



HAL
open science

Distributed Model Predictive Control of a Hydro-Power Valley by Dual Decomposition

Alexandra Grancharova, Sorin Olaru, Guillaume Sandou, Cristina Stoica Maniu, Pedro Rodriguez-Ayerbe

► **To cite this version:**

Alexandra Grancharova, Sorin Olaru, Guillaume Sandou, Cristina Stoica Maniu, Pedro Rodriguez-Ayerbe. Distributed Model Predictive Control of a Hydro-Power Valley by Dual Decomposition. Automatics and Informatics'2014, Oct 2014, Sofia, Bulgaria. hal-01073332

HAL Id: hal-01073332

<https://hal.science/hal-01073332>

Submitted on 16 Mar 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

DISTRIBUTED MODEL PREDICTIVE CONTROL OF A HYDRO-POWER VALLEY BY DUAL DECOMPOSITION¹

A. Grancharova¹, S. Oлару², G. Sandou², C. Stoica², P. Rodriguez²

¹ Institute of System Engineering and Robotics, Bulgarian Academy of Sciences, Acad G. Bonchev str., Bl.2, P.O.Box 79, Sofia 1113, Bulgaria, e-mail: alexandra.grancharova@abv.bg

² SUPELEC E3S Automatic Control Department, 3 rue Joliot-Curie, 91192 Gif-sur-Yvette Cedex, France e-mail: {sorin.olaru; guillaume.sandou; cristina.stoica; pedro.rodriguez}@supelec.fr

Abstract: In this paper, a suboptimal distributed MPC approach for linear interconnected systems is considered, where it is assumed that the systems are coupled through their control inputs and an optimal reference tracking problem for the overall system is solved. The approach is applied to distributed MPC of a hydro-power valley case study.

Key words: Distributed Model Predictive Control, Interconnected Systems, Constraints.

INTRODUCTION

Model predictive control (MPC) involves the solution at each sampling instant of a finite horizon optimal control problem subject to the system dynamics, and state and input constraints. Solving in a centralized way MPC problems for large-scale systems may be impractical due to the topology of the plant and data communication and the large number of decision variables. Recently, several approaches for decentralized and parallel implementation of MPC algorithms have been proposed, [1] - [4].

In [5] - [8], approaches for distributed/decentralized MPC for systems consisting of linear interconnected subsystems have been developed. The approach in [7] is based on the dual decomposition method [9], where large-scale optimization problems are handled by using Lagrange multipliers to relax the couplings between the sub-problems. In [8], a distributed optimization algorithm based on accelerated gradient methods using dual decomposition is proposed and its performance is evaluated on optimization problems arising in distributed MPC. Also, approaches for distributed MPC for systems composed of several nonlinear subsystems have been proposed (e.g. [10], [11], [12]).

In this paper, a modification of the suboptimal distributed MPC approach [7] is considered, where it is assumed that the interconnected linear systems are coupled through their control inputs and an optimal reference tracking problem for the overall system is solved. The approach is applied to distributed MPC of a hydro-power valley case study, described in [13].

THE HYDRO-POWER VALLEY CASE STUDY

Description

The hydro-power valley is a set of hydroelectric production plants, depending on interconnected water resources (typically along a river) and aiming at producing a given amount of electricity according to a daily power-generation program for each power plant. In each plant, the generated power is a function of the turbine flows, which at the same time influence the reservoir levels that need to be controlled such that they

are within the specified limits [13]. The control system design for a hydro-power valley should take into account the fact that the valley can face unpredictable events, such as plant failures or meteorological changes.

The configuration of the considered hydro-power valley is depicted in Fig. 1 [13].

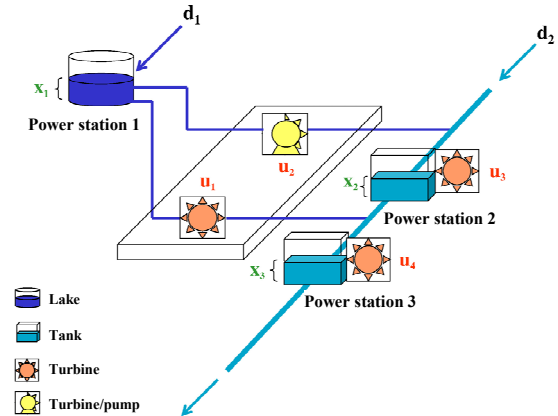


Fig. 1. The hydro-power valley [13].

