Zernike ultrasonic tomography for fluid velocity imaging based on pipeline intrusive time-of-flight measurements

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***Abstract*—In this paper we propose a novel Ultrasonic Tomography method for pipeline flow field imaging, based on Zernike polynomial series. Having intrusive multipath time-of-flight ultrasonic measurements (difference in flight time and speed of ultrasound) at the input, we provide at the output tomograms of the fluid velocity components (axial, radial and orthoradial velocity). Principally, by representing these velocities as Zernike polynomial series, we reduce the tomography problem to an ill-posed problem of finding the coefficients of the series, relying on the acquired ultrasonic measurements. Thereupon, this problem is treated by applying comparatively the Tikhonov regularization and the Quadratically constrained ℓ_1 minimization. In order to enhance the comparative analysis, we additionally introduce sparsity, by employing the SVD based filtering in selecting Zernike polynomials which are to be included in the series. The first approach - Tikhonov regularization without filtering, imposes as the most suitable one. The performances are quantitatively tested by considering a residual norm and by estimating the flow using the axial velocity tomogram. Finally, the obtained results show the relative residual norm and the error in flow estimation, respectively, $\sim 0.3\%$ and $\sim 1.6\%$ for the less turbulent flow and, $\sim 0.5\%$ and $\sim 1.8\%$ for the turbulent one. Additionally, a qualitative validation is performed by proximate matching of the derived tomograms with a flow physical model.**

I. Introduction

Ultrasonic flow metering (UFM) is considered to be one of the most representative non-destructive testing (NDT) measurement techniques for the pipeline flow estimation [1], [2]. Transmission intrusive flowmeters are placed inside the pipeline in order to measure the difference of the transit time of ultrasonic pulses propagating in and against flow direction. Typically, this allows calculating both the speed of sound and the average fluid velocity [3]. The most important constraint is the position of the UFM in the line with respect to discontinuities [4]. However, measuring flow rates inside a pipe flow with the classical time-of-flight method, based on a single ultrasonic propagation path, raises the problem of precision and robustness with respect to flow and geometry parameters [5], [6]. Thus, the improvement in measurement accuracy had to be achieved through the integration of several paths on different sections of flow, resulting in far better performances with respect to the estimation of average fluid velocity [7]. As a consequence, the multi-path ultrasonic systems are increasingly appearing as industrial replacement of classical single-path systems [8]. In moving the limits of the range of ultrasonic flowmeters use, RADAR (RAdio Detection And Ranging) tools such as matched filtering are used as well, given that the wide-band signals are taken into consideration [9].

However, the particular interest lies in estimating the flow field in a cross or an oblique section of the pipeline. The mean appearing as most suitable for this purpose is the Ultrasonic Tomography (UT) [10]. Already used in several different domains, UT is ordinarily conceived as a reconstruction method using iterative algebraic methods, spatial Fast Fourier Transform (FFT), Zernike polynomials etc. [11], [12]. In this paper, we propose the Ultrasonic Tomography method for the flow field imaging, based on Zernike polynomials [13].

Zernike polynomials, a set of orthogonal basis functions defined on a continuous unit circle, are firstly introduced by Zernike in his phase contrast method, the improved version of the knife-edge test [14]. In the context of tomography, they are mostly used to describe the wavefront aberrations [15], [16], [17], where they eventually exhibit even better performances than Fourier transform [18].

By representing fluid velocity components as Zernike polynomial series, we derive a method for obtaining corresponding velocity tomograms using intrusive ultrasonic multipath time-of-flight measurements. This approach reduces the ultrasonic tomography to an ill-posed problem, defined by the acquired measurements and the series of appropriate integrated Zernike polynomials. The problem which assumes finding series coefficients, is treated by comparatively using the Tikhonov regularization [19] and the Quadratically constrained ℓ_1 minimization [20]. Under certain constraints, the first method could be treated as ℓ_2 minimization. In order to enhance the comparative analysis by additionally introducing sparsity, we employed, as well, the SVD (Singular Value Decomposition) based filtering in selecting polynomials which have to be integrated in the series.

It appears that the most suitable mean is the Tikhonov reg-

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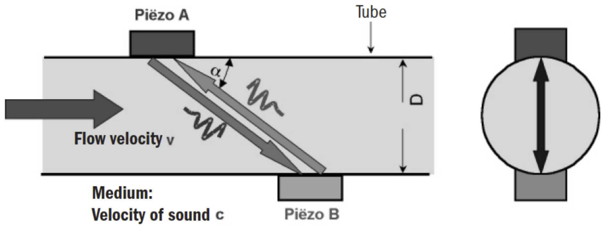

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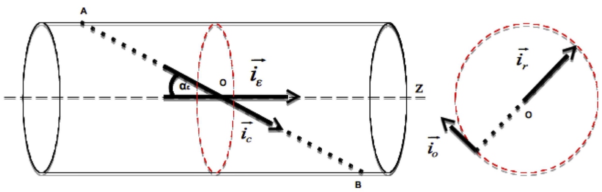


