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# Distance-Based Measures of Spatial Concentration: Introducing a Relative Density Function

Gabriel Lang<sup>a</sup>, Eric Marcon<sup>b</sup>, Florence Puech<sup>c\*</sup>

## Abstract

For a decade, distance-based methods have been widely employed and constantly improved in the field of spatial economics. These methods are a very useful tool for accurately evaluating the spatial distribution of plants or retail stores, for example (Duranton and Overman, 2008). In this paper, we introduce a new distance-based statistical measure for evaluating the spatial concentration of economic activities. To our knowledge, the  $m$  function is the first relative density function to be proposed in the economics literature. This tool supplements the typology of distance-based methods recently drawn up by Marcon and Puech (2012). By considering several theoretical and empirical examples, we show the advantages and the limits of the  $m$  function for detecting spatial structures in economics.

**JEL Classification:** C10, C60, R12

## Keywords

Spatial concentration, Aggregation, Point patterns, Agglomeration, Economic geography

<sup>a</sup>AgroParisTech, UMR 518 Mia, 19 avenue du Maine, F-75732 Paris Cedex 15, France.

<sup>b</sup>AgroParisTech, UMR EcoFoG, BP 709, F-97310 Kourou, French Guiana.

<sup>c</sup>Université de Paris-Sud, RITM, 54 Boulevard Desgranges, F-92330 Sceaux, France

\*Corresponding author: Florence.Puech@u-psud.fr

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## Introduction

Industrial agglomerations are doubtless the main feature of today's economic geography (Krugman, 1991; Henderson and Thisse, 2004). Thus, it is not surprising that much recent research has attempted to improve the measurement of the spatial concentration of activities.<sup>1</sup> Distance-based methods

are the latest statistical measures to be proposed in the field of spatial economics for detecting spatial structures (geographic concentration or dispersion). By treating space as continuous, distance-based methods provide a very detailed analysis with robust results.<sup>2</sup> Consequently, many authors consider them to be very promising techniques (Combes and Overman, 2004; Combes *et al.*, 2008; Duranton, 2008) and that they open the way for new explanations of the spatial concentration of activities (Ellison *et al.*, 2010; Alfaro and Chen, 2014). Today, the Duranton and Overman's  $K_d$  function (Duranton and Overman, 2005) is the most used distance-based method in economics. It is a density function that evaluates absolute concentration (Marcon and Puech, 2012, 2015). In this article for the first time we shall introduce a relative density function in spatial economics, in a similar vein to the well-known location quotient (Florence, 1972). We have developed this for two main reasons. First, this new function, called  $m$ , supplements the typology of distance-based methods recently drawn up by Marcon and Puech (2012). Second, as Brühlhart and Traeger (2005) have stressed, the nature of the spatial concentration (absolute, relative) matters. In this paper we shall show that this still applies in a continuous-space framework by providing various comparisons between the results for  $K_d$  and  $m$  obtained with theoretical and empirical examples.

There is growing evidence that distance-based measures are now preferred in spatial economics since Duranton and Overman's seminal paper (Duranton and Overman, 2005). One of the main reasons for this is that they preserve the rich-

<sup>1</sup>See Duranton and Overman (2005); Marcon and Puech (2003, 2010); Arbia *et al.* (2012); Mori and Smith (2013); Jensen and Michel (2011), among others.

<sup>2</sup>Duranton and Overman (2008) provide many concrete examples of the problems such functions can solve.

ness of individual data. The explanation for this is simple and as follows. In contrast to the Gini (1912) or the Ellison and Glaeser (1997) indices, distance-based methods do not rely on any predefined zoning (regions, counties. . .). Distance-based methods are implemented directly using the position of entities (stores, plants. . .). The analysis of the spatial distribution of entities is based on the distance between them. This is very important because it has been proved that the Gini or Ellison and Glaeser indices for example which aggregate data at a zonal level are sensitive to the zoning chosen (Arbia, 2001; Briant *et al.*, 2010) as described by the Modifiable Areal Unit Problem – MAUP (Openshaw and Taylor, 1979; Arbia, 1989). One way of solving MAUP issues is to treat space as “continuous” as suggested by Duranton (2008).<sup>3</sup> This is the main feature of distance-based measures. Consequently, MAUP issues vanish. These techniques therefore allow an exact and unbiased analysis of the spatial structure of the distribution at all scales simultaneously (and not at only one level of observation as is the case with spatial zoning). Multiple patterns can be detected: for example aggregation or repulsion between entities according to the distance considered. These distance-based measures are now deemed to be very powerful and we can easily understand why many studies now employ them to evaluate spatial patterns (Arbia and Espa, 1996; Sweeney and Feser, 1998; Ó hUallacháin and Leslie, 2007; Bonneu, 2007; Arbia *et al.*, 2008; Nakajima *et al.*, 2012; Barlet *et al.*, 2013; Behrens and Bougna, 2015; Koh and Riedel, 2014; Giuliani *et al.*, 2014).

In what follows, we shall introduce a new distance-based method: the  $m$  function. Our motivation is twofold. First, as far as we know no relative density function has yet been proposed to gauge the spatial concentration in continuous space, as Marcon and Puech (2012) pointed out. We shall devote a great deal of attention to defining our proposed  $m$  function in order to respect a maximum number of the good criteria for measuring spatial concentration in economics (Combes and Overman, 2004). In particular, the  $m$  function satisfies Duranton and Overman’s five important criteria (Duranton and Overman, 2005): (1) the results are comparable across industries, (2) it controls for the overall agglomeration of manufacturing, (3) it controls for industrial concentration in the sense of Ellison and Glaeser (1997), (4) it is unbiased across geographic scales (this is related to the MAUP issues) and lastly (5) it gives an indication of the significance of the results. Second, various theoretical and empirical examples are provided to show that the results provided by the  $m$  function are complementary to Duranton and Overman’s widely used  $K_d$  function. The latter is also a density function but not a relative one. As a result, the  $m$  function is an appropriate statistical tool for detecting spatial structures in economics. However

<sup>3</sup>More precisely, Gilles Duranton wrote the following in the article on “Spatial Economics” in The New Palgrave Dictionary of Economics: “*On the empirical front, a first key challenge is to develop new tools for spatial analysis. With very detailed data becoming available, new tools are needed. Ideally, all the data work should be done in continuous space to avoid border biases and arbitrary spatial units.*”

the results from  $m$  and  $K_d$  do not converge systematically, so we recommend that the nature of the spatial concentration analyzed should be studied carefully to avoid any erroneous conclusions.