Each power plant in the valley is characterized by the dynamics of its water storage, described as follows [13]:

$$S \frac{dh}{dt} = f_{in} - f_{out} \quad (1)$$

where h denotes the water level, S the water surface, and f_{in} , f_{out} the input and output water flows, respectively. These water flows depend on the interconnections between the plants as well as the turbine flows, considered as the control inputs, and disturbance flows. By considering a sampled-data version of equation (1) for a purpose of discrete-time control, and gathering all equations of this form, the whole system is modeled as a classical state space representation [13]:

$$x(t+1) = Ax(t) + Bu(t) + Ed(t) \quad (2)$$

¹ This work was financed by the National Science Fund of the Ministry of Education and Science of Republic of Bulgaria, contract №ДРила 01/12 and the Partenariats Hubert Curien (PHC) Programme Rila of the French Government, contract №29401YJ “Robust distributed predictive control of complex systems”

In (2):

$$x(t) = [x_1(t), x_2(t), x_3(t)]^T, u(t) = [u_1(t), u_2(t), u_3(t), u_4(t)]^T \quad (3)$$

$$d(t) = [d_1(t), d_2(t)]^T \quad (4)$$

are the state, control input, and the disturbance input vectors. All state variables are assumed measurable. The matrices of the model (2) are the following [13]:

$$A = I_{3 \times 3}, B = \begin{bmatrix} -T_s S_1^{-1} & -T_s S_1^{-1} & 0 & 0 \\ 0 & T_s S_2^{-1} & -T_s S_2^{-1} & 0 \\ T_s S_3^{-1} & 0 & T_s S_3^{-1} & -T_s S_3^{-1} \end{bmatrix} \quad (5)$$

$$E = \begin{bmatrix} T_s S_1^{-1} & 0 \\ 0 & T_s S_2^{-1} \\ 0 & 0 \end{bmatrix} \quad (6)$$

where T_s is the sampling time and S_1, S_2, S_3 are the cross-sectional areas of the three tanks (see Fig. 1). The following constraints are imposed on the levels in the three tanks and on the flows through the four turbines [13]:

$$\begin{bmatrix} 0 \\ -6.6 \\ 0 \\ 0 \end{bmatrix} [\text{m}^3/\text{s}] \leq \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} \leq \begin{bmatrix} 60 \\ 0 \\ 60 \\ 60 \end{bmatrix} [\text{m}^3/\text{s}] \quad (7)$$

$$\begin{bmatrix} 716.2 \\ 282 \\ 259.5 \end{bmatrix} [\text{m}] \leq \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \leq \begin{bmatrix} 729.5 \\ 287.5 \\ 268.5 \end{bmatrix} [\text{m}] \quad (8)$$

Decomposition

The hydro-power valley system, described above, can be decomposed into the following three subsystems:

Subsystem 1:

$$x_1(t+1) = A_1 x_1(t) + B_{11} \tilde{u}_1(t) + E_1 d_1(t) \quad (9)$$

with:

$$\tilde{u}_1(t) = [u_1(t), u_2(t)]^T, A_1 = 1, B_{11} = [-T_s S_1^{-1} \quad -T_s S_1^{-1}], E_1 = T_s S_1^{-1} \quad (10)$$

Subsystem 2:

$$x_2(t+1) = A_2 x_2(t) + B_{22} \tilde{u}_2(t) + B_{21} \tilde{u}_1(t) + E_2 d_2(t) \quad (11)$$

with:

$$\tilde{u}_2(t) = u_3(t), A_2 = 1, B_{22} = -T_s S_2^{-1}, B_{21} = [0 \quad T_s S_2^{-1}], E_2 = T_s S_2^{-1} \quad (12)$$

Subsystem 3:

$$x_3(t+1) = A_3 x_3(t) + B_{33} \tilde{u}_3(t) + B_{31} \tilde{u}_1(t) + B_{32} \tilde{u}_2(t) \quad (13)$$

with:

$$\tilde{u}_3(t) = u_4(t), A_3 = 1, B_{33} = -T_s S_3^{-1}, B_{31} = [T_s S_3^{-1} \quad 0], B_{32} = T_s S_3^{-1} \quad (14)$$

It can be seen from equations (9), (11) and (13) that the three subsystems are coupled through their control inputs.

