}) V_{t+1} \right] \quad (48)$$

The discounted value of an open vacancy for a firm in terms of current consumption is equal to the real total hiring costs of the vacancy, plus the expected discounted gain to fill or not the vacancy during the following period. The vacancy will be filled at the period  $t + 1$  with a probability  $p_{t+1}$ , if a match occurs during the matching of this period. Otherwise, with a probability  $(1 - p_{t+1})$ , the job remains vacant in  $t + 1$ .

### *Nash bargaining*



The Nash wage bargaining consists of maximizing the net surplus of the employment relationship for a representative firm and worker's pair, depending on the workers' bargaining power,  $\eta_t$ . This latter is assumed to be time-varying and to follow an auto-regressive process given by:

$$\log \eta_t = (1 - \rho_\eta) \log(\eta) + \rho_\eta \log \eta_{t-1} + u_t^\eta, \quad \rho_\eta \in [0, 1) \quad \text{where } u_t^\eta \stackrel{iid}{\sim} N(0, \sigma_\eta^2)$$

As a consequence, the chosen real hourly wage is the one that maximizes:

$$\max_{W_t} (\mathcal{W}_t^N - \mathcal{W}_t^U)^{\eta_t} (J_{n,t} - V_t)^{(1-\eta_t)}$$

where  $(\mathcal{W}_t^N - \mathcal{W}_t^U)$  is the net surplus of workers (expressed in 46) and  $(J_{n,t} - V_t)$  is the net surplus of firms, for an employment relationship.

Since there is a free entry, at the equilibrium, the vacancy posting condition,  $V_t = 0$ , is satisfied. Indeed, if  $V_t > 0$ , a firm has an incentive to post vacancies as the value of a vacant job is positive. As the number of vacancies increases relative to the number of unemployed workers, the probability to fill a vacancy,  $p_t$ , decreases. Indeed, the labor market tightness,  $\theta_t$ , increases, thus  $p_t$  decreases since  $\partial p_t / \partial \theta_t < 0$ . That reduces the incentive to post vacancy jobs and diminishes the value of  $V_t$ , until it equals zero. Therefore, the first-order necessary condition for the Nash bargaining solution is given by:

$$\eta_t J_{n,t} = (1 - \eta_t)(\mathcal{W}_t^N - \mathcal{W}_t^U) \quad (49)$$

### *Wage setting*

The following Nash real hourly wage is obtained:

$$W_t H_t = \eta_t \frac{P_t^{ws}}{P_t} \frac{(1 - \alpha) Y_t^{ws}}{n_t} + (1 - \eta_t) \left[ b + \frac{\epsilon_t^H H_t^{1+\tau}}{(1 + \tau) \lambda_t} \right] + (1 - \eta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - \delta_t) p_{t+1} \theta_{t+1} (\mathcal{W}_{t+1}^N - \mathcal{W}_{t+1}^U) \quad (50)$$

The wage shares costs and benefits from the match between workers and firms according to the parameter  $\eta_t$ . Workers obtain a fraction  $\eta_t$  of the firm's revenues and are compensated for a fraction  $(1 - \eta_t)$  for the disutility they suffer from supplying hours of work and for the foregone unemployment benefits (workers' outside opportunities). A new element is the expected labor market tightness. If a matching is broken, workers and entrepreneurs have to look for another partners in next periods, which is costly. This cost is thus incorporated in the wage. Rearranging equation (50) by taking into consideration the equilibrium prices given by equations (27) and (28), the following new wage equation is obtained:

$$W_t H_t = \frac{1 - \eta_t}{1 - \eta_t R_t S_t} \left[ b + \frac{\epsilon_t^H H_t^{1+\tau}}{(1 + \tau) \lambda_t} \right] + \frac{1 - \eta_t}{1 - \eta_t R_t S_t} \left[ \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - \delta_t) p_{t+1} \theta_{t+1} (\mathcal{W}_{t+1}^N - \mathcal{W}_{t+1}^U) \right] \quad (51)$$

As a consequence, asymmetric information in financial market altered significantly the real bargaining power of firms and workers.

