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# EIKONAL-BASED VERTICES GROWING AND ITERATIVE SEEDING FOR EFFICIENT GRAPH-BASED SEGMENTATION

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## ABSTRACT

In this paper we propose to use the Eikonal equation on graphs for generalized data clustering. We introduce a new potential function that favors the creation of homogeneous clusters together with an iterative algorithm that place seeds vertices at smart locations. Oversegmentation application shows the effectiveness of our approach and gives results comparable to the state-of-the-art methods.

**Index Terms**— Graphs, Eikonal equation, Seeds positioning, Oversegmentation.

## 1. INTRODUCTION AND CONTEXT

With the increasing amount of available data, and the need for fast and accurate processings, the simplification or clustering of data becomes a crucial point for many applications. A convenient way to address this task is to consider that organized or non-organized data can be modeled by a graph that inherently handle interactions between vertices. Exhibiting clusters of this graph leads to a simplification of the domain and decreases the size of the problem.

Many graph-based clustering techniques have been proposed in the litterature, such as cut-based, spectral or random walks methods (see [1, 2] for a review of these techniques).

Recent works try to adapt well-known signal processing tools to the discrete domain of the graphs [3, 4]. Recently, [5] introduces an adaptation to graphs of the Eikonal equation that generalizes front propagation methods to data of arbitrary dimensions. In this paper, we propose to use this Eikonal-based framework for data clustering, and more particularly for image oversegmentation. Together with a new potential function, we propose an iterative algorithm that automatically produces a desired number of clusters. Oversegmentation comparisons with state-of-the-art dedicated methods show the effectiveness of our approach.

**Notations and context.** We assume that any discrete domain can be modeled by a weighted graph. Let  $G = (V, E, w)$  be a weighted graph composed of a finite set  $V = \{v_1, \dots, v_n\}$  of  $n$  vertices and  $E \subset V \times V$  a set of weighted edges. An edge  $(u, v) \in E$  connects two adjacent vertices  $u$  and  $v$  and is weighted by the function  $w : V \times V \rightarrow \mathbb{R}^+$ . In the

following such a weight is denoted by  $w_{uv}$ . We denote by  $N_v$  the number of neighbors of  $v$ , i.e. the subset of vertices that share an edge with  $v$ . The degree  $\delta_v$  of the vertex  $v$  is the sum of the weights of the edges connected to  $v$ :  $\delta_v = \sum_{u \in V | (u,v) \in E} w_{uv}$ .

Let  $f : V \rightarrow \mathbb{R}$  be a real-valued function that assigns a real value  $f(u)$  to each vertex  $u \in V$ . We denote by  $\mathcal{H}(V)$  the Hilbert space of such functions. Graphs are assumed to be simple, connected and undirected, implying that the function  $w$  is symmetric.

The Eikonal equation transposed on graph makes use of the Partial difference Equations (PdE) framework [6] and can be written as:

$$\begin{cases} \|(\nabla_w^- f)(u)\|_p = P(u) & \forall u \in V \\ f(u) = \phi(u) & \forall u \in V_0 \end{cases} \quad (1)$$

where  $P$  is a positive function,  $\phi$  is an initialization function (see [5] and references therein for details), and  $V_0$  is the set of initial seeds. Solving this equation for a graph gives for each vertex  $v \in V \setminus V_0$  the geodesic distance  $U(v)$  to the closest seed vertex  $u \in V_0$ .

The term  $\|(\nabla_w^- f)(u)\|_p$  denotes the  $\mathcal{L}_p$  norm of the weighted morphological gradient at a vertex  $u$  and is defined as:

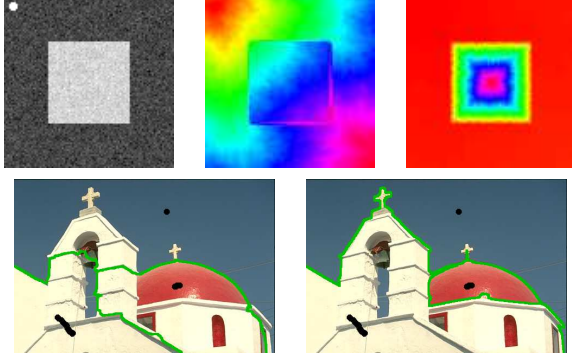
$$\|(\nabla_w^- f)(u)\|_p = \left[ \sum_{v | (u,v) \in V} w_{uv}^{p/2} |(Df(u))^-|^p \right]^{1/p}$$

with  $(Df(u))^- = -\min(f(u) - f(v), 0)$ .