In the following sections, first of all, the background is explained (Section 1). Then, the  $m$  function is presented (Section 2) and some simple illustrative examples are given (Section 3). In the last section of the paper (Section 4) we propose some comparisons with the  $K_d$  function. We also provide a simple theoretical example and an analysis of the spatial distribution of pharmacies in the Lyon area (France) to illustrate the advantages and limits of the use of the  $m$  function in our field.

## 1. Background

As we have already mentioned, distance-based methods are particularly attractive for economists because they provide a complete and unbiased analysis of the location patterns of industries. The basic idea of distance-based methods is simple. Let us consider the case of the textile industry. Evaluating the spatial distribution of plants in this sector depends on an assessment of the surroundings of textile plants for all the distances that are considered. If there are locally more textile plants around textile plants than is generally the case in the whole of the area under investigation, a phenomenon of concentration is detected (“textile plants attract textile plants”). On the other hand, if there are fewer textile plants in the surroundings of the textile plants than there are in the area as a whole, we talk of a phenomenon of dispersion (in which case “textile plants repel textile plants”). Finally, if there is no relationship between the entities, independence is identified (“textile plants are randomly and independently distributed”). The significance of the results is provided by the confidence interval of the null hypothesis. More technically, all distance-based methods explore the spatial structure of point patterns. Their mathematical framework is that of point processes (Møller and Waagepetersen, 2004). Two concepts require additional explanations: the definition of the surroundings of plants and the nature of the spatial concentration (topographic, relative or absolute). Let us now examine these two important factors in greater depth.

Firstly, the notion of the **surroundings of plants** is central because it defines the type of function applied *i.e.* a cumulative or a density function. In practice, the evaluation of the neighboring plants is done for all distances, for example every 100 meters up to the median distance between all pairs of plants. The spatial distribution can be estimated up to a given distance or at a given distance. If the first option is chosen, it calls for a cumulative function. On the other hand, if the second option is selected, a density function is appropriate. The choice between one type of function and the other depends on the issue under study (Marcon and Puech, 2010).

The second clarification concerns **the nature of the spatial concentration**. In order to evaluate the spatial concentration of economic activities, it is necessary to choose a

reference value with which to compare the observed distribution of activities (see Brülhart and Traeger, 2005, among others):

- The first possibility is to use a topographic reference. In this case, physical space is chosen as the benchmark. One possible example is the number of neighboring plants per unit of space (that is on a disk of radius  $r$  for a cumulative function or on the ring at distance  $r$  for a density function). Space may be homogeneous or not. The homogeneity of space implies a constant density all over the study area (in our previous case this means that all the plants in the distribution have the same probability of being located anywhere in the study area). Some authors (Duranton and Overman, 2005; Marcon and Puech, 2003) consider this hypothesis to be too strong in the field of spatial economics and that a non-homogeneous space framework is better.
- The second possibility is a relative reference. In this case, another variable is taken as a benchmark. Any variable can be used except space (if it is, the concentration is topographic). For instance, if we evaluate the spatial distribution of textile plants, we can detect in the plant's near environment the deviations of this distribution of plants from another distribution. The benchmark plants can be all plants at the aggregate industrial level.
- The last possibility is to have no reference. In this case, an absolute measure is defined. For example, the number of neighboring textile plants located at a given distance from a textile plant.

The growing number of measures in continuous space recently prompted us to provide a typology of such functions (Marcon and Puech, 2012). They proved that a classification of statistical measures can be drawn up by considering only the nature of the geographic concentration and the definition of the type of function. Table 1 gives an overview of all the distance-based measures that have been used to evaluate the spatial distribution of economic activities:

- the  $K$  function of Ripley (1976, 1977),
- the  $g$  function of Ripley (1976, 1977),
- the  $K_{mm}$  function introduced by Penttinen (2006) and Penttinen *et al.* (1992),
- the  $D$  function of Diggle and Chetwynd (1991),
- the  $K_{inhom}$  function of Baddeley *et al.* (2000),
- the  $g_{inhom}$  function of Baddeley *et al.* (2000),
- the  $K_d$  function of Duranton and Overman (2005),
- the  $M$  function of Marcon and Puech (2010),

- the (unnamed) cumulative function of  $K_d$  proposed by Behrens and Bougna (2015).

We can easily see that one cell in the table is empty: no relative density function has yet been proposed for the field of spatial economics. The present paper fills this gap. In the next section we shall complete Table 1 by proposing a new density function, named the  $m$  function, which expresses relative spatial concentration.

Finally, it should be noted that the application of distance-based methods is not confined to spatial economics. They were first developed and applied in other disciplines. Much empirical research has thus been conducted in the fields of ecology (Law *et al.*, 2009) and epidemiology (Waller, 2010), for example.

## 2. Presentation of the $m$ function

### 2.1 An intuitive presentation

The idea of the  $m$  function is as follows. Consider an area in which various plants belonging to several industrial sectors are located. The  $m$  function is a relative measure that compares the proportion of plants of interest in the neighborhood of the reference plants to the proportion of neighbors of interest in the area as a whole. If plants are agglomerated the proportion of neighbors of interest in the neighborhood of reference plants is greater than in the area as a whole. On the contrary, if plants are dispersed then the proportion of plants of interest in the neighborhood of the reference plants is lower than in the area as a whole. These proportions (ratios) are estimated from observed data. If the neighbors of interest are in the same sector as the reference plants, the  $m$  function helps to detect agglomeration phenomena. If the neighbors of interest do not belong to the same sector as the reference plants, the  $m$  function identifies co-agglomeration.

