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# Auctioning divisible goods: does every bidder need to win to achieve an equilibrium price ?

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Does every bidder need to win to achieve an equilibrium price?"

Titre en français : "Enchères de biens divisibles:  
Faut-il que tous les enchérisseurs gagnent pour atteindre un prix d'équilibre?"

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Résumé :

Cette note sur les enchères d'un bien divisible soulève une question à propos des perdants dans de telles enchères. Je mets en évidence les limites des modèles théoriques des enchères d'un bien divisible à valeur commune lorsque les joueurs possèdent une information privée sur le prix de revente du bien, c'est-à-dire en présence d'asymétrie d'information. Jusqu'à maintenant, personne n'est parvenu à expliciter les stratégies d'équilibre dans un cadre d'analyse général. Le problème est particulièrement complexe, car il faut optimiser la stratégie du joueur en tenant compte, à la fois, de tous les points de la courbe de demande, et de l'information qu'il peut inférer sur les signaux de ses concurrents à partir de la condition d'équilibre. Les stratégies d'enchère peuvent être n'importe quelle fonction décroissante, ainsi les exemples proposés dans la littérature spécifient les distributions des variables aléatoires et limitent l'espace des stratégies des enchérisseurs. Wang et Zender (2002) notamment restreignent, *ex ante*, les stratégies d'enchères des joueurs à une certaine classe de fonctions de demande linéaires. Cependant, je montre que ces auteurs n'envisagent pas tous les cas possibles. La faille réside dans l'écriture même de la contrainte de marché qui les amène à une mauvaise détermination du prix limite d'équilibre. En effet, selon leur résolution du jeu, les auteurs supposent implicitement que tous les enchérisseurs gagnent une quantité positive à l'équilibre. Ainsi, le prix limite d'équilibre obtenu n'est correct que si l'on suppose *ex ante* que tous les enchérisseurs gagnent une fraction non nulle du bien. Or, cette hypothèse est particulièrement gênante. Dans la pratique, il est fréquent que des enchérisseurs n'obtiennent rien du tout dans une enchère de bien divisible, notamment dans les enchères du Trésor. Je propose ici un contre exemple numérique et des simulations pour illustrer ce problème.

Champs d'expertise : théorie des enchères, enchères des valeurs du Trésor, enchères de bois

# Auctioning divisible goods: Does every bidder need to win to achieve an equilibrium price?★

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## Abstract

This note on divisible good auctions raises a question about the losers in such share auctions. Until now, no one has put explicit equilibrium bidding strategies in a general framework. In fact bidding strategies may be any weakly downward sloping bid schedules, so examples specifying distributions of the random variables and limiting the bidding strategy space are presented. I claim that the equilibrium stop-out price derived in the case of linear bidding strategies, the example developed in Wang and Zender (2002), is correct only if it is assumed ex-ante that every bidder wins a positive fraction of the good. But that assumption is particularly dubious. In practice, it is unusual for there to be no losers in a divisible good auction; especially in a Treasury auction.

**Keywords:** Divisible good auctions, Treasury auctions, Equilibrium bidding.

**JEL Classification:** D44

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# 1 Introduction

Despite the widespread use of divisible good auctions, the understanding of that mechanism is far from complete. We still do not know explicitly what are the optimal bidding strategies and thus, the equilibrium result, of such a game in a general framework.

In a divisible good auction, the seller puts on sale a fixed amount of a good that is assumed to be perfectly divisible and the bidders submit as many price-quantity pairs as they want. Because of the quantity dimension combined with the price dimension, the strategy space is much larger than in unit auctions: bidders may submit any weakly downward sloping bid schedule. The stop-out price in a divisible good auction is the marginal equilibrium price that equalizes the aggregate demand with the available supply. All the bids at or above the stop-out price are winning bids. The payment rule depends on the auction format. The two most used in practice are the uniform-price auction in which every winning bidder pays the stop-out price, and the discriminatory auction in which every winning bid is paid at the price submitted.