## 4.6 Monetary and fiscal policy

### *Monetary policy*

The monetary policy is carried out following a Taylor-type rule, by revising the nominal interest rate in response to deviations in output growth and inflation from its steady-state level. So the gross nominal interest rate,  $R_t$ , follows:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[ \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\rho_Y} \right]^{1-\rho_R} u_t^R \quad (52)$$

where  $R$  is the steady state nominal gross interest rate,  $\pi$  is the steady state inflation rate,  $\rho_R$  is the degree of interest rate smoothing and  $\rho_Y$  and  $\rho_\pi$  are the response coefficients to output and inflation variables.  $u_t^R$  is the monetary policy shock, following a first-order auto-regressive process:

$$\log \xi_t^R = \rho_{\xi_R} \log \xi_{t-1}^R + u_t^R, \quad \rho_{\xi_R} \in [0, 1) \quad \text{where } u_t^R \stackrel{iid}{\sim} N(0, \sigma_{\xi_R}^2)$$

### ***Fiscal policy***

The government receives lump-sum taxes,  $T_t$ , from households to finance real unemployment benefits for unemployed workers,  $(1 - n_t)b$ , and real exogenous government spending,  $G_t$ , such that:

$$G_t + (1 - n_t)b = \frac{T_t}{P_t} \quad (53)$$

where  $G_t$  obeys to:

$$G_t = \left(1 - \frac{1}{\epsilon_t^G}\right) Y_t^f \quad (54)$$

Following [Gertler et al. \(2008\)](#), the government spending shock,  $\epsilon_t^G$ , follows a first-order auto-regressive process given by:

$$\log \epsilon_t^G = (1 - \rho_G) \log \epsilon_t^G + \rho_G \log \epsilon_{t-1}^G + u_t^G, \quad \rho_G \in [0, 1) \quad \text{where } u_t^G \stackrel{iid}{\sim} N(0, \sigma_G^2)$$

where  $\epsilon^G$  is the long-run target level of government spending,  $\epsilon_t^G$ .

## **5 Equilibrium and log-linearized model**

The model describes the behavior of variables  $\{Y_t, C_t, D_t, K_t, n_t, H_t, U_t, V_t, R_t, P_t, S_t, RP_t, \omega_t, W_t, \theta_t, \pi_t, G_t, T_t\}$ . The equilibrium is then defined by equations and from representative household optimization, a job-creation condition, the optimal condition for the threshold, the definition of market tightness, the lender's participation constraint, a wage rule, aggregate entrepreneurial net worth, the monetary authority policy rule, the aggregate resource constraint and laws of motion for asset accumulation and aggregate unemployment and employment, and the specifications of the disturbances for the preference shock, the labor supply shock, the technological shock.

The equilibrium conditions do not have an analytical solution. The model will then be characterized by log-linearising it around the steady state.

As no analytical solutions are obtained for equilibrium conditions, the previous model is log-linearized around the deterministic stationary steady-state (around a zero inflation steady state). The dynamic nonlinear equations are thus replaced by dynamic linear equations. Interpretation and calculation are indeed made easier if the equations are linear in percent deviations from the steady state. Log-linearisation is indeed used to solve non-linear dynamic stochastic models by reducing the equations characterizing equilibrium with log-linear approximations (Taylor approximation) around the steady-state. The hat above variables denotes their logarithmic deviation from the steady state or the percentage deviation of variables from their steady state ( $\hat{x}_t = \log(\frac{x_t}{x})$ ). The steady-state of the model and the complete log-linearisation is founded in [7](#).