### 2.2 Definition of the $m$ function

Let us now turn to the mathematical definition of the  $m$  function. Plants are defined as points. All points belong to a point pattern denoted by  $\mathcal{X}$ . Two subsets are considered: that of the reference points  $\mathcal{R}$  (e.g. a sector of activity) and that of the neighboring points of interest  $\mathcal{N}$  (belonging to the same or another sector of activity).

The estimator of  $m$  is:

$$\hat{m}(r) = \frac{\sum_{x_i \in \mathcal{R}} \frac{\sum_{x_j \neq x_i, x_j \in \mathcal{N}} k(\|x_i - x_j\|, r) w(x_j)}{\sum_{x_j \neq x_i, x_j \in \mathcal{X}} k(\|x_i - x_j\|, r) w(x_j)}}{\sum_{x_i \in \mathcal{R}} \frac{W_{\mathcal{N}} - w(x_i)}{W - w(x_i)}} \quad (1)$$

where  $x_i$  denotes the reference points, and  $x_j$  the neighbors.  $w(x_i)$  is the weight of point  $x_i$ .  $W_{\mathcal{N}}$  is the total weight of the neighboring points of interest and  $W$  is the total weight of all the points. If points represent industrial establishments or shops, the weight can be the number of employees working in those entities.  $k(\cdot)$  is a kernel estimator whose sum can be used to estimate the number of neighbors of point

**Table 1.** Choice of the appropriate function to describe a point pattern structure

Function choice	Topographic, homogeneous	Topographic, inhomogeneous	Absolute	Relative
Probability density functions	$g$	$g_{inhom}$	$K_d$ $K^{emp}$	
Cumulative functions	$K$ $K_{mm}$	$K_{inhom}$ $D_i$	Cumulative of $K_d$ Cumulative of $K^{emp}$	$M$ Case-control $K_{inhom}$

$x_i$  at distance  $r$ . We have followed Duranton and Overman (2005) and used a Gaussian kernel of optimal bandwidth as described by Silverman (1986). The kernel estimator considers the neighbors of the reference point and gives them a maximum weighting if they are exactly  $r$  apart. Their weighting decreases according to a Gaussian distribution of the standard deviation  $h$ . The choice of the bandwidth  $h$  is arbitrary but important (Illian *et al.*, 2008). The wider it is, the smoother the estimator. In this paper, we shall only analyze the spatial distribution of one sector, so we shall focus on the intratype (or univariate)  $m$  function taking  $\mathcal{R} = \mathcal{N}$ . But we can extend the  $m$  function for the analysis of inter-industrial spatial distributions (co-agglomeration): the intertype (or bivariate) function can be defined in the same way, choosing different point types as the reference and neighbors.<sup>4</sup>

The equation of the  $m$  function reads as follows. The numerator is the local ratio *i.e.* the relative weight of the neighbors of interest at distance  $r$  from all the reference points. This is averaged over all the reference points (actually, it is simply summed because the number of reference points is simplified with the denominator). The denominator is the same ratio over the whole data set, *i.e.* the global ratio. It is not just  $W_{\mathcal{N}}/W$  because the reference points are never counted as neighbors. An unbiased estimator of the global ratio is thus the average local ratio considering all points are neighbors to each other. For this reason, the denominator is slightly different in the intertype function:  $\sum_{x_i \in \mathcal{R}} \frac{W_{\mathcal{N}}}{W - w(x_i)}$ .

The reference value of the  $m$  function is 1 for any distance  $r$ . This value is obtained when the points are independent.  $m$  values greater than 1 indicate the spatial concentration of points while  $m$  values lower than 1 express dispersion.  $m$  values can be interpreted. For example, if the  $m$  function is 1.5, at distance  $r$ , the proportion of the neighbor points of interest at this distance is 50% higher than in the area as a

<sup>4</sup>To give an example, if the aim is to evaluate the spatial distribution of the textile industry, the analysis of the distribution of textile plants around textile plants is relevant. In the case of intra-industrial analysis, the intratype function should be used. If, on the other hand, the focus is on for instance the co-agglomeration of the textile and clothing sectors, analysis will deal with the distribution of textile plants around clothing plants or the distribution of clothing plants around textile plants. Only the intertype function is relevant in the second case.

whole.

The significance of the estimates of  $m$  is given by the confidence interval of the null hypothesis (Monte-Carlo simulations). This technique is widely employed in the case of distance-based methods. In practice, random distributions of points are generated by permuting the marks (type and weighting pairs) of the actual points on the actual spatial positions of points (coordinates). We have generated only a global confidence interval, following Duranton and Overman (2005).

### 2.3 Discussion

Like the location quotient (Florence, 1972), the  $m$  function is a continuous space function that exists in a discrete space approach (with predefined zoning). In continuous space, the definition of  $m$  is similar to that of the cumulative  $M$  function (Marcon and Puech, 2010) except that the local ratio is defined at distance  $r$  and not up to it. In contrast with the topographic functions  $g$  and  $K$ , the cumulative function is not the integral of the density function over  $r$  (Ripley, 1977) because relative functions are not derived from a measure of space.

The  $m$  function fulfills all of Duranton and Overman's criteria mentioned in the introduction: (i) it compares the geographic concentration results across industries, (ii) it controls for industrial concentration (indirectly, comparing its values to the confidence envelope of the appropriate null hypothesis), (iii) it controls for the overall aggregation patterns of industries, (iv) it enables the significance of the results to be tested (using the confidence interval) and, (v) it keeps the empirical results unbiased across geographic scales. This is very important because only a few continuous-space based methods respect all of these criteria (Marcon and Puech, 2012).

## 3. Theoretical examples

We shall now provide simple examples for three theoretical cases. In every example we have considered a 1-by-1 window and a maximum distance for the  $m$  function equal to one-third of the diagonal of the window ( $\approx 0.471$ ). We have systematically used 512 regular intervals to calculate the  $m$  function. All points have a weighting of 1. A global confidence interval (CI) at the 1% risk level was generated after 10,000 simula-

tions. All simulations are made with the help of the R package (R Development Core Team, 2014). **spatstat** (Baddeley and Turner, 2005) was used to realize the point processes and the **dbmss** package (Marcon *et al.*, 2014) was used to compute the  $m$  function.