The best known example of divisible good auctions is Treasury auctions. The divisible good auction mechanism is used in many countries to issue new government securities. Treasury auctions are generally modeled as common value auctions since it is often assumed that the real value of the issued security is its future resale price on the market. That value is the same for all the bidders but is imperfectly known at the time of the auction. Although there is a large literature on Treasury auctions, the analysis of these auctions presents quite a challenge, not only because Treasury auctions are divisible good auctions, but also because the Treasury auction environment is quite complex. Treasury auctions take place regularly among the same set of bidders (the primary dealers); moreover there is a when-issued market and a secondary market which have complex interactions with the auctions. Besides, some institutional features (such as the noncompetitive bids which are guaranteed to be filled, or the French practice of issuing securities of different maturities simultaneously in two distinct

auctions, etc.) may have significant impacts on the auction result.

The important debate concerning the choice of the best auction format for maximizing the Treasury's expected revenue leads to many theoretical, empirical and experimental works on Treasury auctions (see for example: Daripa, 2001; Das and Sundaram, 1996; Goswami, Noe and Rebello, 1996; Nyborg, Rydqvist and Sundaresan, 2002; etc.) Very few articles analyze deeply the multi unit demand aspect, and yet it is the most important feature of Treasury auctions. From a theoretical point of view, it is fundamental to be able first to solve the game of a stand alone auction in which a fixed and commonly known number of symmetric risk-neutral bidders participate.

Since Wilson's seminal model on share auctions in 1979, there have been only two major contributions on the theoretical issue of divisible good auctions: Back and Zender (1993) who compare discriminatory and uniform-price auctions and, more recently, Wang and Zender (2002) who characterize the equilibrium strategies in a general framework.

A first step is to consider that every bidder has the same information. In this symmetric bidder information case, all the bidders receive the same signal as to the real value of the auctioned good. Since it is often assumed that all the bidders are symmetric ex-ante, it is reasonable to assume in Treasury auctions they get the same information from the market. Nevertheless, this symmetric bidder information case is limiting and frustrating since one of the advantages of an auction procedure is to select the bidders who have the greatest willingness to pay. Unfortunately, introducing private signals in such a game makes it much more difficult to solve. Bidders must simultaneously take into account in their bids an informational inference problem combined with a strategic component of bidding.

The most general results produce an equilibrium condition (see Wilson, 1979 and Wang and Zender, 2002), but it seems very difficult to go beyond that stage to explicit equilibrium bidding strategies without imposing some restrictions on the space of the bidding strategies. The difficulty is to determine the equilibrium stop-out price from the clearing market condition when the form of the bidding strategies is unknown. Besides, even when the form of bidding strategies is imposed, it is not always easy to specify the market clearing condition. Indeed, I show in this note that even with linear bidding strategies such as the one used in the example given in Wang and Zender (2002) the

resolution of the game is not convincing.

More precisely, my point is to question an issue that is somewhat obscured in the divisible good auction literature: the fact that it is implicitly assumed that every bidder wins a positive fraction of the good at the equilibrium stop-out price.

In the next section I recapitulate the general framework of Wang and Zender's divisible good auction model, and I present the characteristics of the particular case they propose to study. In section 3, I highlight the case of losers, and show with a numerical counter example and simulations that the way Wang and Zender express the clearing market condition is not always correct. Section 4 gives some concluding remarks.

## 2 Wang and Zender's example

Given the difficulties of obtaining an explicit solution to the general problem, Wang and Zender (2002) propose to gain insights into the informational properties of divisible good auction with a parametric example. So, I first briefly recapitulate the general model before presenting the particular assumptions, and then the result, of the example proposed by Wang and Zender.

### 2.1 The general model

Most of the Treasury auction models in the literature are based on usual assumptions of a common value auction. There are  $N$  symmetric competitive bidders. To keep the framework as simple as possible and since the risk aversion is not the most important feature in Treasury auctions, the agents (the seller and all the potential buyers) are assumed to be risk neutral.<sup>1</sup>

The objective of the seller and the bidders is to maximize their expected revenue.