(b)

Fig. 1: Principle of ultrasonic flow measurement (time-of-flight method): (a) schema [23], (b) fluid propagation velocity components.

ularization without filtering. The quantitative decision criteria are the regularization relative residual norm and the comparison of the flow measured by the reference electromagnetic flowmeter with the one estimated using the derived axial tomogram. The obtained results show the relative residual norm and the average error in the flow estimation, respectively, $\sim$ 0.3% and $\sim$ 1.6% for the less turbulent flow (before the elbow curvature) and, $\sim$ 0.5% and $\sim$ 1.8% for the turbulent one (after the elbow curvature). Significantly lower residual norm and bigger dynamic range in case of Tikhonov regularization outcome even better flow estimation results obtained by the Quadratically constrained ℓ_1 minimization. Qualitative validation is performed through the comparison with the reference physical model of flow in a pipe with an elbow curvature [21], [22].

The article is organized as follows: in Section II we provide a brief theoretical background by introducing the basic principles of the intrusive ultrasonic time-of-flight flowmeter. Further, section III, the core of the article, describes the proposed method in details. In section IV we present the results followed by the appropriate discussion and finally, we conclude the article with section V.

II. Ultrasonic measurements

This section aims to briefly provide few details concerning basic principles of intrusive ultrasonic time-of-flight measurements.

Short ultrasonic pulses propagating through the liquid flowing in the pipeline, are emitted and received by piezoelectric converters (piëzos) [23], placed in a longitudinal direction on two opposite sides of a pipe with a certain offset (Fig. 1a). The propagation of ultrasonic pulses is influenced by the pipeline flow in the same manner as a canoe crossing diagonally a river: propagation time (time-of-flight) will be shorter when crossing in river flow direction than against it. These two propagation times depend on the velocity of river flow and the velocity of canoe. It is exactly equivalent for the ultrasonic time-of-flight measurements with the fluid velocity and the sound speed. The propagation times are to be calculated as [24]:

$$t_{AB} = \frac{||\vec{AB}||}{c + \vec{v} \cdot \frac{\vec{AB}}{||\vec{AB}||}} \approx \frac{1}{c^2}(c||\vec{AB}|| - \vec{v} \cdot \vec{AB}), \tag{1}$$

and

$$t_{BA} = \frac{||\vec{BA}||}{c - \vec{v} \cdot \frac{\vec{BA}}{||\vec{BA}||}} \approx \frac{1}{c^2}(c||\vec{BA}|| + \vec{v} \cdot \vec{BA}), \tag{2}$$

with c being the average speed of sound and $\vec{v}$ the fluid velocity vector. Given the non-uniform velocity distribution in a pipeline, the difference between times-of-flight can be further expressed as a line integral:

$$\Delta t = t_{BA} - t_{AB} = 2 \int_C \frac{\vec{v} \cdot d\vec{s}}{c^2}, \tag{3}$$

where C is the domain of integration, representing a straight line defined by vector $\vec{AB}$. Although it depends on the type of the liquid in the pipeline, the average speed of sound can be found as:

$$c = \frac{2||\vec{AB}||}{t_{AB} + t_{BA}}. \tag{4}$$

The velocity of the fluid propagating along the pipeline is expressed as a vector:

$$\vec{v} = \begin{pmatrix} \vec{v_r} \\ \vec{v_o} \\ \vec{v_\varepsilon} \end{pmatrix}, \tag{5}$$

composed of the radial (v_r), orthoradial (v_o) and axial (v_ε) component. Therefore, finally, the fluid's flow impacts the ultrasound propagation through the difference in time flight is:

$$\begin{aligned} \Delta t &= 2 \int_C \frac{\vec{v} \cdot d\vec{s}}{c^2} \qquad (6) \\ &= \frac{2}{c^2} \int_C (|\vec{v_\varepsilon}| \cos\alpha_\varepsilon + |\vec{v_r}| \cos\alpha_r + |\vec{v_o}| \cos\alpha_o) ds. \end{aligned}$$

The derived equation are based on the plane wave assumption, which represents a sort of simplification. The wave propagation in the pipeline flow is generally far more complex [25], [26]. The accuracy of a velocity estimation using time flight measurements, depends on the fluid flow Reynolds number and the mode of the ultrasonic wave, rather than on the radius of the pipeline and the wave frequency [27]. The influence of the pipeline vibrations cannot be neglected, neither [28].

Although the method derivation was motivated by the existing set of intrusive measurements, the application of the method is not by any means restricted to this kind of measurements.