## 6 Model estimation

### 6.1 Data

The model is estimated using Bayesian techniques. Quarterly data are used for the sample period 1960Q1 through 2007Q4. Output is defined as the real gross domestic product; consumption as the real personal consumption expenditures; the nominal interest rate is defined as quarterly averages of Federal Funds rate; inflation is defined as the growth rate of the GDP deflator; unemployment and employment are defined as the civilian unemployment and employment rate (as a percentage of the civilian labor force); real wages are defined as the nonfarm business sector real compensation per hour and hours worked per employee are nonfarm business sector average weekly hours worked per employee. Investment is defined as the real private nonresidential fixed investment; default rate is defined as the default rate for rated US speculative-grade corporate bonds. Finally, government expenditures are defined as the real government consumption expenditures and gross investment<sup>20</sup>.

The data are demeaned, seasonally adjusted when necessary and the output, consumption, investment and government expenditures series are expressed in per capita terms.

### 6.2 Calibration

The literature is followed to calibrate some parameters. The household discount factor,  $\beta$ , is set to 0.99 in order to reach an annual real interest rate of 4 % as observed in data. The inverse of the elasticity of worked hours to real hourly wage (or the Frish elasticity),  $\tau$ , is equal to 1. The external habit persistence parameter is fixed to 0.57, between the estimation 0.5 of [Trigari \(2009\)](#), 0.5, and [Christiano et al. \(2005\)](#), 0.63.

For the wholesale-good sector, the capital share in output,  $\alpha$ , is standard 0.36 (based on calculation of [Kydland and Prescott \(1982\)](#) using US time series data). The capital depreciation rate,  $\delta_K$ , is fixed to 0.025 corresponding to an annual depreciation rate of 10 per cent, as in [King and Rebelo \(1999\)](#) and [Gertler et al. \(2008\)](#).

For the intermediate-good sector, the monopolistic mark-up or the elasticity of substitution across inputs,  $\epsilon$ , is fixed to 11 to have a conventional price-mark-up on marginal costs at 10 % as in [Walsh \(2005\)](#) and [Trigari \(2009\)](#). The Calvo stickiness of prices,  $\varrho$ , is set to 0.75 as in [Smets and Wouters \(2007\)](#). Thus, the average period between price adjustments is about 1.3 quarters. This is compatible with the estimates of [Álvarez et al. \(2006\)](#). This timing is important because it will determine the adjustment speed of inflation to shocks. Finally, the entrepreneurial survival rate is equal to 0.96 in line with [Bernanke and Gertler \(1995\)](#) and [Zanetti and Mumtaz \(2013\)](#).

The elasticity of matches to unemployment,  $\rho$ , is set to 0.5 as in [Trigari \(2009\)](#). This value is within the range of plausible values of 0.5 to 0.7 reported by [Petrongolo and Pissarides \(2001\)](#) in their survey of the literature on the estimation of matching function<sup>21</sup>. The conventional value, 0.5, is given to the bargaining power,  $\eta$ , as in [Gertler and Trigari \(2009\)](#). The vacancy posting cost,  $\gamma$ , is set to 0.01, as in [Andolfatto \(1996\)](#), [Walsh \(2005\)](#) and [Gertler et al. \(2008\)](#), which represents 1 percent of output. The vacancy-filling rate is set to 0.7 following [Trigari \(2009\)](#). A vacancy is then filled on average on 1.4 quarters. The job-finding rate is set to 0.45 according to [Shimer \(2005\)](#) and [Gertler et al. \(2008\)](#). The average time until a worker finds a job is 5 quarters. The exogenous destruction rate is set to 0.08 according to [Davis et al. \(1998\)](#) and [Trigari \(2009\)](#). It is compatible with those used in the literature which range from 0.07 in [Merz \(1995\)](#) to 0.15 in [Andolfatto \(1996\)](#). Finally, the unemployment benefits,  $b$ , is equal to 0.4.

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<sup>20</sup>More details about data used are available in Appendix 7.

<sup>21</sup>Others values are used such as 0.4 in [Blanchard and Diamond \(1989\)](#)), [Merz \(1995\)](#), [Andolfatto \(1996\)](#) and [Mortensen and Nagypal \(2007\)](#) or 0.72 in [Shimer \(2004\)](#).