The intrinsic value  $v$  of the good is the realization of a random value  $\tilde{v}$  which is

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<sup>1</sup> Wang and Zender (2002) consider also the case of risk averse bidders.

the market resale price. So  $v$  is the same for all the bidders but it is unknown at the time of the auction. It is assumed that the prior distribution is common knowledge.

Each bidder  $i$  receives a private signal  $s_i \in S$  which is the realization of a random variable  $\tilde{s}_i$ . Bidder  $i$ 's strategy is a schedule  $x_i(p, s_i)$  that gives the quantity (or the fraction) of the good demanded at different price levels,  $p$ , given the received signal  $s_i$ . Bid schedules are assumed to be piece-wise continuously differentiable with respect to price. Furthermore, only bid schedules that are weakly decreasing in price are considered. Let us write as  $x_A(p, s) = \sum_{i=1}^N x_i(p, s_i)$  the aggregate demand of all the bidders.

In Wang and Zender (2002), the quantity put on sale by the Treasury is normalized to one, but they assume there is a random noncompetitive demand  $\tilde{z} \in [0,1]$ , so competitive bidders compete for an uncertain quantity,  $\tilde{Q} = 1 - \tilde{z}$ .

Given that in France the practice of noncompetitive bidding is quite different<sup>2</sup> and that it is not appropriate to assume that the noncompetitive bids are exogenous, I prefer to justify a random quantity for the competitive bidders by the fact that the Treasury does not announce the exact amount of assets there will be in a particular auction.<sup>3</sup> It is shown later that it is important to keep some uncertainty as to the quantity supplied for the competitive bidders in this model. But the different justifications for a random quantity do not have any qualitative impact on what I want to show; the results are the same.

The equilibrium stop-out price is:

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<sup>2</sup> First, only primary dealers are allowed to submit a predetermined amount of noncompetitive bids. Before each auction an *attribution coefficient* is calculated for each primary dealer according to his competitive participation at the three last auctions. The total quantity a dealer can demand on a noncompetitive basis depends on his attribution coefficient. Second, there are two kinds of noncompetitive bid: one that is submitted at the same time as competitive bids, and one that primary dealers can submit after the auction result.

<sup>3</sup> I assume it is common knowledge that the Treasury does not have any strategy when fixing the quantity  $Q$  ex post.

$$p(s) = \begin{cases} \max\{p | x_A(p, s) \geq Q; p \geq 0\} & \text{if } \{p | x_A(p, s) \geq Q; p \geq 0\} \neq \emptyset \\ 0 & \text{if } \{p | x_A(p, s) \geq Q; p \geq 0\} = \emptyset \end{cases}$$

In the uniform-price auction with a stop-out price  $p$ , bidder  $i$ 's actual profit is:

$$\tilde{\pi}_i(p, s_i) = (\tilde{v} - p)x_i(p, s_i)$$

Only symmetric, pure strategy Bayesian-Nash equilibria are considered.

## 2.2 Specifications of Wang and Zender's example

Given the complex inference problem of solving the model in a general framework when bidders receive private information on the good, Wang and Zender (2002) characterize two aspects of their model in section 4.4: the distributions of random values and the form of the bidding strategies.

On the one hand, it is usual to characterize distributions of random values, but on the other hand, it is more critical to impose a specific form to the bidding strategies. In fact the form of the equilibrium strategies should result from the game resolution without imposing ex-ante any restriction on bidding strategies. But, without specifying the form of the bid schedule  $x_i(p, s_i)$ , the strategy space is large and we cannot specify the clearing market condition that makes it possible to determine the equilibrium stop-out price. So it is natural to develop a special case when it seems particularly difficult to solve the general framework. Consequently, Wang and Zender (2002) limit the possibilities given to the bidders to a certain type of linear strategy:

$$x_i(p, s_i) = \mu_i + \beta_i s_i - \gamma_i p, \quad (1)$$

with  $\mu_i$ ,  $\beta_i$  and  $\gamma_i$  positive parameters and  $i = 1, 2, \dots, N$ .