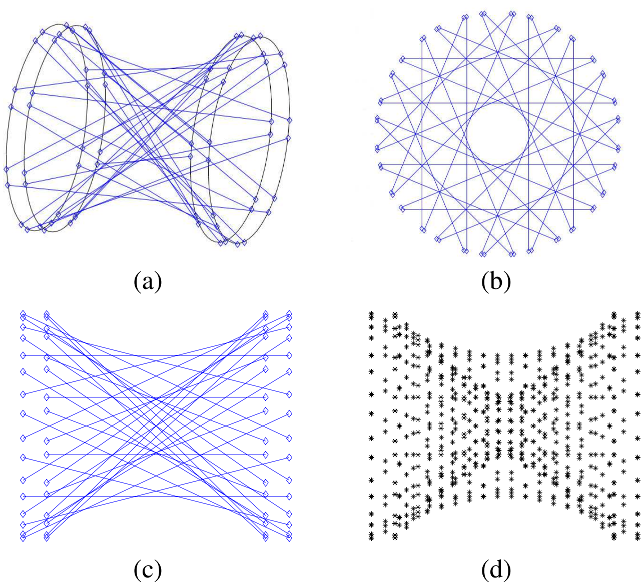


Fig. 2: The paths of the ultrasound propagation (multiple measurements) in the pipeline: (a) three-dimensional representation, (b) continuous cross section, (c) continuous longitudinal section, (d) discrete longitudinal section.

III. Ultrasonic Tomography Method

The velocity components in Eq. 6 are functions of distance s (along the line C). Given the flowmeter configuration (Fig. 1b), if we assume that the velocity distribution doesn't change along the z direction, each s corresponds to a particular point in the pipeline cross-section. Thus, a conversion to a cylindrical coordinate system appears to be suitable. This way, velocities themselves will be defined in a polar coordinate system, which is even intuitive, given the defined purpose - deriving the tomogram.

In suitable cylindrical coordinate system, Eq. 6 can be rewritten in the following manner:

$$\begin{aligned}\frac{\Delta t_i\, c_i^2}{2} &= \int_C |\vec{v_\varepsilon}| \cos\alpha_\varepsilon^i ds + \int_C |\vec{v_r}| \cos\alpha_r^i ds + \\ &+ \int_C |\vec{v_o}| \cos\alpha_o^i ds = \\ &= \int_\theta \int_r |\vec{v_\varepsilon}(\theta, r)| \cos\alpha_\varepsilon^i dr d\theta + \\ &+ \int_\theta \int_r |\vec{v_r}(\theta, r)| \cos\alpha_r^i dr d\theta + \\ &+ \int_\theta \int_r |\vec{v_o}(\theta, r)| \cos\alpha_o^i dr d\theta \quad i = 1, 2, ...N.\end{aligned} \tag{7}$$

where N is the total number of measurements, or the ultrasound paths. This system of equations, coming from the multipath measurements is the only way to derive the tomogram. Having classical, single-path measurement could lead to the estimation of the average fluid velocity, but not the flow field. This is obvious from the Eq. 6 which can not be, by any means, solved in the context of deriving $\vec{v}_\varepsilon(s)$, $\vec{v}_r(s)$ and $\vec{v}_o(s)$. Even if it was possible, we cannot derive the entire tomogram, but simply velocities corresponding to one diameter of the pipeline cross-section.

As it can be seen in Fig. 2, these multiple paths have to be discretized along z in order to computationally solve the double integrals in Eq. 7. However, this sort of system, in this form, can not be solved neither. Therefore, in our method, in order to be able to derive velocities in polar coordinates (tomograms), we firstly represent each of them as the Zernike polynomial expansion. That way, we reduce the problem to the ill-posed problem of finding expansion coefficients for each of the addends (velocities).

A. Zernike polynomials

Zernike polynomials represent a set of polynomials defined on a unit circle [29] (Fig. 3). In polar coordinate system, they are given as a product of radial polynomials and angular functions, with the former being developed from the Jacobi polynomials [30], and the latter being the basis functions for the two-dimensional rotation group:

$$\left.\begin{aligned} Z_j^{even} &= \sqrt{n+1} R_n^m(r) \sin m\theta \\ Z_j^{odd} &= \sqrt{n+1} R_n^m(r) \cos m\theta \end{aligned}\right\} m \neq 0, \qquad Z_j = \sqrt{n+1} R_n^0(r), \quad m = 0, \tag{8}$$

where radial polynomial is defined as:

$$R_n^m(r) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s! \left(\frac{n-m}{2} - s\right)! \left(\frac{n+m}{2} - s\right)!} r^{n-2s}. \tag{9}$$

The principal advantages of Zernike polynomials, in terms of mathematical properties, are their orthogonality over the continuous unit circle, and the fact that they represent a complete set. Consequently, they can represent arbitrarily complex continuous surfaces, given enough terms [31]. Due to this, they are commonly used in a polynomial expansion of an arbitrary wave front over a circular aperture [30]:

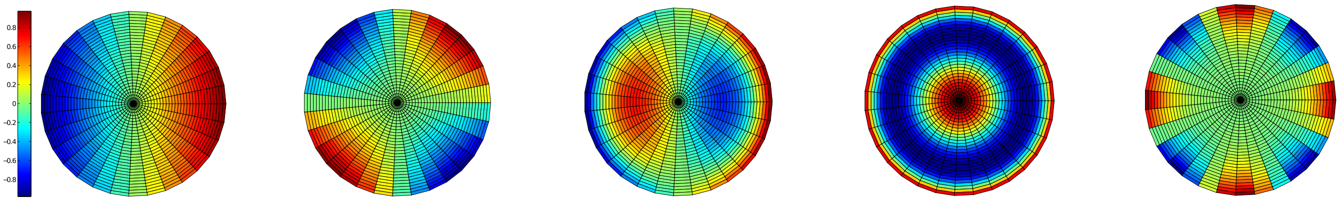
$$f(\theta, r) = \sum_{j=0}^{n} k_j Z_j(\theta, r), \tag{10}$$

with $f(\theta, r)$ being the arbitrary function and k_j the expansion coefficient. Therefore, they appear to be a suitable representation of our velocity components. By replacing Eq. 10 in Eq. 7 we are able to express the acquired ultrasound measurements as:

$$\begin{aligned}\frac{\Delta t_i\, c_i^2}{2} &= \sum_{j=0}^{n} a_j \int_\theta \int_r Z_j(\theta, r) \cos\alpha_\varepsilon^i dr d\theta + \\ &+ \sum_{j=0}^{n} b_j \int_\theta \int_r Z_j(\theta, r) \cos\alpha_r^i dr d\theta + \\ &+ \sum_{j=0}^{n} c_j \int_\theta \int_r Z_j(\theta, r) \cos\alpha_o^i dr d\theta.\end{aligned} \tag{11}$$

That allows us to define the system of equations in the matrix form:

$$m_i = \frac{\Delta t_i\, c_i^2}{2} = \mathbf{T}_i^a \mathbf{a}' + \mathbf{T}_i^b \mathbf{b}' + \mathbf{T}_i^c \mathbf{c}', \quad i = 1, 2, ...N, \tag{12}$$



(a) (b) (c) (d) (e)

Fig. 3: Zernike polynomials (Z_j): (a) Z_2, (b) Z_5, (c) Z_8, (d) Z_{11}, (e) Z_{14}.

where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the vectors containing the unknown coefficients and $\mathbf{T}_i^a$, $\mathbf{T}_i^b$ and $\mathbf{T}_i^c$ the rows of the transformation matrices i.e. solutions of the previously defined integrals for each j, where i is the index of measurement (N in total). $\mathbf{m}$ is the vector of measurements. The discrete ill-posed problem is finally defined as:

$$\begin{bmatrix}\mathbf{T}^a & \mathbf{T}^b & \mathbf{T}^c\end{bmatrix}\begin{bmatrix}\mathbf{a}' \\ \mathbf{b}' \\ \mathbf{c}'\end{bmatrix} = \mathbf{T}\mathbf{x} = \mathbf{m}. \tag{13}$$

B. Tikhonov Regularization

In order to be able to reconstruct velocity distribution, we ought to find coefficients $\mathbf{x}$ by solving discrete ill-posed problem [19] designated in Eq. 13. The useful and stable solution of this problem is obtained through the regularization [32]. Certainly, the most common form of the regularization and therefore the first applied method, is Tikhonov regularization [33], [34], where the regularized solution $\mathbf{x}_\lambda$ is defined as a minimizer of the following weighted combination of the residual norm and the side constraint:

$$\mathbf{x}_\lambda = argmin\left(||\mathbf{T}\mathbf{x} - \mathbf{m}||_2^2 + \lambda^2||\mathbf{L}(\mathbf{x} - \mathbf{x_0})||_2^2\right), \tag{14}$$

with λ being the regularization parameter, $\mathbf{L}$ the identity matrix and $\mathbf{x_0}$ the vector of initial conditions. The regularization parameter has significant impact on solution properties, through controlling the weight of minimization of the side constraint and the sensitivity of the solution with respect to perturbations in $\mathbf{T}$ and $\mathbf{m}$.

As it appears, the most utilized algorithm for the stated least square problem (Eq. 14) is the one based on QR factorisation [35]. However, we rather rely on the MATLAB toolbox proposed by Hansen [36], [37], who chooses another approach, based on Singular Value Decomposition (SVD) of the matrix $\mathbf{T}$:

$$\mathbf{T} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_{i=1}^{n}\mathbf{u}_i\sigma_i\mathbf{v}_i^T. \tag{15}$$

By assuming $\mathbf{L} = I_N$ the regularized solution is given as:

$$\mathbf{x}^{reg} = \sum_{i=1}^{n} f_i\frac{\mathbf{u}_i^T\mathbf{m}}{\sigma_i}\mathbf{v}_i, \tag{16}$$

where f_i are filter factors assuring that the addends corresponding to the smaller singular values are filtered out. In case of a Tikhonov regularization, they are defined as:

$$f_i = \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2}. \tag{17}$$

The regularization parameter λ is determined using Generalized Cross Validation (GCV) [38], by minimizing the GCV function:

$$G = \frac{||\mathbf{T}\mathbf{x}^{reg} - \mathbf{m}||_2^2}{(trace(\mathbf{I}_N - \mathbf{T}\mathbf{T}'))^2}. \tag{18}$$