DISTRIBUTED MPC OF LINEAR SYSTEMS COUPLED THROUGH THEIR INPUTS

Consider a system composed by the interconnection of M local subsystems described by the following linear discrete-time models:

$$x_i(t+1) = A_i x_i(t) + \sum_{j=1}^M B_{ij} u_j(t) + E_i d_i(t), i = 1, 2, \dots, M \quad (15)$$

where $x_i(t) \in \mathfrak{R}^{n_i}$, $u_i(t) \in \mathfrak{R}^{m_i}$, and $d_i(t) \in \mathfrak{R}^{s_i}$ are the state, control input, and disturbance input vectors, related to the i -th subsystem, and $A_i \in \mathfrak{R}^{n_i \times n_i}$, $B_{ij} \in \mathfrak{R}^{n_i \times m_j}$, and $E_i \in \mathfrak{R}^{n_i \times s_i}$ are constant matrices. It can be seen that the subsystems (15) are coupled through their control inputs. Let the overall state, control input, and disturbance input be denoted by:

$$x(t) = [x_1^T(t), x_2^T(t), \dots, x_M^T(t)]^T \in \mathfrak{R}^n, n = \sum_{i=1}^M n_i \quad (16)$$

$$u(t) = [u_1^T(t), u_2^T(t), \dots, u_M^T(t)]^T \in \mathfrak{R}^m, m = \sum_{i=1}^M m_i \quad (17)$$

$$d(t) = [d_1^T(t), d_2^T(t), \dots, d_M^T(t)]^T \in \mathfrak{R}^s, s = \sum_{i=1}^M s_i \quad (18)$$

The following constraints are imposed on the subsystems:

$$u_{\min, i} \leq u_i(t) \leq u_{\max, i}, x_{\min, i} \leq x_i(t) \leq x_{\max, i}, i = 1, 2, \dots, M \quad (19)$$

The following assumptions are made:

A1. Reference trajectories $X^{ref} = [x_t^{ref^T}, x_{t+1}^{ref^T}, \dots, x_{t+T}^{ref^T}]^T$ and $U^{ref} = [u_t^{ref^T}, u_{t+1}^{ref^T}, \dots, u_{t+T}^{ref^T}]^T$ of the overall state and control input for a specified finite horizon T are generated by a supervisory optimization algorithm and supposed to be admissible with respect to the state and input constraints.

A2. A forecast $D = [d_t^T, d_{t+1}^T, \dots, d_{t+T}^T]^T$ of the overall disturbance input realization is available for the finite horizon T .

It is supposed that a full measurement $x = [x_1^T, x_2^T, \dots, x_M^T]^T$ of the overall state is available at the current time t . The optimal reference tracking problem is considered where the goal is to have the overall state $x(t)$ and control input $u(t)$ following the reference signals $x^{ref}(t)$ and $u^{ref}(t)$ by minimizing certain performance index. For the current $x = [x_1^T, x_2^T, \dots, x_M^T]^T$, the reference tracking MPC solves the optimization problem:

Problem P1 (Centralized MPC):

$$V^{opt}(x) = \min_U J(U, x) \quad (20)$$

subject to $x_{t|t} = x$ and:

$$x_{i,t+k|t} \in X_i, i = 1, \dots, M, k = 1, \dots, N \quad (21)$$

$$u_{i,t+k} \in U_i, i = 1, \dots, M, k = 0, 1, \dots, N-1 \quad (22)$$

$$x_{i,t+k+1|t} = A_i x_{i,t+k|t} + \sum_{j=1}^M B_{ij} u_{j,t+k|t} + E_i d_{i,t+k|t} \quad (23)$$

$$i = 1, \dots, M, k = 0, 1, \dots, N-1$$

$$x_{t+k|t} = [x_{1,t+k|t}^T, x_{2,t+k|t}^T, \dots, x_{M,t+k|t}^T]^T, k = 1, \dots, N \quad (24)$$

$$u_{t+k} = [u_{1,t+k}^T, u_{2,t+k}^T, \dots, u_{M,t+k}^T]^T, k = 0, 1, \dots, N-1 \quad (25)$$

with $U = [u_t^T, u_{t+1}^T, \dots, u_{t+N-1}^T]^T$ and the cost function given by:

$$J(U, x) = \sum_{k=0}^{N-1} \sum_{i=1}^M l_i(x_{i,t+k+1|t}, u_{i,t+k}) \quad (26)$$