The gross external finance premium,  $s_1$ , is set to 1.03, either 3 percent annualized.

For the Taylor rule, conventional value are also taken. The interest rate smoothing coefficient,  $\rho_r$ , is set to 0.85. Coefficients for the responses of interest rate to inflation,  $\rho_{pi}$  and to the output gap as in are fixed respectively to 1.5 and 0.125 as in Trigari (2009), who use these values because they are consistent with the estimates presented in Clarida, Gali and Gertler (2000)..

**Table 1:** Baseline calibration

Parameter	Value	Description
Preferences		
$\beta$	0.99	Households discount factor
$\tau$	1	Inverse of the elasticity of worked hours to real wage
$h$	0.57	External habit persistence
$\epsilon^H$		Steady-state of supplied hours shock
Technology		
$\alpha$	0.36	Capital share in production
$\delta_K$	0.025	Capital depreciation rate
$\varrho$	0.5	Calvo stickiness of prices
$\epsilon$	11	Elasticity of substitution across inputs
$\varsigma$	0.96	Entrepreneurial survival rate
Job market		
$\rho$	0.5	Elasticity of matches to unemployment
$\eta$	0.5	Steady-state worker bargaining power
$\gamma$	0.01	Unit cost of job vacancies
$b$	0.4	Unemployment benefits
$p$	0.45	Job-finding rate
$q$	0.7	Vacancy-filling rate
$\epsilon^M$	1	Steady-state value of matching process efficiency
$\delta$	0.06	Job destruction rate
Financial market		
$s_1$	1.03	Gross external finance premium
$s_0$	0.9	
$\sigma^\omega$		Steady-state value of the standard deviation of entrepreneurs riskiness
Monetary and fiscal policy		
$\rho_R$	0.85	Interest rate smoothing coefficient
$\rho_\pi$	1.5	Response to inflation
$\rho_Y$	0.125	Response to output gap

### 6.3 Estimation

Structural parameters  $\{\rho_H, \rho_C, \rho_I, \rho_\delta, \rho_M, \rho_\sigma, \rho_A, \rho_{s_0}, \rho_\epsilon, \rho_G, \rho_\eta, \rho_{\xi_R}, \sigma_H, \sigma_C, \sigma_I, \sigma_\delta, \sigma_M, \sigma_\sigma, \sigma_A, \sigma_{s_0}, \sigma_\epsilon, \sigma_G, \sigma_\eta, \sigma_{\xi_R}\}$  are estimated by using Bayesian methods. These methods use information from existing microeconomic and calibration evidence on behavioral parameters and update it with new information as captured by the likelihood. The solution of the linearized model results in a state-space representation of the reduced form.

The Bayesian estimation technique enables to address the identification problems of reduced-form models and the potential misspecification problem in the comparison of DSGE models. Furthermore, it performs better than GMM and maximum likelihood methods for small data samples, as it is our case. Finally, Bayesian estimation technique permits to see which frictions are empirically important by comparing likelihood of various models.

The technique is the following. First, a parameter space of the model,  $\Theta$  and data observed,  $S^T = (s_t)_{t=1}^T$ , are defined. A relationship between the prior distribution of parameters,  $P(\Theta)$  and the conditional distribution of the likelihood function,  $P(\Theta|S^t)$ , is derived by using the joint probability distribution  $P(S^t, \Theta)$ . Thanks to Bayesian theory the posterior distribution of parameters  $P(S^t|\Theta)$  is defined. The likelihood function is combined with a prior density to obtain the posterior distribution of parameters.

Since the model exhibits balanced growth, all non stationary variables have to be detrended in order for the model to be solvable by linear quadratic approximation around the stationary steady-state.