Given a set of seed vertices  $V_0$ , the solution of Eq.1 processed by the adaptation of the *Fast Marching* algorithm [7] on graphs leads to a graph-based clustering of the datas.

## 2. CONTRIBUTIONS

Our contributions are twofold : First, we extend our previous work [8] by considering the adaptation of the *Eikonal-based Region Growing Clustering* (ERGC) algorithm on graphs. Given a set of seeds, ERGC processes to a label diffusion via a dynamic potential function. Second, we propose a greedy algorithm that adds iteratively labeled seeds at specific locations. The only parameter of this algorithm is the desired



**Fig. 1.** Top: Comparison of the behaviors of the diffusion on a synthetic image. Geodesic distances map with a heat color map for the gradient-based potential function (middle), and the proposed one (right). The initial seed is depicted on the left figure by the white dot. Bottom: Segmentation result from a set of seeds (depicted in black) with the gradient-based potential function (left) and the proposed one (right).

number of clusters. Let  $F_v \in \mathbb{R}^n$  be the data attached to  $v$ , and  $\hat{F}_{C_i}$  the mean feature vector of a subset  $C_i$  of  $V$ .

**Proposed potential function.** Classical potential functions are based on the gradient of the graph computed at each vertex [5]. In case of graph-based image segmentation, the results using this static potential function heavily depend on the location of the seeds. Fig. 1 (top middle) shows the geodesic distances computed with such a potential function: the front propagates on the white square before having recovered all the black area. Then, a good segmentation of the square based on these geodesic distances cannot be obtained. Such a bad result is shown in Fig. 1 (bottom left) that presents important leaks. In this work, we propose a dynamic potential function that favors the grouping of perceptually and adjacent vertices. Given a vertex  $v$  belonging to the evolving front, its local potential is computed as the distance between its feature  $F_v$  and the mean feature vector  $\hat{F}_{C_j}$  of the adjoining cluster  $C_j$ :  $P(v, C_j) = \|F_v - \hat{F}_{C_j}\|_2$ . Each time a vertex  $v_i$  is incorporated to a cluster  $C_j$ , its mean feature  $\hat{F}_{C_j}$  is updated:

$$\begin{cases} \hat{F}_{C_j} \leftarrow \frac{\hat{F}_{C_j} \times \text{Card}(C_j) + F_v}{\text{Card}(C_j) + 1} \\ \text{Card}(C_j) \leftarrow \text{Card}(C_j) + 1 \end{cases}$$

This potential function is clearly dynamic since it relies on continuously updating of the cluster features. It also favors the diffusion of the front to vertices whose features are close to the mean features of the expanding cluster. Fig. 1 (top right) shows the geodesic distances computed with this potential function: the front recovers entirely the black area before propagating through the white square. Applied to image, it favors the grouping of perceptually and adjacent pixels, while preserving much more the contours, see Fig. 1 (bottom right).

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#### Algorithm 1: Automatic seeds positioning

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**Data:** A Graph  $G$ ,  $n$  the number of desired clusters.