C. Quadratically constrained ℓ_1 minimization

The alternative approach assumes using the quadratically constrained ℓ_1 minimization [20], [39] in deriving unknown Zernike coefficients. In this case, we search minimal value of $\mathbf{x}$ satisfying the inequality:

$$\frac{1}{2}\left(||\mathbf{T}\mathbf{x} - \mathbf{m}||_2^2 - \epsilon^2\right) \leq 0. \tag{19}$$

The problem is reformulated as the second order cone problem [40]:

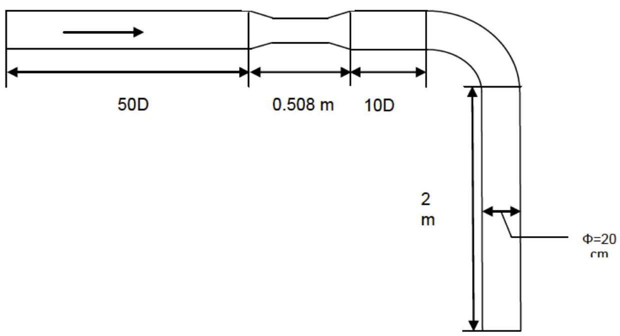
$$\begin{aligned} \min_{\mathbf{x},\mathbf{u}} \sum_i u_i \quad \text{subject to} \quad \mathbf{x} - \mathbf{u} &\leq 0, \\ -\mathbf{x} - \mathbf{u} &\leq 0 \\ \frac{1}{2}\left(||\mathbf{T}\mathbf{x} - \mathbf{m}||_2^2 - \epsilon^2\right) &\leq 0, \end{aligned} \tag{20}$$

where the first two inequalities assure that $\mathbf{u}$ is positive, therefore that the cost-function is non-negative. The problem is solved using the log-barrier method [41].

D. SVD based filtering of Zernike polynomials

The filtering is applied in deriving matrices $\mathbf{T}$. The contribution of Zernike polynomial to the integration is estimated by applying the Singular Value Decomposition (SVD) to the matrix $\mathbf{T}$, after adding a new polynomial to the expansion (new column). As the criterion we take a minimal singular value, which is compared with the predefined threshold ($t = 0.001$). This way, we obtain sparse Zernike representation of the velocity.

This preprocessing is combined both with the Tikhonov Regularization and the Quadratically constrained ℓ_1 minimization. However, given that the former can be equalized with ℓ_2 minimization, in this case imposing "more strict" sparsity of the Zernike representation should influence the results more than in case of ℓ_1 minimization.



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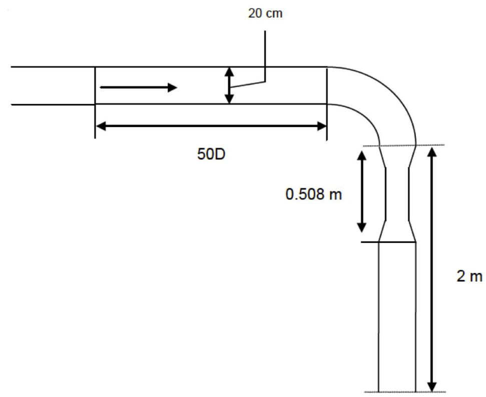


(b)

Fig. 4: Measurements configuration ($\delta = 1.5$): (a) before the knee, (b) after the knee.

E. Velocity distribution reconstruction

Finally, knowing the expansion coefficients, velocity tomograms are calculated using Zernike polynomial expansion:

$$\begin{aligned} |\widehat{\vec{v_\varepsilon}(\theta,r)}| &= \sum_{j=0}^{n} \mathbf{a}_j^{reg} Z_j(\theta,r), \\ |\widehat{\vec{v_r}(\theta,r)}| &= \sum_{j=0}^{n} \mathbf{b}_j^{reg} Z_j(\theta,r), \\ |\widehat{\vec{v_o}(\theta,r)}| &= \sum_{j=0}^{n} \mathbf{c}_j^{reg} Z_j(\theta,r). \end{aligned} \tag{21}$$

The results we present in the following section are based on the polynomials expansion up to the order $n = 13$. This choice is based on the empirical assertion that, in the considered case, the inclusion of higher orders does not influence the obtained tomograms.

IV. Results

We applied the proposed method to the measurements acquired using DFX MM US flowmeter [42] in a pipeline with the radius $R \approx 100mm$ over a distance (z) of $D \approx 508mm$. Radius of curvature to pipeline diameter ratio is $\delta = 1.5$. Multipath measurements assume in this case $N = 32$ different paths. Each of the paths is discretized in 1024 points.

The first set of measurements was acquired before the elbow, where the propagating fluid can be considered as a turbulent fully-developed flow, while the second set, taken after the elbow, represent a turbulent disturbed flow (Fig. 4).