So as to compute the equilibrium strategies, Wang and Zender (2002) also assume that the random values are normally distributed, which makes it possible to express density functions.

The bidders' private signals take the form:  $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$ , where  $\tilde{v}_i \sim \mathbf{N}(\bar{v}, \tau_v^{-1})$  and  $\tilde{\varepsilon}_i$  are identically distributed:  $\tilde{\varepsilon}_i \sim \mathbf{N}(0, \tau_e^{-1})$ ,  $i = 1, 2, \dots, N$ . Thus the prior distribution of



the signals is:  $\tilde{s}_i \sim N(\bar{v}, \tau_v^{-1} + \tau_\varepsilon^{-1})$  and their distribution conditional on the realization  $\tilde{v} = v$  is also normally distributed but with the mean  $v$  and a standard deviation of  $\tau_\varepsilon^{-1}$ . In addition, Wang and Zender (2002) assume the noncompetitive demand distribution is  $\tilde{z} \sim N(0, \tau_z^{-1})$ . So I assume directly that  $\tilde{Q} \sim N(\bar{Q}, \sigma_Q^{-1})$ .

The assumption of uncertain supply is important here, because without it there would be no equilibrium with linear demand such as the one defined in (1). Indeed, what is important to bidders is the information they can infer from the market clearing condition. But, without any noise as to the supplied quantity (or in the absence of noncompetitive bids) for each realized stop-out price every bidder infers the sum of the signals received by all the bidders. However, with normally distributed random values,

$$E \left[ \tilde{v} \left| s_i, \frac{\sum_{j \neq i} s_j}{N-1} \right. \right] = E \left[ \tilde{v} \left| \sum_{i=1}^N s_i \right. \right]$$

and one can show that there is no equilibrium bidding strategy of the type defined by (1). The intuition of that result is that the relevant information contained in the signals is entirely revealed by the equilibrium stop-out price. Consequently, all the bidders have the same information which leads them to exactly the same conditional expected resale price. But then, they can not submit the same demand unless  $\beta = 0$ . Hence, the form defined by (1) is not appropriate when there is no supply uncertainty.

In the example of Wang and Zender, all bidders have CARA utility with the risk aversion coefficient  $\rho$ . However, that feature is not essential to what follows, and since the risk neutral assumption is more convenient, I transpose the example to the risk neutral case:  $\rho = 0$ .

### 2.3 Wang and Zender's solution of their example

In their proposition 4.5, Wang and Zender (2002) claim that : *In a uniform-price auction with  $N$  bidders there exists a unique symmetric equilibrium in linear strategies, and they characterize this equilibrium. In the case of risk neutral bidders, this equilibrium corresponds to:*

$$\tilde{x}_i = \mu + \beta \tilde{s}_i - \gamma \tilde{p}, \quad i = 1, 2, \dots, N,$$

$$\text{where } \mu = \frac{2\bar{v}}{N\sigma_v^{-1}} \sqrt{\frac{N-2}{N(N-1)} \sigma_\varepsilon^{-1} \sigma_Q^{-1}} + \frac{N-2}{N(N-1)} \bar{Q}$$

$$\beta = \sqrt{\frac{(N-2)\sigma_Q^{-1}}{N(N-1)\sigma_\varepsilon^{-1}}}$$

$$\gamma = \sqrt{\frac{(N-2)\sigma_Q^{-1}}{N(N-1)\sigma_\varepsilon^{-1}}} \left( 1 + \frac{2\sigma_\varepsilon^{-1}}{N\sigma_v^{-1}} \right)$$

Next, in corollary 4.3 they show that *for any realized stop-out price, the competitive bidder who values the good the most receives the largest share*. So it is natural to think that the competitive bidder who gets the worst signal receives the smallest share. Thus a question arises: is it possible for a bidder to receive such a low signal that he prefers to obtain nothing at the stop-out price?