### 3.1 Random distribution

Figure 1a shows two distributions: one for the cases (diamonds) and the other for the controls (crosses). The cases and controls were simulated under the hypothesis of complete spatial randomness (CSR), under which points are distributed randomly and independently from each other. To achieve this, we generated two distributions from a homogeneous Poisson process with a parameter respectively equal to 25 and 100. The parameter of the Poisson process is the expectation of the number of points for each distribution<sup>5</sup>. In figure 1a, 29 points were simulated for the cases and 103 for the controls. Figure 1b depicts the  $m$  function results for this case.

No significant result was observed:  $m$  fluctuates for all distance ranges but stays inside the confidence interval of the null hypothesis. As expected for random distributions of cases and controls, figure 1b provides no evidence of any attraction or repulsion between cases. Two additional minor comments should be made. First, the global confidence interval is quite large at small radii: there is a small number of neighbors at very small radii. Second, the  $m$  function is not defined for very small distances: this indicates that cases are separated by gaps of more than 0.01.

### 3.2 Aggregate distribution

Figure 2a shows a multiple pattern: an aggregate distribution of cases (diamonds) and a completely random distribution of controls (crosses). For the clusters of cases, we generated simulations from a Matérn process with the following parameters: 2 for the density of the Poisson process that generates cluster centers, 0.05 for the radius of clusters and 50 for the average number of points per cluster. Controls were simulated from a homogeneous Poisson process with a density of 100. 203 points were plotted on figure 2a: 102 cases shared between two clusters and 101 controls were randomly distributed over the entire domain. Figure 2b shows the results for the  $m$  function.

On figure 2b, two significant concentration peaks are apparent. They occur at distances for which the relative local density of cases is the greatest. The first distance at which a peak is observed corresponds approximately to the radius of the clusters (around 0.05). This is due to the presence of controls in the ring at this distance, the peak is not reached exactly at a distance of 0.05 but only approximately. The second peak identifies the distance between clusters (approximately

0.5) and has a lower value. The local relative density of cases over controls is greater for the first peak because the presence of controls in the cluster is possible but rare.

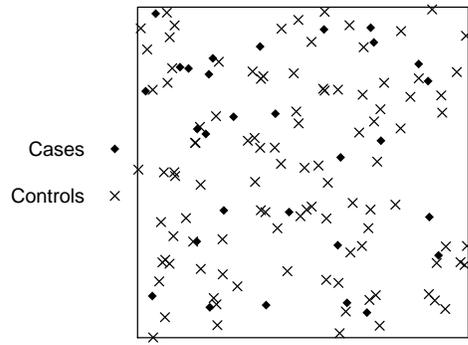
Three additional comments have to be made. First, by construction there are no cases between the aggregates. For these distances, the maximum dispersion is detected: between the clusters the  $m$  function attains its lowest possible value (zero). The rapid decrease in the gradient of  $m$  is a feature of density functions. In contrast to cumulative functions, the values are very sensitive and large ranges of results may be observed over small intervals of distance. Second, one can observe that the  $m$  plot takes on its highest values in the case of small distances (first concentration peak) and then decreases. The explanation for this is simple. In the first radii, the local relative density is the greatest because the maximum number of cases is observed around these distances. Around a distance of 0.3, the  $m$  function detects the first cases located at the periphery of the (other) cluster: as a result, the  $m$  function rises. Then the local relative density continues to increase rapidly because of the large number of cases inside the cluster. Third and last, it is interesting to note that the confidence interval of the null hypothesis is narrower in the inter-distance clusters, because more points are observed at these distances.

### 3.3 Regular distribution

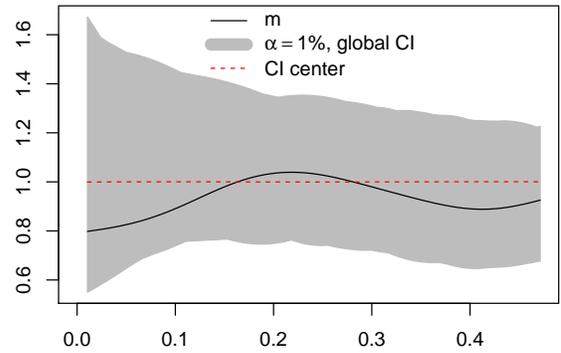
Figure 3a shows another multiple pattern. A regular distribution of cases (diamonds) is clearly visible. 100 cases are positioned on a square grid measuring  $0.1 \times 0.1$ . The completely random distribution of controls (crosses) is a realization of a homogeneous Poisson process whose parameter is equal to 200. Figure 3a shows 209 controls. The  $m$  function estimates are given in figure 3b.

Up to the size of the square grid (0.1), the cases have no case neighbor: for small distances there is a significant amount of dispersion. Then, a large number of peaks can be observed but results are not significant. The reason is simple. In this example as we previously said we retained the optimal bandwidth as described by Silverman (1986). A thinner bandwidth should have shown significant positive and negative peaks. The choice of the bandwidth is important, unfortunately “*in general, however, no simple recipe for the choice of the bandwidth exists*” (Illian *et al.*, 2008, p.115). Let us take for example half of the previous bandwidth to better explain the spatial structure under study. Results of the  $m$  function are given in figure 3c. Up to the size of the square grid (0.1),  $m$  plots are the same whatever the definition of the bandwidth. At a distance equal to the size of the grid, the cases have four neighbors: a significant positive peak is observed in figure 3c, indicating the spatial concentration of cases at this distance. At this distance, a positive peak is also detected in figure 3b but the  $m$  plot stays within the confidence interval. Due to smoothing, significant values of  $m$  can also be observed just below the grid size (0.1). The  $m$  plot then plummets when the radius increases: no cases are located in the close environment of cases, the  $m$  value

<sup>5</sup> The Poisson process is commonly used for simulating CSR distributions. As Diggle (1983) wrote the Poisson process “*is the cornerstone on which the theory of spatial point processes is built. It represents the simplest possible stochastic mechanism for the generation of spatial point patterns, and in applications is used as an idealized standard of complete spatial randomness* (...)” (p.50).