The quality of the proposed imaging method was analysed with respect to two criteria:

- *residual norm*: indicator of the suitability of the employed regularization/minimization method:

$$R = \frac{||\mathbf{Tx} - \mathbf{m}||_2^2}{||\mathbf{m}||_2^2}. \tag{22}$$

- *flow*: the volumetric flow rate, computed using derived axial component:

$$Q = \int_\theta \int_r |\widehat{\vec{v_\varepsilon}(\theta,r)}| r dr d\theta. \tag{23}$$

In Table I, we provide the quantitative results achieved with four tested approaches: Tikhnov regularization and Quadratically constrained ℓ_1 minimization, with and without filtering of polynomials. The obtained velocity tomograms are presented in Fig. 5 and 6.

The residual norm reflects the capability of the employed regularization/minimization method to face the ill-possessedness of the problem. This parameter implies the error in the Zernike polynomial expansion coefficients' estimation. It can be considered as the error in the reconstruction of time-of-flight measurements, using derived tomograms.

The flow relative error ("flow error") is calculated by comparing the estimated flow (Eq. 23) to the one measured using reference electromagnetic flowmeter [43].

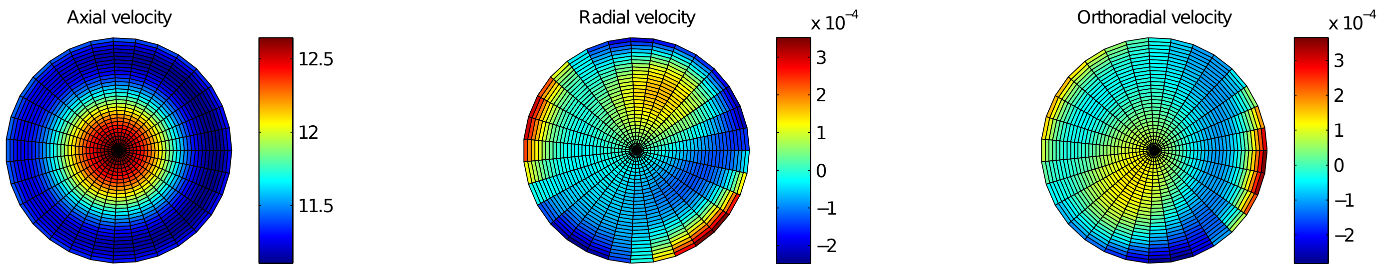
As a mean of qualitative validation the obtained tomograms are compared with the reference flow physical model (Fig. 7), as well [43]. This validation is reinforced through the additional comparison with the computational fluid dynamics (CFD) calculation for the pipeline characterised with the same δ, provided in Fig. 18 in [22].

On one side, the relative residual norm indicates the superiority of the Tikhonov regularization method in facing the ill-possessedness of the problem. On the other side, the flow relative error shows that the more accurate estimation is achieved with Quadratically constrained ℓ_1 minimization.

Employed filtering does not improve the performances for none of two approaches, with respect to both of the defined criteria. It almost does not affect the flow estimation while degrading the relative residual norm in the former case. In the latter one, the results are virtually identical, which was the reason for not illustrating them in Fig. 5 and 6.

However, significantly bigger dynamic range in case of the Tikhonov regularization (Fig. 5 and 6), closer to the one expected by a simulated flow physical model, imposes this approach as the most appropriate choice for the proposed method.

Very low magnitudes of the estimated radial and orthoradial velocity tomograms lead to the eventual ambiguity with the



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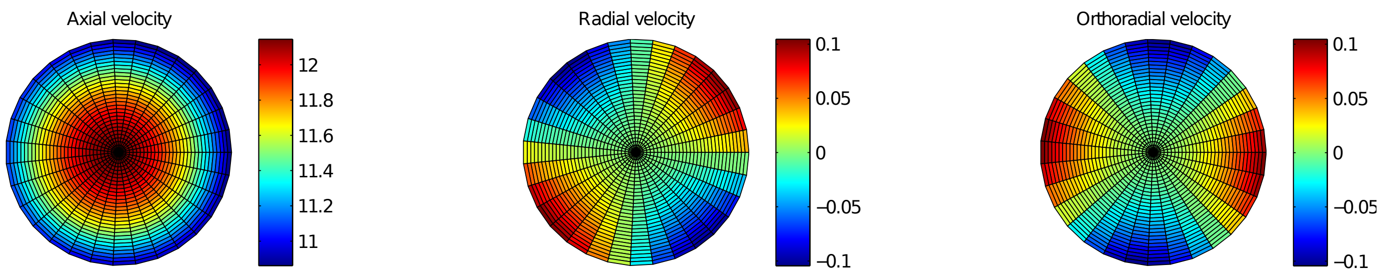