### 6.3.1 Prior distributions

Priors on the stochastic processes are set as follow. The standard errors of innovations are supposed to follow an inverse-gamma distribution with a mean of The autoregressive parameter in the law of motion for technology is set to the standard 0.95 and  $\sigma = 0.007$  as in Paustian (2004), Merz (1995). Or  $\sigma = 0.01$  as in Lechtaler and al. Or  $\sigma = 0.009$  as in Petrosky.

$\rho_{s0} = 0.95$   $\sigma_{s0}^2 = 0.16$  from Petrosky.

### 6.3.2 Posterior distributions

**Table 2:** Estimated parameters. Auto-correlation of shocks and standard deviation of innovations

Parameter	Description	Prior			Posterior		5%	95%
		Distr.	Mean	s.d.	Mean	s.d		
Auto-correlation								
$\rho_H$	Labor supply shock	$\beta$		0.075				
$\rho_C$	Consumption shock	$\beta$	0.85	0.075				
$\rho_\eta$	Bargaining power shock	$\beta$		0.075				
$\rho_I$	Investment-specific technological shock	$\beta$		0.075				
$\rho_\delta$	Separation rate shock	$\beta$		0.075				
$\rho_M$	Matching efficiency shock	$\beta$		0.075				
$\rho_\sigma$	Time-varying uncertainty	$\beta$		0.075				
$\rho_A$	Technology shock	$\beta$	0.95	0.075				
$\rho_{s_0}$	Recovery rate shock	$\beta$	0.95	0.075				
$\rho_\epsilon$	Monopolistic mark-up shock	$\beta$		0.075				
$\rho_G$	Government spending shock	$\beta$	0.9	0.075				
$\rho_{\xi_R}$	Monetary policy shock	$\beta$	0.5	0.075				
Standard deviations								
		Distr.	Mean	d.f	Mean	s.d		
$\sigma_H$	Labor supply	Inv- $\Gamma$						
$\sigma_C$	consumption	Inv- $\Gamma$						
$\sigma_I$	Investment-specific technology	Inv- $\Gamma$						
$\sigma_\delta$	Separation rate	Inv- $\Gamma$						
$\sigma_M$	Matching efficiency	Inv- $\Gamma$						
$\sigma_\sigma$	Time-varying uncertainty	Inv- $\Gamma$						
$\sigma_\eta$	Bargaining power shock	Inv- $\Gamma$						
$\sigma_A$	Technology	Inv- $\Gamma$	0.008					
$\sigma_{s_0}$	Recovery rate	Inv- $\Gamma$						
$\sigma_\epsilon$	Monopolistic mark-up	Inv- $\Gamma$						
$\sigma_G$	Government spending	Inv- $\Gamma$	0.008					
$\sigma_{\xi_R}$	Monetary policy	Inv- $\Gamma$	0.006					

### 6.3.3 Results

A new channel of monetary policy may be founded. Worker flows are seen as a new capital goods, as hiring is conditional of state of credit markets, and at the end it could have an impact on inflation dynamics.

## 7 Conclusion

# Appendices

## Appendix A

In this appendix, we prove that  $S_t > 1$  for  $\bar{\omega} \in [0, \infty)$ , where  $S_t = \{1 - \mu_t[\Gamma(\bar{\omega}_t) + \bar{\omega}_t h(\bar{\omega}_t) f(\bar{\omega}_t)]\}^{-1}$ . Using the assumption that  $\bar{\omega}_t h(\bar{\omega}_t)$  is increasing in  $\bar{\omega}_t$  and taking derivatives, we obtain

$$S'_t = \frac{\mu_t}{1 - \Phi(\bar{\omega}_t)} \frac{d(\bar{\omega}_t h(\bar{\omega}_t))}{d\bar{\omega}_t} \frac{1}{S_t^2} > 0.$$

Given this result,  $S_t$  is an increasing function of  $\bar{\omega}_t$ .