### 3 What about the losers?

It is easy to see that Wang and Zender (2002) do not consider the case of losers, *i.e.*, bidders who do not want any quantity at the stop-out price. They assume that every bidder wins a positive fraction of the good at the equilibrium stop-out price. This can be seen from their market clearing condition. Next, I present a numerical example in which Wang and Zender's theoretical stop-out price fails because a bidder receives a relatively low signal compared to the other bidders. I then show by simulations that this case is not necessarily rare.

#### 3.1 The market clearing condition

In the first step of their proof (page 700), Wang and Zender (2002) express the market clearing condition as if every bidder has a positive demand at the stop-out price:

$$\tilde{Q} = (N-1)\mu + \beta \sum_{j \neq i} (\tilde{v} + \tilde{\varepsilon}_j) - (N-1)\tilde{\gamma}\tilde{p} + x_i$$

In fact, if  $\tilde{p}$  is high and  $\tilde{\varepsilon}_j$  is sufficiently negative,  $(\mu + \beta\tilde{s}_j - \tilde{\gamma}\tilde{p})$  may be negative. Since in a Treasury auction it is not permitted to sell to the Treasury, bids cannot be negative, therefore  $x_j(\tilde{p}, \tilde{s}_j) = 0$  and thus the market clearing condition is incorrect.

It is important to take into account the fact that:

$$x_i(\tilde{p}, \tilde{s}_i) = \begin{cases} \mu_i + \beta_i \tilde{s}_i - \gamma_i \tilde{p} & \text{if } \tilde{p} \leq \frac{\mu_i + \beta_i \tilde{s}_i}{\gamma_i} \\ 0 & \text{if } \tilde{p} > \frac{\mu_i + \beta_i \tilde{s}_i}{\gamma_i} \end{cases}$$

### 3.2 A numerical counter example

The following numerical counter example shows that the market clearing condition does not always correspond to the expression given by Wang and Zender (2002). That means that the theoretical stop-out price is not the one obtained.

Let us give a numerical value to all the parameters. These parameters are roughly calibrated so as to fit the French Treasury auction case:

$$\begin{array}{lll} N = 20 & \bar{v} = 100 & \bar{Q} = 10 \\ \sigma_v^{-1} = 1 & \sigma_\varepsilon^{-1} = 1 & \sigma_Q^{-1} = 1 \end{array}$$

From the preceding results, we obtain:

$$\mu = 2.65011296, \beta = 0.21764288 \text{ and } \gamma = 0.23940716$$

The individual bidder's demand quantity, depending on the price and the received signal, is:

$$x(p, s_i) = 2.65011296 + 0.21764288 s_i - 0.23940716 p.$$

For simplicity, let us assume that the exact quantity for competitive bidders is  $Q = \bar{Q} = 10$ , and that 19 bidders ( $j$ ) receive the signal 100 and that only one bidder ( $i$ ) receives the signal 97.5. From the preceding market clearing condition we should get:

$$p = \frac{N\mu + \beta \left( \sum_{j \neq i} s_j + s_i \right) - Q}{N\gamma} = \mathbf{99.776443}$$

But at that price bidder  $i$ 's demand is negative, which is forbidden.

$$x(p; s_i = 97.5) = \begin{cases} 23.8702933 - 0.2394072p & \text{if } p < 99.7058443 \\ 0 & \text{if not} \end{cases}$$

Similarly, the strategy of the 19 other bidders  $j$  is:

$$x(p; s_j = 100) = \begin{cases} 24.4144005 - 0.2394072p & \text{if } p < 101.978572 \\ 0 & \text{if not} \end{cases}$$

The aggregate demand received by the Treasury is thus:

$$x_A(p) = \begin{cases} 19x(p; s_j = 100) + x(p; s_i = 97.5) & \text{if } p < 99.7058443 \\ 19x(p; s_j = 100) & \text{if } 99.7058443 \leq p < 101.978572 \\ 0 & \text{if } p \geq 101.978572 \end{cases}$$

In this way, the real<sup>4</sup> stop-out price is **99.7801587**, since at that price, the demand of the 19 bidders who receive the signal 100 reaches the quantity supplied by the Treasury:  $19 \times x(p = 99.7801587; s_j = 100) = 19 \times 0.52631579 = 10$ .