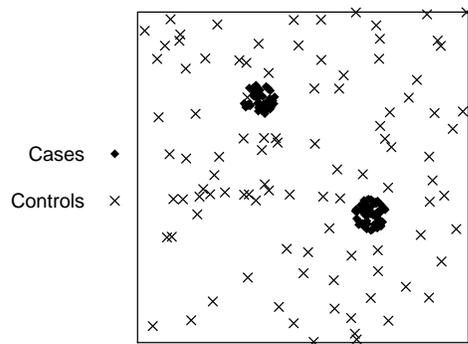


(a) Completely random distributions for cases and controls

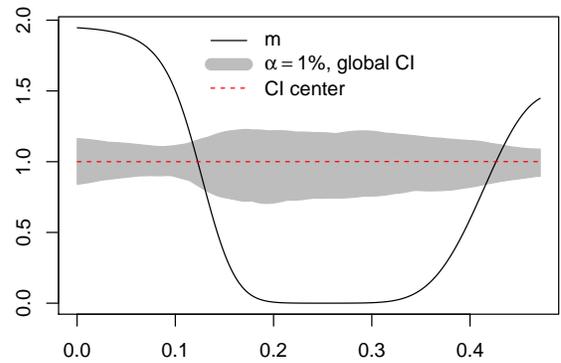


(b)  $m$  function results

**Figure 1.** Random distribution

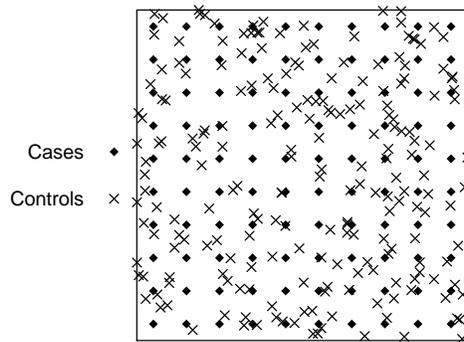


(a) Aggregate distribution of cases, complete spatial randomness for controls

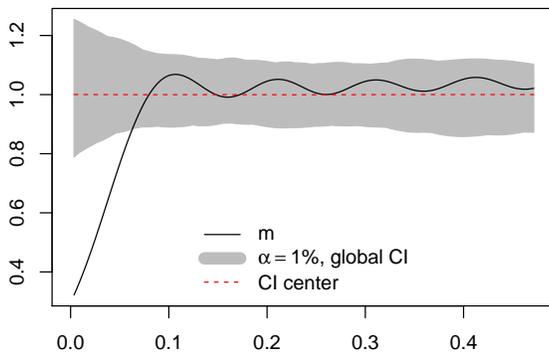


(b)  $m$  function results

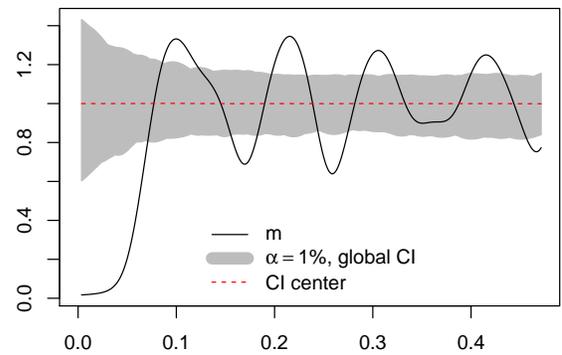
**Figure 2.** Aggregate distribution



(a) Regular distribution of cases, complete spatial randomness for controls



(b)  $m$  function results, original bandwidth of Silverman (1986)



(c)  $m$  function results, half of the original bandwidth of Silverman (1986)

**Figure 3.** Regular distribution

returns rapidly to below the confidence interval, indicating dispersion. On figure 3c, the irregularity in the gradient of  $m$  between a distance of 0.10 and 0.15 is interesting. At a distance equal to the diagonal of the grid (around 0.141) new neighbors are present. The irregularity in the gradient of  $m$  shows the existence of these new neighboring points. However, there is no positive peak because the smoothing we applied was too strong, but weaker smoothing would have generated positive  $m$  values. On the opposite, the original bandwidth appears too weak in that case: the irregularity in the gradient of  $m$  is not visible between a distance of 0.10 and 0.15 in figure 3b. Note at larger distances, the observed positive peaks appear at around twice the grid size (0.2), three times the grid size (0.3) etc. Between these peaks,  $m$  indicates the absence of neighboring cases: depending on the distance, significant dispersion may (as in the case for a distance of 0.25) or may not (as in the case for a distance around 0.35) be observed.

## 4. Discussion

This section provides some comparisons with the most used density function in spatial economics, Duranton and Overman's  $K_d$  function (Duranton and Overman, 2005). Keeping the notations previously used in equation 1 and using  $n$  to denote the total number of points, the  $K_d$  function is defined by:

$$\hat{K}_d(r) = \frac{1}{n(n-1)} \sum_{x_i \in \mathcal{R}} \sum_{x_j \neq x_i, x_j \in \mathcal{N}} k(\|x_i - x_j\|, r) \quad (2)$$

The weighted version of the  $K_d$  function, called the  $K_{emp}$  function (Duranton and Overman, 2005), is given by:

$$\hat{K}^{emp}(r) = \frac{\sum_{x_i \in \mathcal{R}} \sum_{x_j \neq x_i, x_j \in \mathcal{N}} w(x_i) w(x_j) k(\|x_i - x_j\|, r)}{\sum_{x_i \in \mathcal{R}} \sum_{x_j \neq x_i, x_j \in \mathcal{N}} w(x_i) w(x_j)} \quad (3)$$

The  $K_d$  function is very popular in spatial economics (see Marcon and Puech, 2012, for details). Thus, it is therefore interesting to compare the results from the  $m$  function with those provided by the  $K_d$  function in order to understand the main differences between the two statistical measures. In what follows, we have used the R package **dbmss** to estimate the  $K_d$  and  $m$  functions.

### 4.1 Comparisons of $K_d$ and $m$ results on the previous three theoretical cases

The  $K_d$  function was estimated for the three theoretical cases considered above. The  $K_d$  plots are shown on figure 4a for the random distribution of cases, on figure 4b for the aggregate distribution and on figure 4c for the regular distribution. For all the cases, we have used the same maximum distance, *i.e.* one-third of the diagonal of the window ( $\approx 0.471$ ).