Here:

$$l_i(x_{i,t+k+1|t}, u_{i,t+k}) = \|x_{i,t+k+1|t} - x_{i,t+k+1|t}^{ref}\|_{Q_i}^2 + \|u_{i,t+k} - u_{i,t+k}^{ref}\|_{R_i}^2 \quad (27)$$

is the stage cost for the i -th subsystem with corresponding weighting matrices $Q_i, R_i \succ 0$, and N is a finite horizon. It is supposed that $N < T$ (T being the specified finite horizon of the reference trajectories and the disturbance forecast, cf. assumptions A1 and A2). The sets X_i and U_i are defined by:

$$X_i = \{\lambda_i \in \mathfrak{R}^{n_i} \mid x_{\min, i} \leq \lambda_i \leq x_{\max, i}\} \quad (28)$$

$$U_i = \{\eta_i \in \mathfrak{R}^{m_i} \mid u_{\min, i} \leq \eta_i \leq u_{\max, i}\} \quad (29)$$

and they represent convex (polyhedral) sets.

Problem P1 can be decomposed by using the dynamic dual decomposition approach [7], [9]. The following *decoupled* state equations can be formulated:

$$x_i(t+1) = A_i x_i(t) + B_{ii} u_i(t) + v_i(t) + E_i d_i(t), i = 1, \dots, M \quad (30)$$

with the additional constraints that:

$$v_i(t) = \sum_{j=1, j \neq i}^M B_{ij} u_j(t), \quad i=1, \dots, M \text{ for all } t \quad (31)$$

The variable $v_i \in \mathfrak{R}^{n_i}$ can be interpreted as the influence of the other subsystems in the update of x_i . Then, the constraints (31) are relaxed by introducing the corresponding Lagrange multipliers $p_i \in \mathfrak{R}^{n_i}$ (also referred to as prices) in the cost function (26) and the following *distributed* reference tracking MPC problem is formulated:

Problem P2 (Distributed MPC):

$$V^*(x) = \max_P \sum_{i=1}^M \left(\min_{U_i, X_i, V_i} \sum_{k=0}^{N-1} [U_i^T(x_{i,t+k+1|t}, u_{i,t+k}) + p_{i,t+k}^T v_{i,t+k} - u_{i,t+k}^T \sum_{j=1, j \neq i}^M B_{ji}^T p_{j,t+k}] \right) \quad (32)$$

$l_i^P(x_{i,t+k+1|t}, u_{i,t+k}, v_{i,t+k}, P)$

subject to $x_{i|t} = x$, constraints (21), (22) and:

$$x_{i,t+k+1|t} = A_i x_{i,t+k|t} + B_{ii} u_{i,t+k|t} + v_{i,t+k|t} + E_i d_{i,t+k|t} \quad (33)$$

$$i = 1, \dots, M, \quad k = 0, 1, \dots, N-1$$

$$p_{t+N-1} = 0 \quad (34)$$

Here:

$$P = [p_t^T, p_{t+1}^T, \dots, p_{t+N-1}^T]^T$$

with $p_{t+k} = [p_{1,t+k}^T, p_{2,t+k}^T, \dots, p_{M,t+k}^T]^T, k = 0, 1, \dots, N-1$

$$U = [u_t^T, u_{t+1}^T, \dots, u_{t+N-1}^T]^T, U_i = [u_{i,t}^T, u_{i,t+1}^T, \dots, u_{i,t+N-1}^T]^T \quad (35)$$

$$X = [x_{t+1|t}^T, \dots, x_{t+N|t}^T]^T, X_i = [x_{i,t+1|t}^T, \dots, x_{i,t+N|t}^T]^T$$

$$V = [v_t^T, v_{t+1}^T, \dots, v_{t+N-1}^T]^T, V_i = [v_{i,t}^T, v_{i,t+1}^T, \dots, v_{i,t+N-1}^T]^T$$

with $v_{t+k} = [v_{1,t+k}^T, v_{2,t+k}^T, \dots, v_{M,t+k}^T]^T, k = 0, 1, \dots, N-1$

Since the problem P1 is convex and the Slater's condition holds for the inequality constraints (21)-(22), then $V^{\text{opt}}(x) = V^*(x)$ (there will be no duality gap between the primal and the dual problem [14]).