Furthermore, although this counter example is just a particular case, it is not improbable. In fact, the probability that the bidder who receives the lowest signal out of 20 gets a signal under 97.5 is greater than 11%.

$$\begin{aligned} \text{Prob} \left[ \min_{i=1, \dots, 20} s_i \leq 97.5 \right] &= \text{Prob} \left[ \min_{i=1, \dots, 20} \varepsilon_i \leq -2.5 \right] \\ &= 1 - (\text{Prob}[\varepsilon_i > -2.5])^{20} = 1 - 0.9938^{20} = 0.11696 \end{aligned}$$

The assumption that the 19 other bidders all receive a signal of 100 is made only to simplify the calculus. Nevertheless, to relax this constraint, I propose to make simulations to estimate the frequency of this problem of negative demand.

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<sup>4</sup> By real stop-out price I mean here the price that results from the auction if the bidders follow (2). This does not mean it is the correct equilibrium stop-out price.

### 3.3 Simulations

Given the difficulty of showing the extent of the problem analytically, I evaluate it by simulation.

The market constraint is incorrect ex-post only when, in a given auction, one or more bidders demand a negative theoretical quantity at the stop-out price defined as if there were no losers. This equilibrium quantity is:

$$x(p, s_i) = \mu + \beta s_i - \mathcal{P} = \frac{Q}{N} + \frac{1}{N} \sqrt{\frac{(N-2)\sigma_Q^{-1}}{N(N-1)\sigma_\varepsilon^{-1}}} \left( (N-1)s_i - \sum_{j \neq i} s_j \right).$$

In this expression, three parameters,  $N$ ,  $\sigma_\varepsilon^{-1}$  and  $\sigma_Q^{-1}$ , directly affect the frequency of the problem. To take into account the fact that the value of these parameters may change the probability of the problem occurring, I simulate one million auctions with random values for those three parameters.

After few tries, I set  $\bar{v}$ ,  $\bar{Q}$  and  $\sigma_v^2$  to 100, 1 and 1 respectively. The number of bidders  $N$  is drawn from a uniform distribution on [5, 35] and the standard deviations  $\sigma_\varepsilon^{-1}$  and  $\sigma_Q^{-1}$  are drawn from a gamma distribution of parameter 1 (for  $\sigma_Q^{-1}$  the value is divided by 100). I choose a gamma distribution because it gives only positive values.

In accordance with the theoretical model, the value  $v$  of the good is drawn from a normal distribution  $N(\bar{v} = 100, \tau_v^{-1} = 1)$ . Next, bidders' signals are drawn from a normal distribution  $N(v, \tau_\varepsilon^{-1})$ . Then I compute the equilibrium theoretical price and check if at that price every bidder demands a positive quantity. As soon as there is one bidder who demands a negative quantity, I consider there is a problem in the auction. The statistic I am interested in is the ratio between the number of auctions in which there is a problem and the total number of simulated auctions.

Over one million simulated auctions, there are 186,109 auctions in which there is a problem, or 18.6%. Table 1 summarizes the values of the parameters. Variable  $k$  corresponds to the number of negative demands at the stop-out price in an auction.

**Table 1**

Variables	Mean	Std dev.	Minimum	Maximum
$k$	0.495871	1.370632	0	19
$N$	20.01197	8.940794	5	35
$\tau_\varepsilon^{-1}$	1.001069	0.9984041	9.57e-07	12.98552
$\tau_Q^{-1}$	0.0100056	0.0100039	1.11e-08	0.1729886

First note that the number of negative demands at the stop-out price can be relatively high in an auction. For example,  $k = 10$  in 1069 auctions. If we take into consideration the fact that to get  $k = 10$  at least 11 participants are needed, this value is not negligible.