For the random distributions of cases and controls (figure 4a),  $m$  and  $K_d$  give identical results. No significant level of dispersion or concentration was detected.

For the aggregate distribution (figure 4b), like the  $m$  function,  $K_d$  detects the first peak of concentration occurring at a distance of approximately 0.05. The main difference relates to the shape of the concentration peak. At very small distances the  $K_d$  values increase up to a distance of 0.05 which corresponds to the radius of the cluster. After this, the value of  $K_d$  starts to fall. The increase in  $K_d$  contrasts with the shape of the  $m$  function at short distances. The explanation for this lies in the definition of  $K_d$  which evaluates the probability of finding one case neighbor at a given distance. At short distances this probability increases because more case-neighbors are observed. This probability reaches its maximum at the radius of the circle. Then, it progressively decreases since the radius of the circle increases.

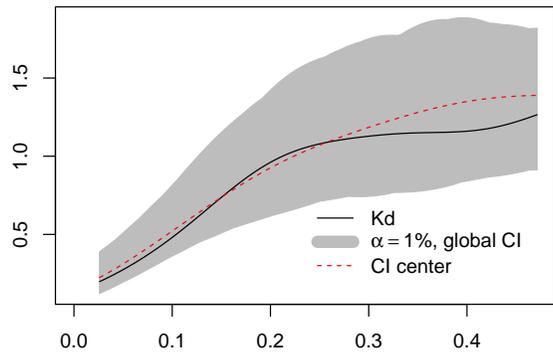
Let us now turn to the regular theoretical example. With the same original smoothing of Silverman (1986) also chosen by Duranton and Overman (2005), the results for this spatial pattern with the  $m$  (figure 3b) or the  $K_d$  (figure 4c) function are totally in accordance. The only difference appears in the first radii: estimates of the  $K_d$  function begin at a distance of  $r = 0.1$  while  $m$  values can be estimated at lower distances. This is explained by the definition of  $K_d$ : no case-neighbor is located at a distance of less than 0.1 thus  $K_d$  is not defined. This is not the case for  $m$ : if there are no case neighbors but there are some control neighbors, the  $m$  function is defined and takes its lowest value (zero) as shown in figure 3b. Finally, it should be noted that we applied the same smoothing when we compared the  $m$  and  $K_d$  functions. This is of great importance, as shown when we modified the smoothing by choosing a narrower bandwidth, that is half of the original bandwidth of Silverman (1986) (while retaining the same risk level, number of simulations and type of confidence interval). The results for the  $K_d$  function are given in figure 4d and should be compared with the  $m$  results given in figure 3c. As one would expect, there are a large number of positive and negative significant peaks in comparison to weaker smoothing of the results.

### 4.2 Comparisons between the results from the $K_d$ and $m$ functions on a more complex example

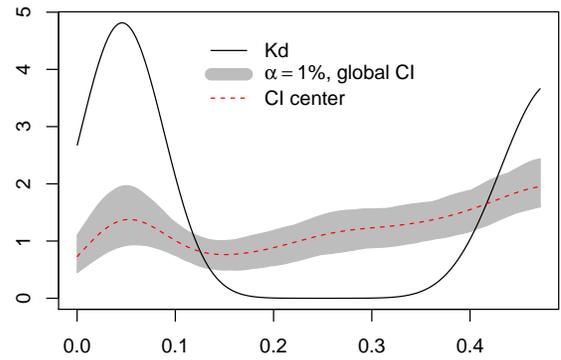
In the three above theoretical cases, there is not a great deal of difference between the results with the  $K_d$  and  $m$  functions. However, this is not always the case. In real life, the distribution of activities may be more complex. In this sub-section, we shall draw attention to the type of concentration that the  $m$  function can identify. To make things clearer, if we return to table 1 we can see that the  $m$  function evaluates the relative concentration while  $K_d$  appraises the absolute concentration. This distinction may be crucial to an understanding of spatial structures.

#### 4.2.1 Divergence in the $K_d$ and $m$ results on a theoretical case

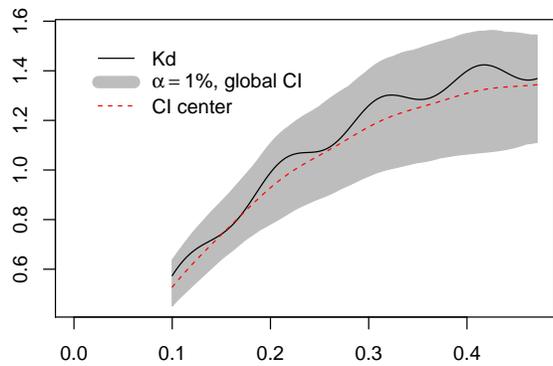
Consider the following theoretical example. A city is delimited by a 1-by-1 window. For the sake of simplicity "cases" and "controls" are the only two types of shops located in the city. Figure 5 shows the distribution of cases (diamonds) and



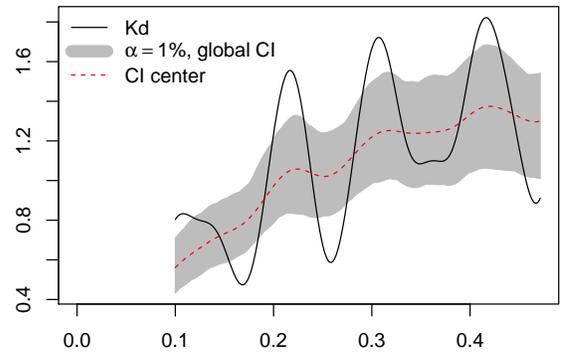
(a)  $K_d$  results for completely random distributions of cases and controls (map on figure 1a)



(b)  $K_d$  results for aggregate distribution of cases (map on figure 2a)