The inner *decoupled* optimization problems in problem P2 are Quadratic Programming (QP) sub-problems, since the constraints (33) are linear and the stage cost function $l_i^P(x_{i,t+k+1|t}, u_{i,t+k}, v_{i,t+k}, P)$ is quadratic:

Problem P3ⁱ (i-th QP sub-problem):

$$V_i^*(P, x_i) = \min_{U_i, X_i, V_i} \sum_{k=0}^{N-1} l_i^P(x_{i,t+k+1|t}, u_{i,t+k}, v_{i,t+k}, P) \quad (36)$$

subject to $x_{i|t} = x_i$ and:

$$x_{i,t+k|t} \in X_i, \quad k = 1, \dots, N \quad (37)$$

$$u_{i,t+k} \in U_i, \quad k = 0, 1, \dots, N-1 \quad (38)$$

$$x_{i,t+k+1|t} = A_i x_{i,t+k|t} + B_{ii} u_{i,t+k|t} + v_{i,t+k|t} + E_i d_{i,t+k|t} \quad (39)$$

$$k = 0, 1, \dots, N-1$$

Denote by $U_i^* = [u_{i,t}^{*T}, u_{i,t+1}^{*T}, \dots, u_{i,t+N-1}^{*T}]^T,$

$X_i^* = [x_{i,t+1|t}^{*T}, \dots, x_{i,t+N|t}^{*T}]^T$ and $V_i^* = [v_{i,t}^{*T}, v_{i,t+1}^{*T}, \dots, v_{i,t+N-1}^{*T}]^T$ the optimal solution of P3ⁱ. It should be noted that the optimal solution to these sub-problems depends on the values of the prices P .

From the formulation of the distributed MPC problem P2 it can be seen that the computation of U_i^*, X_i^* and V_i^* for given prices P is completely decentralized. However, finding the optimal prices requires coordination [7]. According to the duality theory [14], $V_i^*(P, x_i), i = 1, \dots, M$ are concave functions of P . Therefore, the optimal price sequence $P^* = [p_t^{*T}, p_{t+1}^{*T}, \dots, p_{t+N-1}^{*T}]^T$ can be found as the limits of a

gradient iteration [7]. Given a price prediction sequence $P_i^l = [p_{i,t}^{lT}, \dots, p_{i,t+N-1}^{lT}]^T$ for the l -th iteration, the corresponding sequences $U_i^{*l} = [u_{i,t}^{*lT}, \dots, u_{i,t+N-1}^{*lT}]^T,$ $X_i^{*l} = [x_{i,t+1|t}^{*lT}, \dots, x_{i,t+N|t}^{*lT}]^T$ and $V_i^{*l} = [v_{i,t}^{*lT}, \dots, v_{i,t+N-1}^{*lT}]^T$ are computed locally by solving problem P3ⁱ. Then, the prices can be updated *distributedly* by a gradient step:

$$p_{i,t+k}^{l+1} = p_{i,t+k}^l + \gamma_i^l (v_{i,t+k}^{*l} - \sum_{j=1, j \neq i}^M B_{ij} u_{j,t+k}^{*l}), \quad k = 0, 1, \dots, N-2 \quad (40)$$

$$\text{with } p_{i,t+N-1}^{l+1} = p_{i,t+N-1}^l = 0$$

In [7], a suboptimal algorithm for on-line computation of distributed MPC for linear interconnected systems (coupled through their states) has been suggested, where the prices are updated iteratively by a gradient step (similar to (40)). Also, a stopping criterion for the iterative updates is proposed in [7] that can be locally verified by each subsystem and that guarantees closed-loop suboptimality above a pre-specified level and asymptotic stability of the overall system. The algorithm in [7] is adapted to the case considered here, where the linear systems are coupled through their inputs.