Now taking limits of  $\Gamma(\bar{\omega}_t)$  and  $\bar{\omega}_t h(\bar{\omega}_t) f(\bar{\omega}_t)$  at the lower bound of  $\bar{\omega}$ , we get

$$\lim_{\bar{\omega} \rightarrow 0} \Gamma(\bar{\omega}_t) = 0, \quad \lim_{\bar{\omega} \rightarrow 0} \bar{\omega}_t h(\bar{\omega}_t) f(\bar{\omega}_t) = 0$$

Thus,  $\lim_{\bar{\omega} \rightarrow 0} S_t = 1$ .

Combining the previous results, then,  $S_t \geq 1$ ,  $\forall \bar{\omega} \in [0, \infty)$ .

## Appendix B

**Data documentation: USA from 1960Q1 to 2007Q4, unless notified**

Real output per capita	=	$LN(GDPC09/CNP16OVIndex) * 100$
Real consumption per capita	=	$LN((PCEC/GDPDEF)/CNP16OVIndex) * 100$
Real investment per capita	=	$LN((PNFI/GDPDEF)/CNP16OVIndex) * 100$
Inflation	=	$LN(GDPDEF/GDPDEF(-1)) * 100$
Nominal interest rate	=	$FEDFUNDS/4$
Average weekly hours worked per employee	=	$LN((PRS85006023/CE16OVIndex)/CNP16OVIndex) * 100$
Unemployment rate	=	$UNRATE$
Employment rate	=	$(CE160V/CLF16OV) * 100$
Real wage per hour	=	$LN(COMPRNFB) * 100$
Total hours worked	=	$LN(PRS85006023) * 100$
Labor market tightness	=	$Vacancies\ rate/UNRATE$
Real government expenditure per capita	=	$LN(GCEC1/CNP16OVIndex) * 100$
Default rate	=	Moody's default rate
GZ Spread	=	Gilchrist and Zakrajsek data

### Sources and explanation

- GDP09: Real gross domestic product. Billions of chained 2009 dollars. Seasonally adjusted. Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- PCEC: Personal consumption expenditures. Billions of dollars. Seasonally adjusted. Source: U.S. Department of Commerce, Bureau of Economic Analysis.

- PNFI: Private nonresidential fixed investment. Billions of dollars. Seasonally adjusted annual rate.  
Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- GDPDEF: Gross domestic product implicit price deflator. Index 2009 = 100. Seasonally adjusted.  
Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- CNP16OV: Civilian non-institutional population: Sixteen years and over. Thousands of persons.  
Not seasonally adjusted.  
Source: U.S. Bureau of Labor Statistics.
- CNP16OVIndex:  $LNS10000000(1991:3) = 1$
- FEDFUNDS: Effective Federal Funds rate. Averages of daily figures. Percent.  
Source: Board of Governors of the Federal Reserve System.
- CE16OV: Civilian employment. Thousands of persons. Seasonally adjusted.  
Source: U.S. Bureau of Labor Statistics.
- UNRATE: Civilian unemployment rate. Percent. Seasonally adjusted.  
Source: U.S. Bureau of Labor Statistics.
- CLF16OV: Civilian labor force. Thousands of persons. Seasonally adjusted.  
Source: U.S. Bureau of Labor Statistics.
- COMPRNFB: Nonfarm business sector: real compensation per hour. Index 2009 = 100. Seasonally  
adjusted.  
Source: U.S. Bureau of Labor Statistics.
- PRS85006023: Nonfarm business sector: average weekly hours. Index 2009 = 100. Seasonally  
adjusted.  
Source: U.S. Bureau of Labor Statistics.
- Vacancies rate:
- Job creation rate: from 1972Q2 to 1998Q4.
- Job destruction rate: from 1972Q2 to 1998Q4.
- GCEC1: Real government consumption expenditures and gross investment. Billions of Chained 2009  
Dollars. Seasonally adjusted annual rate.  
Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- Default rate: from 1970Q1 to 2007Q4. Default rate for rated US speculative-grade corporate bonds.  
Percent.  
Source: Moody's.
- GZ Spread: from 1973Q1 to 2007Q4. Average credit spread on senior unsecured corporate bonds  
issued by non-financial firms. Percent.  
Source: Gilchrist and Zakrajsek.