$V_0 \leftarrow \arg \min_{v \in V} (\delta_v / N_v)$ ;

Solve Eq. 1 with  $V_0$  as the seed vertices set;

Save geodesic distances  $U_0$ ;

$it = 1$ ;

**while**  $it < n$  **do**

$V_0 \leftarrow V_0 \cup \arg \max_{v \in V} (U_{it}(v))$ ;

Solve Eq. 1 with  $V_0$ ;

$it \leftarrow it + 1$ ;

Save geodesic distances  $U_{it}$ ;

---

**Automatic seeds positioning.** The set  $V_0$  of initial seeds vertices is a critical aspect for many front propagation algorithms. As shown in Fig. 1, a not carefully well-chosen location for seeds leads to important leaks and results in bad clusterings. Together with our dynamic potential function, we propose a simple scheme (similar to the Farthest point seeding method [9]) that iteratively adds new seeds at smart locations. First, one proceeds to a complete diffusion with an initial first seed located at the vertex of minimal normalized degree  $V_0 \leftarrow \arg \min_{v \in V} (\delta_v / N_v)$ . From the resulting geodesic distances map, one places a new seed at the location of the highest geodesic distance, and proceeds to a new diffusion. One iterates until a predefined stopping criterion is reached. In our experiments described in the sequel, the stopping criterion is simply the final number of desired clusters. This procedure is summarized in Algorithm 1.

Fig. 2 illustrates the process: the first seed vertex (depicted in red) is located in the flat area of the sky, and the corresponding geodesic distance map exhibits high values in the non-sky areas. Two seeds (depicted in green and blue) are then successively added to the locations of highest geodesic distance (shown with a heat color map).

### 3. IMAGE OVERSEGMENTATION

This paper focuses on the image oversegmentation application of our proposal. In the following color images are considered in the Lab colorspace; so for a given vertex  $v$  corresponding to a pixel  $p$ ,  $F_v$  reduces to  $[l, a, b]^T$ . The proposed oversegmentation scheme makes use of both contributions of the paper. Although the iterative process described above could be applied directly on the whole image domain via a 4-grid graph, it is very time consuming in practice since the dimension is large (i.e. the number of pixels). We then proceed to an initial oversegmentation of the image domain that reduces drastically the dimension of the graph. This initial oversegmentation is performed by considering a small number of seeds (1% of the number of pixels) placed on a regular grid, and the  $\mathcal{L}_2$  norm is used to weight the edges. The iterative algorithm proposed above is then applied on the underlying Region Adjacency Graph (RAG) of the initial oversegmenta-

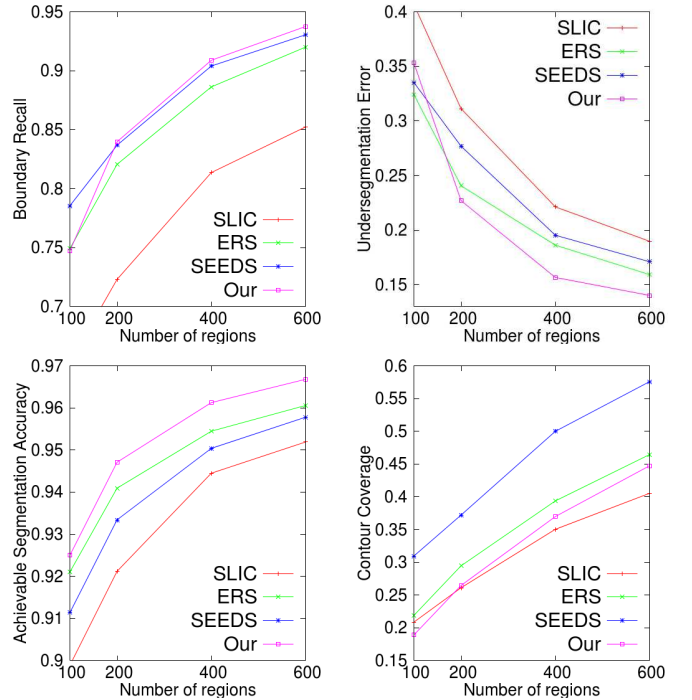


**Fig. 2.** Illustration of our automatic seeds positioning scheme on graph. First column: the image and the initial dense oversegmentation. Then from left to right, 3 seeds are added iteratively on the RAG (depicted in red, green and blue respectively), and the corresponding geodesic distance map are shown with a heat color map.

tion that contains far much less vertices than the number of pixels of the image. The feature vector  $F_v$  attached to a vertex  $v$  corresponding to a region  $C_i$  is the mean color (in the Lab colorspace) of this region. As for the 4-grid graph, the  $\mathcal{L}_2$  norm is used to weight the edges of the RAG. Throughout the experiments, the stopping criterion of the iterative process is simply the desired number of final clusters in order to provide fair comparisons with other oversegmentation algorithms with equal number of clusters.