**Table 2**

	small	medium	large
$N$	$5 < N < 15$ 4.56% (322,209)	$15 \leq N \leq 25$ 18.99% (355,070)	$25 < N < 35$ 32.13% (322,721)
$\tau_\varepsilon^{-1}$	$\tau_\varepsilon^{-1} < 0.5$ 2.79% (391,974)	$0.5 \leq \tau_\varepsilon^{-1} \leq 1.5$ 20.20% (384,395)	$25 < \tau_\varepsilon^{-1} < 35$ 43.62% (223,631)
$\tau_Q^{-1}$	$\tau_Q^{-1} < 0.005$ 2.78% (393,193)	$0.005 \leq \tau_Q^{-1} \leq 0.015$ 20.15% (383,457)	$0.015 < \tau_Q^{-1}$ 43.84% (223,350)

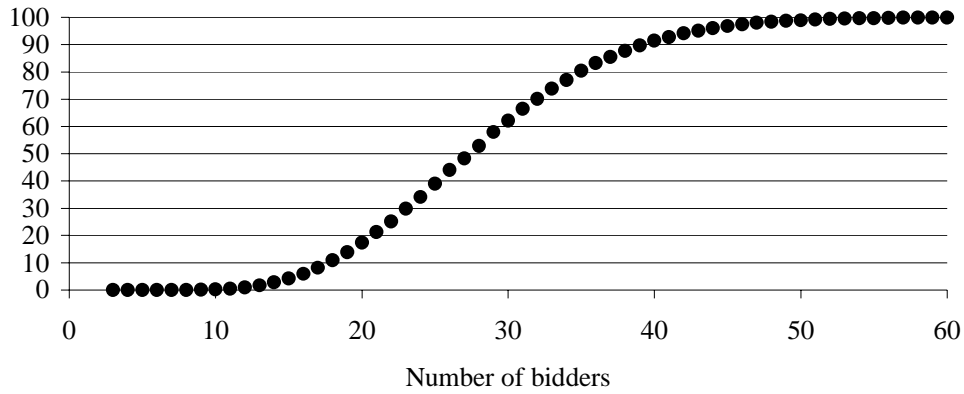
Table 2 gives the auction percentage in which there is a problem depending on the three parameters values. The number in brackets is the number of auctions for which the percentage has been computed. This table shows that the negative demand problem is increasingly frequent as parameters  $N$ ,  $\tau_\varepsilon^{-1}$  and  $\tau_Q^{-1}$  rise. To specify this fact, I make new simulations.

For each value  $N$ , from 3 to 60, I simulate 100,000 auctions keeping the other parameters set at their reference level, *i.e.*:  $\bar{v} = 100$ ,  $\bar{Q} = 1$ ,  $\sigma_v^{-1} = 1$ ,  $\sigma_\varepsilon^{-1} = 1$  and  $\sigma_Q^{-1} = 0,01$ . This allows me to estimate the probability of encountering the problem in an auction according to the number of bidders. I repeat the same type of simulation, varying  $\sigma_\varepsilon^{-1}$  between 0.1 and 5.0 with 0.1 increment, then the parameter  $\sigma_Q^{-1}$  between 0.001 and 0.050 with 0.001 increment and the number of bidders fixed at 20.

Graphs 1 to 3 are constructed from these three series of simulations. For each of these parameters, the probability of encountering a problem tends to zero as the value of the parameter decreases, but the probability tends to one as the value increases.

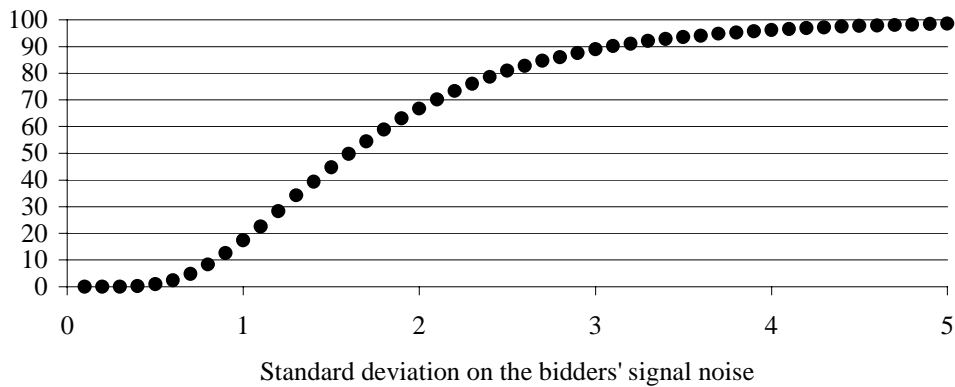
### Graph 1

**Auction percentage in which at least one bidder demands a negative quantity at the stop-out price depending on parameter  $N$**