Note that our estimation period ends in 2007Q4 as justified by Gali, Smets and Wouters (2012) in order to prevent estimations from being disturbed by nonlinearities induced by the lower bound on the Federal Funds rate and binding downward nominal wage rigidities during the recent recession.



## Appendix C

### Steady-state of the model

The variance of shock processes is set to 0, so as to the model converges to the steady-state.

Marginal utility of consumption:  $\lambda = (1 - \beta hE)(C - hC)^{-1}$

Consumption Euler equation:  $\lambda = \beta RE(\frac{\lambda}{\pi})$

Resource constraint:  $Y^f = C + C^e + G + \gamma RSV + \Upsilon(\nu)K$

Matching function:  $M = \epsilon^M U^\rho V^{(1-\rho)}$

Labor market tightness:  $\theta = \frac{V}{U}$

Probability for a firm to fill a vacancy job:  $p = \frac{M}{V}$

Probability for an unemployed worker to find a job:  $q = \frac{M}{U}$

Wholesale-good production:  $Y^{ws} = AK^\alpha N^{1-alpha}$

Number of employed workers (after the matching):  $\delta n = M$

Number of unemployed workers (after the matching):  $U = 1 - n$

Number of unemployed/searching workers (at the end of the period):  $U = 1 - (1 - \delta)n$

Number of hired workers:  $\psi = pV = M$

Vacancy posting equation:

Wage bargaining FOC:

Government spending:  $G = (1 - \frac{1}{\epsilon^G})Y^f$

Market clearing:  $Y^f = C + C^e + I + G + \gamma RSV + \Upsilon(\nu)K^p$

### Log-linearized model

Marginal utility of consumption:  $\hat{\lambda}_t = -(\hat{C}_t - h\hat{C}_{t-1})$

Consumption Euler equation:  $\hat{\lambda}_t = E_t(\hat{R}_t + \hat{\lambda}_{t+1} - \hat{\pi}_{t+1})$

Resource constraint:  $\hat{Y}^f = \hat{C}_t + \hat{C}_t^e + \hat{G} +$

Matching function:  $M = \hat{\epsilon}_t^M + \rho\hat{U}_t + (1 - \rho)\hat{V}_t$

Labor market tightness:  $\hat{\theta}_t = \hat{V}_t - \hat{U}_t$

Probability for a firm to fill a vacancy job:  $\widehat{p}_t = \widehat{M}_t - \widehat{V}_t$

Probability for an unemployed worker to find a job:  $\widehat{q}_t = \widehat{M}_t - \widehat{U}_t$

Wholesale-good production:  $\widehat{Y}^{ws} = \widehat{A}_t + (1 - \alpha)\widehat{N}_t + \alpha\widehat{K}_t$

Number of employed workers (after the matching):  $\widehat{n}_t = (1 - \delta)\widehat{n}_{t-1} + \frac{M}{n}\widehat{M}_t - \delta\widehat{\delta}_{t-1}$

Number of unemployed workers (after the matching):  $\widehat{U}_t = -\frac{n}{U}\widehat{n}_t$

Number of unemployed/searching workers (at the end of the period):  $\widehat{U}_t = -\frac{n}{U}(1 - \delta)\widehat{n}_t + \delta\frac{n}{U}\widehat{\delta}_t$

Number of hired workers:  $\widehat{\psi}_t = \widehat{p}_t + \widehat{V}_t = \widehat{M}_t$

Vacancy posting equation:

Government spending:  $\widehat{G}_t =$

Taylor rule:  $\widehat{R}_t = \rho_R\widehat{R}_{t-1} + (1 - \rho_R)[\rho_\pi\widehat{\pi}_t + \rho_Y(\widehat{Y}_t - \widehat{Y}_{t-1})] + \widehat{u}_t^R$

Market clearing:  $\widehat{Y}_t^f$

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