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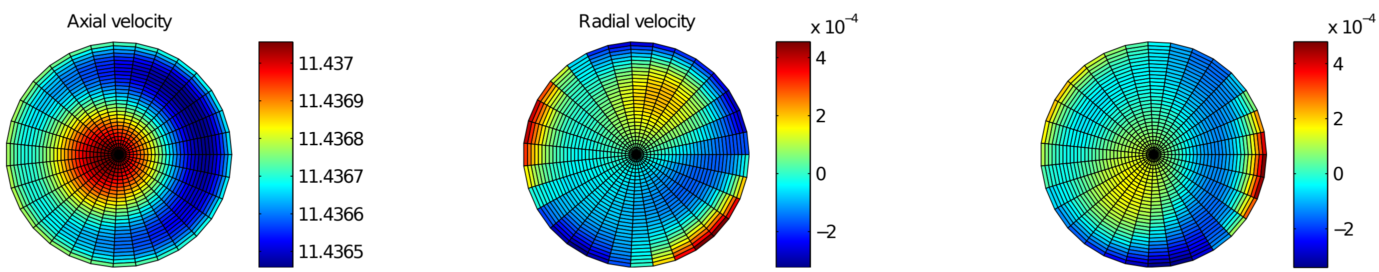

(c)

Fig. 5: Tomograms of the dimensionless velocity for a fully-developed flow (before the elbow): (a) Tikhonov regularization without filtering, (b) Tikhonov regularization with filtering, (c) Quadratically constrained ℓ_1 minimization without filtering.

noise (Fig. 5 and 6), leaving the axial velocity tomogram as the most relevant output in the proposed method.

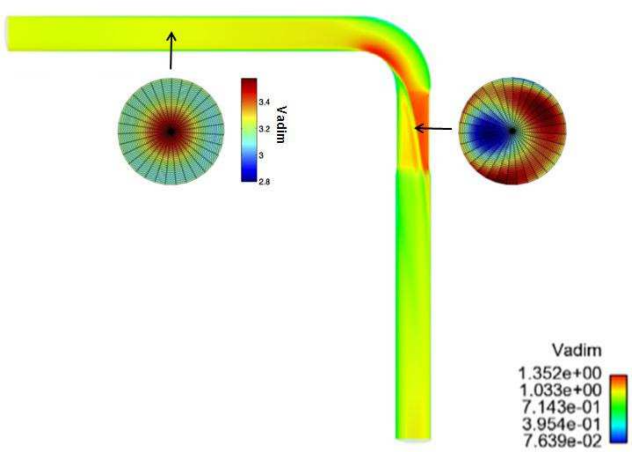

Fig. 7: The comparison of the obtained tomograms with the longitudinal section of the flow physical model ($\delta = 1.5$) [43] ($Q = 300m^3/h$).

The presented quantitative validation demonstrate the plausibility of the proposed method in terms of capability to represent the acquired measurements using ultrasonic tomograms. Furthermore, the comparison with the simulated flow physical model, qualitatively upholds the fidelity of the derived tomograms.

V. Conclusion

This paper presents a method allowing the reconstruction of a transverse flow field in a pipeline, using intrusive time-of-flight ultrasonic measurements. Given the outputs, having a form of velocities images, it genuinely represents one Ultrasonic Tomography (UT) method. By introducing Zernike polynomial expansions instead of velocity components, we reduced the UT problem to one ill-posed problem of finding the coefficients of the expansion. Further, we have applied comparatively two approaches in treating this problem: Tikhonov regularization and Quadratically constrained ℓ_1 minimization. Both of the approaches were eventually reinforced with the SVD filtering of Zernike polynomials, as well. However, the former one, without filtering, proved to be the most suitable with respect to the performed validation, which assumed both the relative residual norm and the flow estimation using obtained tomograms. Finally, the obtained results in terms of the average relative error indicate for a fully-developed flow $\sim 0.3\%$ and $\sim 1.6\%$ in residual norm and flow estimation respectively, and $\sim 0.5\%$ and $\sim 1.8\%$ for a disturbed one.