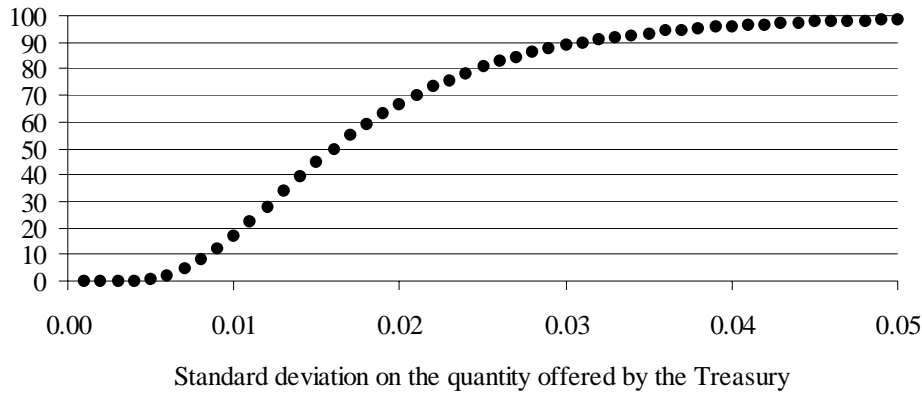
### Graph 2

**Auction percentage in which at least one bidder demands a negative quantity at the stop-out price depending on parameter  $\sigma_\varepsilon^{-1}$**



**Graph 3**

**Auction percentage in which at least one bidder demands a negative quantity at the stop-out price depending on parameter  $\sigma_Q^{-1}$**



## **4 Conclusion**

Divisible good auction models with asymmetrically informed bidders are difficult to handle. Wilson (1979) and Wang and Zender (2002) provide fundamental contributions on this topic, but they only give the first order condition for devising equilibrium bidding strategies. To go beyond that stage, Wilson (1979) and Wang and Zender (2002) are forced to give up the general framework and propose examples. First of all, those examples specify the distribution functions of random variables, but also restrict bidders' strategy space, which is embarrassing and regrettable.

Using the linear bidding strategy example of Wang and Zender, I examine the case of losers in divisible good auctions. The solution given by Wang and Zender (2002) implicitly assumes that every bidder gets a nonnegative fraction of the good at the stop-out price. Through a counter example and simulations I show that the case of losers should be taken into account.

Furthermore, empirically, it is very common for bidders to end up with nothing in divisible good auctions. In Treasury auctions, about one third of the bidders do not win anything. Therefore, it seems important to take this fact into account in theoretical



models.

Unfortunately, it seems to be difficult to study the possibility that some bidders may not win anything, because many cases have to be considered: from the case in which every bidder gets a fraction of the good, to the case in which only one bidder wins all the supplied quantity. Finally, it might be laborious to obtain an equilibrium for the game even when the bidding strategies are linear and separable in price and signal.

Note that in the symmetric information case, it is impossible for some bidders to obtain nothing in symmetric equilibria, because all the bidders submit the same demand and thus win exactly the same fraction of the good. The losers problem is also overcome when the equilibrium bidding quantity turns out to be positive at any equilibrium stop-out price. For instance, this is the case in Wilson's (1979) example 1 page 682.

Nevertheless, it is not persuasive to search for equilibrium strategies such that there are no losers at the equilibrium stop-out price since in practice there are losers. Finally, more convincing theoretical work still needs to be done to devise a more satisfying benchmark model for divisible good auctions.

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