SIMULATION RESULTS FOR THE DISTRIBUTED MPC OF THE HYDRO-POWER VALLEY

The distributed MPC approach is applied to the hydro-power valley, described above. The sampling time in the model (2)-(6) is $T_s = 5$ [min]. The prediction horizon in the MPC problem is $N = 6$ and the weighting matrices in the stage cost (27) are $Q_i = 100, i = 1, 2, 3, R_1 = \text{diag}(1, 10), R_2 = R_3 = 1$. The distributed MPC approach, described in the previous section is used to generate the four control inputs (turbine flows) for an initial state $x(t) = [726.4, 283.85, 264.2]^T$ [m] and a disturbance $d_2(t)$ in the river flow, shown in Fig. 2 (it is assumed that $d_1(t) \equiv 0$). The obtained suboptimal trajectories of the four turbine flows and the water levels in the three tanks are depicted in Fig. 3 to Fig. 9 along with the reference trajectories generated by a supervisory optimization algorithm.

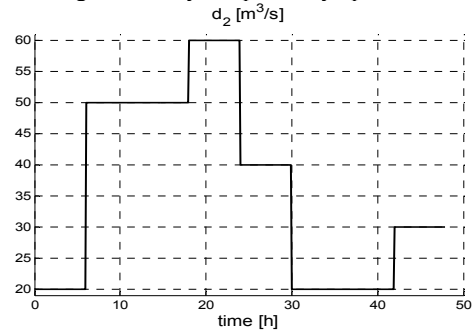


Fig. 2. The disturbance d_2 .

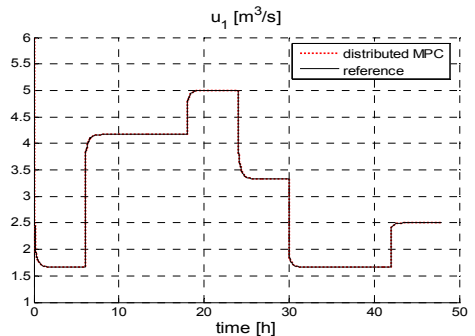


Fig. 3. The control input u_1 .

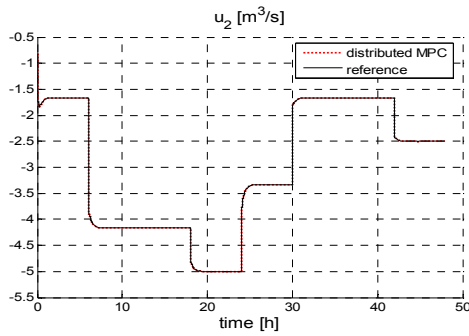


Fig. 4. The control input u_2 .

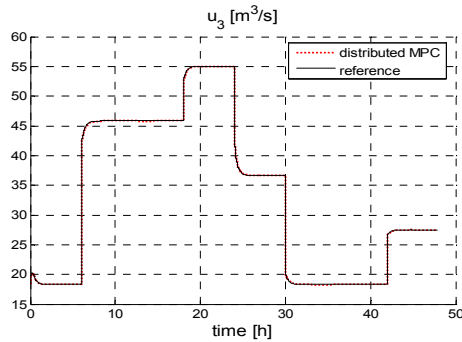


Fig. 5. The control input u_3 .

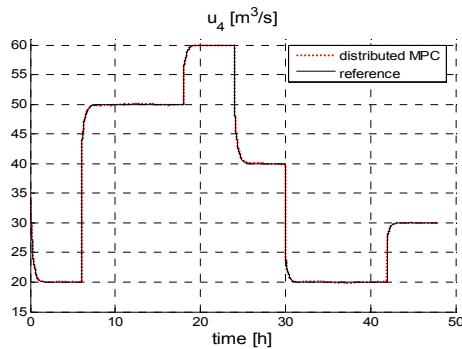


Fig. 6. The control input u_4 .

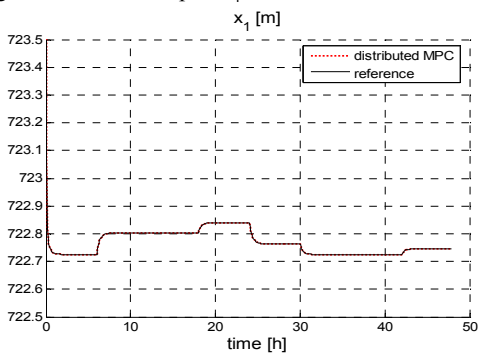


Fig. 7. The water level x_1 .