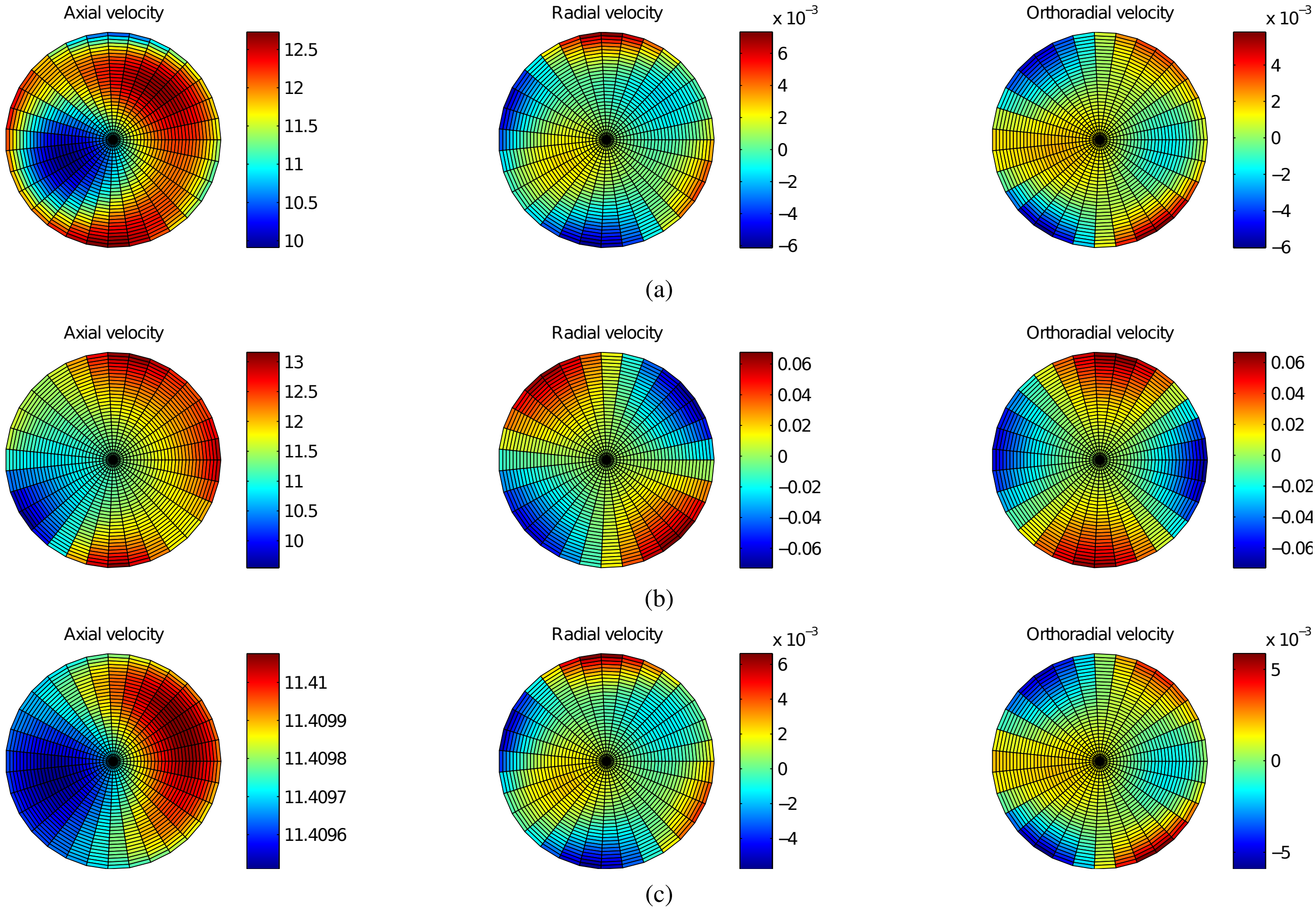

Fig. 6: Tomograms of the dimensionless velocity for a disturbed flow (after the elbow): (a) Tikhonov regularization without filtering, (b) Tikhonov regularization with filtering, (c) Quadratically constrained ℓ_1 minimization without filtering.

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TABLE I: COMPARISON OF DIFFERENT METHODS IN TERMS OF RELATIVE RESIDUAL NORM AND FLOW ERROR.

	Set of measurements	reference flow $[m^3/h]$	Tikhonov regularization				Quadratically const. ℓ_1 minim.			
			without filtering		with filtering		without filtering		with filtering	
			relative residual norm [%]	flow error [%]	relative residual norm [%]	flow error [%]	relative residual norm [%]	flow error [%]	relative residual norm [%]	flow error [%]
before the elbow	1	1073.19	0.3029	1.8384	0.5806	1.8293	2.1218	0.8040	2.1433	0.8067
	2	903.88	0.3050	1.7651	0.5858	1.7559	2.1337	0.7334	2.1555	0.7360
	3	705.66	0.3063	1.6495	0.5894	1.6403	2.1445	0.6201	2.1663	0.6227
	4	503.99	0.3123	1.5954	0.6039	1.5861	2.1815	0.5715	2.2037	0.5741
	5	301.08	0.3243	1.0657	0.6282	1.0562	2.2444	0.0570	2.2650	0.0568
	average	/	0.3102	1.5828	0.5976	1.5736	2.1652	0.5572	2.2638	0.5572
after the elbow	1	1073.7	0.5251	2.1145	1.7347	2.1206	4.5884	0.5192	4.7940	0.5231
	2	905.37	0.5248	2.0029	1.7324	2.0092	4.6008	0.4091	4.8072	0.4131
	3	704.38	0.5235	1.9576	1.7148	1.9639	4.5958	0.3654	4.7985	0.3691
	4	504.83	0.5246	1.7947	1.7229	1.8012	4.6394	0.2009	4.8437	0.2047
	5	305.58	0.4876	1.2481	1.6637	1.2554	4.7424	0.3618	4.9520	0.3577
	average	/	0.5171	1.8226	1.7137	1.8301	4.6334	0.3713	4.8391	0.3713

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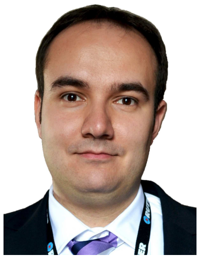

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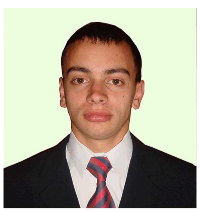

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