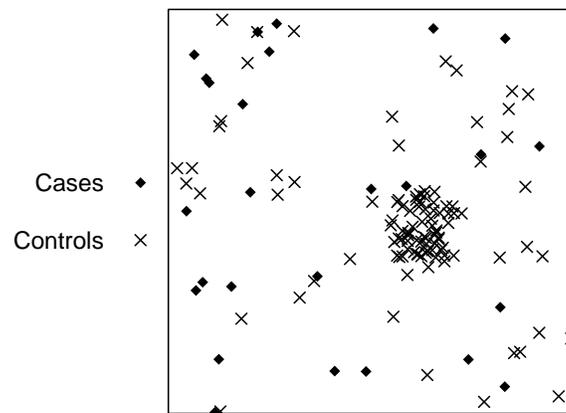


(c)  $K_d$  results for regular distribution of cases (map on figure 3a) with the original Duranton and Overman's smoothing



(d)  $K_d$  results for regular distribution of cases (map on figure 3a) with a more detailed smoothing

**Figure 4.**  $K_d$  results for the three theoretical cases



**Figure 5.** A more complex theoretical case

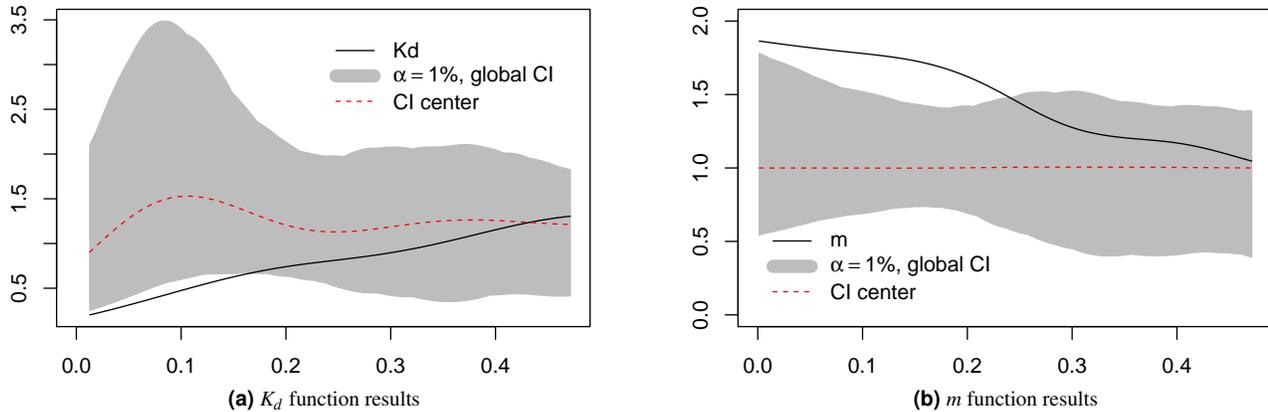


Figure 6.  $K_d$  and  $m$  results for a more complex theoretical case

controls (crosses). A multiple pattern is observed: a completely random distribution of controls (crosses) and cases (diamonds) and also an aggregate distribution of controls (crosses). More technically, for the cluster of controls we generated simulations from a Matérn process with the following parameters: 1 for the density of the Poisson process that generates cluster centers, 0.1 for the radius of clusters and 75 for the average number of points per cluster. Controls were also simulated from a homogeneous Poisson process with a density of 25. 134 points were plotted on figure 5. The cluster is composed of 66 controls: 42 controls are randomly distributed on the area and 26 cases are randomly distributed over the entire domain.

Figures 6a and 6b show the results for the  $K_d$  function and the  $m$  function. In line with our previous examples, all points have a weighting of 1. A maximum distance for the  $m$  function equal to one-third of the diagonal of the window ( $\approx 0.471$ ). A global confidence interval (CI) at the 1% risk level was generated after 10,000 simulations. For both functions, we used a Gaussian kernel of optimal bandwidth (Silverman, 1986; Durantou and Overman, 2005). All simulations were conducted with the help of the R package the **dbmss** package for computing the  $K_d$  and  $m$  functions. The divergence in the results between  $K_d$  function and  $m$  show all the importance of the definition of the nature of the spatial concentration studied.

As we underlined, the  $m$  function detects the relative spatial concentration. Cases are more regularly distributed than the controls in the example. As a consequence, around the cases the presence of cases is *relatively* more important than the one of controls: relative concentration is detected by the  $m$  function up to a distance approximately equal to 0.25 (figure 6b). Now, consider the  $K_d$  results (figure 6a). A phenomenon of a dispersion of cases is detected. This result can be explained in a easy way and there is no contradiction between the spatial structure detected by the  $m$  function. The  $K_d$  function detects the absolute concentration. When we simulate the null hypothesis, the cases will be more concentrated than we can observed in figure 5 because some cases

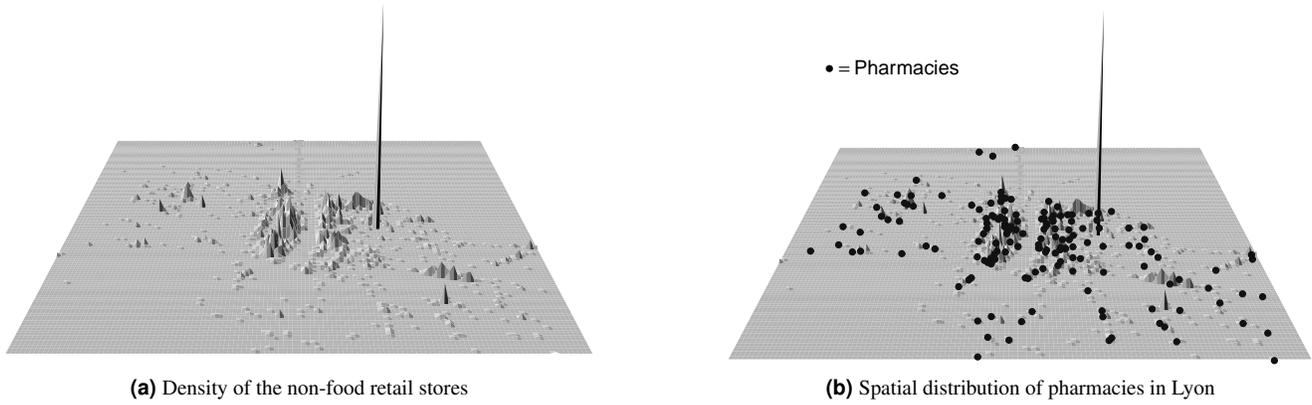
will probably be located in the cluster. As a conclusion, the distribution of cases is less concentrated than under the null hypothesis: absolute concentration is detected in figure 6a (up to a distance approximately equal to 0.17). This should not be considered as only a textbook case, a real case of that example is given in the following section.