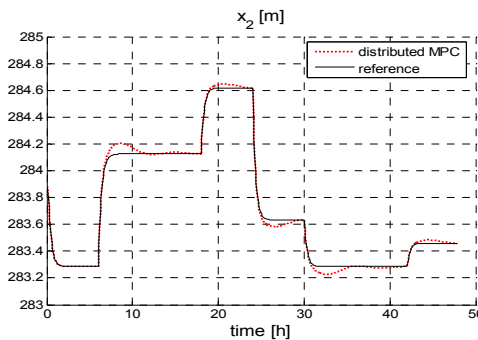


Fig. 8. The water level x_2 .

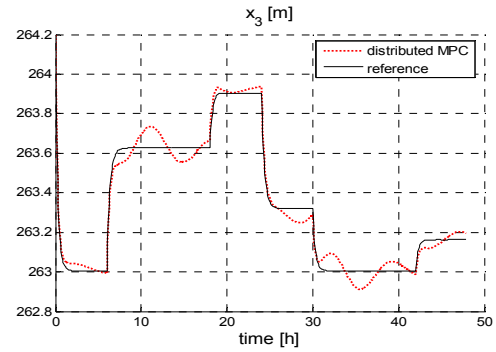


Fig. 9. The water level x_3 .

It can be seen that the closed-loop trajectories obtained with the suboptimal distributed MPC approach deviate slightly from the reference ones. The accuracy of the tracking of water level x_3 can be improved by increasing the requirement for the level of suboptimality of the distributed MPC solution.

REFERENCES

1. Constantinides G. A. Parallel architectures for model predictive control. Proceedings of European Control Conference, Budapest, Hungary, pp. 138-143, 2009.
2. Scattolini R. Architectures for distributed and hierarchical Model Predictive Control - A review. Journal of Process Control, vol. 19, pp. 723-731, 2009.
3. Christofides P. D., Scattolini, R., Muñoz de la Peña, D., Liu, J. Distributed model predictive control: A tutorial review and future research directions. Computers and Chemical Engineering, vol. 51, pp. 21-41, 2013.
4. Maestre J. M., Negenborn R. R. Distributed model predictive control made easy. Series: Intelligent Systems, Control and Automation: Science and Engineering, vol.69, Springer-Verlag, 2014.
5. Venkat, A. N., Hiskens I. A., Rawlings J. B., Wright S. J. Distributed MPC strategies with application to power system automatic generation control. IEEE Transactions on Control Systems Technology, vol. 16, pp. 1192-1206, 2008.
6. Alessio A., Barcelli D., Bemporad A. Decentralized model predictive control of dynamically coupled linear systems. Journal of Process Control, vol. 21, pp. 705-714, 2011.
7. Giselsson P., Rantzer A. Distributed model predictive control with suboptimality and stability guarantees. Proceedings of 49-th IEEE Conference on Decision and Control, Atlanta, GA, 2010, pp. 7272-7277.
8. Giselsson P., Doan M. D., Keviczky T., De Schutter B., Rantzer A. Accelerated gradient methods and dual decomposition in distributed model predictive control. Automatica, vol. 49, pp. 829-833, 2013.
9. Cohen G., Miara B. Optimization with an auxiliary constraint and decomposition. SIAM Journal on Control and Optimization, vol. 28, pp. 137-157, 1990.
10. Raimondo D. M., Magni L., Scattolini R. Decentralized MPC of nonlinear systems: An input-to-state stability approach. International Journal of Robust and Nonlinear Control, vol. 17, pp. 1651-1667, 2007.
11. Heidarnejad M., Liu J., Muñoz de la Peña D., Davis J. F., Christofides P. D. Multirate Lyapunov-based distributed model predictive control of nonlinear uncertain systems. Journal of Process Control, vol. 21, pp. 1231-1242, 2011.
12. Grancharova A., Johansen T. A. Distributed quasi-nonlinear model predictive control by dual decomposition. Proceedings of the 18-th IFAC World Congress, Milano, 2011, pp. 1429-1434.
13. Zárate Flórez J. Control studies by decomposition-coordination for the optimization of the operation of hydro-power valleys. Ph.D. Thesis, University of Grenoble, France, 2012.
14. Boyd S., Vandenberghe L. Convex optimization, Cambridge University Press, 2004.