We compare our algorithm to state-of-the-art methods *Simple Linear Iterative Clustering* [10] (SLIC), *Entropy Rate Superpixels* [11] (ERS), and *SEEDS* [12]. Some oversegmentation results obtained with these algorithms are shown in Fig. 4. The Berkeley dataset [13] is used as a benchmark and contains 500 images of size  $481 \times 321$  (or  $321 \times 481$ ) and about 2700 ground truth manual segmentations. All the experiments have been computed from scratch on these images with the code of state-of-the-art methods available on their respective authors webpage. Our method has an approximate complexity of  $\mathcal{O}(n \log n)$  with an appropriate heap to sort the pixels/vertices according to their distances. Despite this theoretical complexity, the proposed algorithm is very fast in practice, and nearly linear in time. It oversegments an image of this dataset in less than half a second. This processing times are comparable to SEEDS and SLIC algorithms, ERS being quite slower (about 2s per image). Figure 3 plots comparative results on 4 metrics, namely *Boundary Recall*, (*Corrected*) *Undersegmentation Error*, *Achievable Segmentation Accuracy*, and the additional proposed metric *Contour Coverage*.

- *Boundary Recall* measures the fraction of segmented edges that is also present in the ground truth segmentation within a distances threshold  $t$ .  $t$  has been fixed to 2 (as in [10, 12, 11]) to deal with sometimes approximate manual segmentations.
- (*Corrected*) *Undersegmentation Error* proposed in [14] tries to overcome and unify the different definitions [10, 11] of *Un-*



**Fig. 3.** Comparison of state-of-the-art methods with ours on several metrics: *Boundary Recall*, *Undersegmentation Error*, *Achievable Segmentation Accuracy*, and *Contour Coverage*.

*dersegmentation Error*. It measures the fraction of clusters *bleeding* into another cluster according to the ground truth:

$$UE = \frac{\sum_k |C_k - g_{max}(C_k)|}{\sum_i |g_i|}$$

where  $g_{max}(C_k)$  indicates the matching ground truth segment  $g_j$  of  $C_k$  with the largest overlap, and  $|\cdot|$  denotes the size (in pixels) of an element.



**Fig. 4.** Visual comparison between algorithms with 100 clusters per oversegmentation. From top to bottom: SLIC [10], ERS [11], SEEDS [12], and our algorithm. Important leaks for SLIC, ERS and SEEDS can be seen on the muzzle of the bear (left column), on the horse’s mane or on the belly foal (middle column), and on the bear cubs (right column). High *Contour Coverage* of SEEDS is also highlighted (third row).

- *Achievable Segmentation Accuracy* (ASA) is a segmentation upperbound measure that gives the best segmentation accuracy that can be obtained by using the clusters as units:

$$ASA = \frac{\sum_k \max_i |C_k \cap g_i|}{\sum_i g_i}$$

- We introduce the *Contour Coverage* metric (CC) in addition to traditional ones in order to measure the fraction of contour pixels present in the segmentation according to the whole size of the image. Low *Contour Coverage* values reflect high *compactness* of the clusters of an oversegmentation.

As shown in Fig. 3, over 200 clusters our algorithm outperforms state-of-the-art algorithms on UE and ASA metrics, and is competitive with the best algorithm (i.e. SEEDS) on

the BR metric. This last result has to be appreciated under the light of *Contour Coverage* measures, for which SEEDS presents the worst results. The third row of Fig. 4 illustrates such high *Contour Coverage* values of SEEDS results.

#### 4. CONCLUSION

In this paper we have presented a new dynamic potential function for the Eikonal equation on graphs. A simple iterative algorithm was also proposed to deal with the critical step of seeds placement. These two contributions applied to image oversegmentation leads to results comparable to those obtain with dedicated state-of-the-art methods. Further works will investigate the usability of our approach on meshes oversegmentation, automatic databases and point clouds clustering.

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