#### 4.2.2 Confirmation of the previous results for a retail sector in the city of Lyon (France)

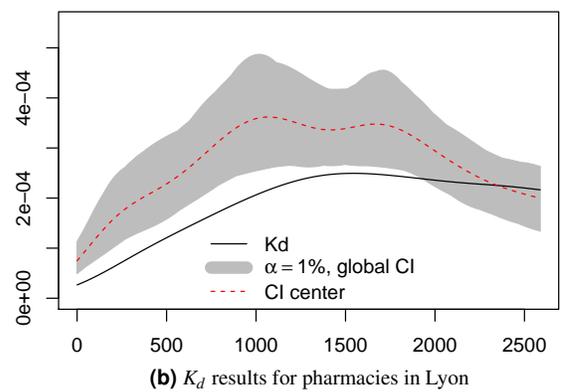
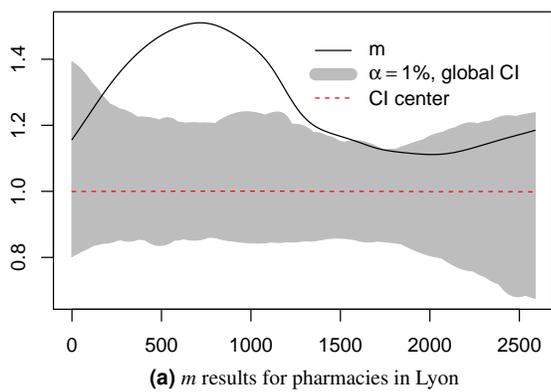
To give a concrete example of the previous theoretical case, we shall consider the spatial distribution of the non-food retail stores in the Lyon area (France). We have exploited a database provided by the Chamber of Commerce and Industry of Lyon. This contains the exact geographic position of 3,124 non-food stores in April 2012. The different types of store have been classified into 26 sectors from 47.30Z to 47.79Z of the French NAF rev. 2 classification of activities. We shall focus on “dispensing chemists in specialized stores” (47.73Z), of which there are 156 in the Lyon area. We shall refer to these stores as pharmacies in what follows.

First of all, a comparison of the density of all non-food stores in Lyon (figure 7a) and the spatial distribution of the pharmacies over the same area (black points on figure 7b) is worthwhile. We can see that many of the city’s non-food stores are located in central Lyon and the left bank of Rhône river (to the east of central Lyon, figure 7a). However, pharmacies are undoubtedly more regularly distributed than non-food stores as a whole. One can easily observe the presence of these activities over the entire Lyon area (black points on figure 6a).

The impacts on the results for  $m$  and  $K_d$  will be of interest. These are given on figures 8a and 8b. Their respective global confidence intervals (CI) were computed at the 1% risk level after 10,000 simulations. All the simulations were conducted with the help of the R package **dbmss** for computing the  $K_d$  and  $m$  functions. All the pharmacies were assigned a weighting of 1 and the maximum distance analyzed was around 2.5 kilometers. The spatial structures detected by  $K_d$  and  $m$  differ. Up to a distance of approximately of 2 kilometers, the plot of



**Figure 7.** Non-food retail stores in the area of Lyon (France)



**Figure 8.** Spatial structure of the pharmacies

$K_d$  indicates that pharmacies are dispersed while that of  $m$  indicates a degree of spatial concentration up to 1.5 kilometers. These results are in line with the type of concentration each function seeks to identify. It should be remembered that  $K_d$  evaluates the absolute concentration: it has no reference value for assessing the spatial concentration because it evaluates the probability of finding one case neighbor at a distance  $r$  (in our example we are evaluating the probability of finding another pharmacy from any pharmacy). What are the implications for our case study? As we have seen, this activity is more regularly distributed than non-food retail activities as a whole in the Lyon area. Thus, under the null hypothesis, pharmacies are more concentrated than in the observed distribution (points are randomly labeled under the null hypothesis). Consequently, as one would expect,  $K_d$  detects an observed dispersion of these stores in that sector. If we now turn to the results for  $m$ , a level of spatial concentration is detected. The reason is that  $m$  identifies the relative spatial concentration (and not the absolute one). While there are pharmacies in high density business areas (central Lyon, the left bank of the River Rhône...), they are particularly present in low density business areas. In the latter, pharmacies are over-represented around other pharmacies than under the null hypothesis. When we simulate distributions, pharmacies will be located in areas where the number of non-retail stores is greater so the relative concentration of these stores will be lower under the null hypothesis. As a result, the observed  $m$  plot is over the confidence interval of the null hypothesis and indicates a relative spatial concentration of pharmacies between approximately 250 m and 1500 m. This comparison of results emphasizes that the nature of the spatial concentration should be studied with care. Our conclusion is in line with Brühlhart and Traeger (2005) or Marcon and Puech (2015) findings.

## 5. Conclusion

The first objective of this article was to introduce a new distance-based function called  $m$ , which is a relative density function.  $m$  is the first relative density function to be proposed in this field. The  $m$  function will therefore undoubtedly be useful for economists in order to detect the relative spatial structures of any distribution. Like any density function,  $m$  depicts local patterns more precisely than any existing cumulative function. The second objective of the paper was to prove that  $m$  is not equivalent to the leading density function  $K_d$ . The reason is that  $m$  evaluates the relative concentration and  $K_d$  the absolute one. We took a real example to show the complementarity of  $m$  and  $K_d$ . More precisely, we illustrated this point with the distribution of pharmacies in the Lyon area in France. Both functions detect irregularities in the spatial distribution of this activity but  $K_d$  identifies the spatial structure as one of dispersion and  $m$  as one of aggregation. In conclusion, a great deal of attention should be given to the statistical measure that is used to gauge the spatial structure of activities.

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