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Demonstration of the rules non-isolability of sets of faulty states

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1 Introduction

This report is an appendix of a research work which will be published soon. A fault detection, isolation and adaptation (FDIA) formalism is introduced and this report demonstrates the two rules of non isolability.

The proposed FDIA framework assumes that one system observation produces two estimates of the same quantity: G and N . These estimates are stored in FDIA memory. K observations of the system are made.

Faults affecting G_i and N_i cause their value to be different from the true one P_i . The index i stands for the i^{th} observation. The state of G_i (resp. N_i) of being faulty or not is denoted by the boolean variable f_{G_i} (resp. f_{N_i}). For instance, $f_{G_i} = 1$ means that a fault affects G_i then $G_i \neq P_i$. At the K^{th} observation, the faulty states of every estimate is summarised by e called the set of faulty states. The purpose of FDIA is to isolate (i.e. determine) e which means to ascertain the faulty state of the estimates G_i and N_i for every observation $i \leq K$.

FDIA is based on the use of the residual vector $R(e)$. The terms of R are the results of boolean operations between the faulty states f_{G_i} and f_{N_i} . Some sets of faulty states produce unique residuals, isolation is then possible. However, some residuals are generated by several sets of faulty states, isolation is not possible.

The aim of this report is to prove the conditions on f_{G_i} and f_{N_i} for e to be not isolable. These are stated by two rules in Proposition 1:

Proposition 1. A set of faulty states is not isolable if and only if, it complies with one of the following rules:

1. $f_{N_i} = 1, \forall i \in \{1, \dots, K\}$ and $\exists! j \in \{1, \dots, K\}$ such as $f_{G_j} = 0$
2. $f_{G_i} = 1, \forall i \in \{1, \dots, K\}$

In other words it is not possible to isolate faults if:

1. Every estimates N is faulty and there is a unique true G .
2. Every G is faulty.

Section 2 introduces the notations required for the demonstration. Section 3 deduces mathematical properties of the proposed formalism. According to these properties, Section 4 reformulates the problem and demonstrate the proposition. Finally Section 5 concludes the demonstration.

2 Notations

A set of faulty states e is defined as $e = (f_{G_i}, f_{N_i})_{1 \leq i \leq K} \in \{0, 1\}^{2K}$ for some $K \geq 1$.

Let E be the set of sets of faulty states e .

$$e \in E \quad (2.1)$$

R is the function that associates a residual to a set of faulty states. $R(e)$ is the residual of e .

$$R = \{r_{G_i G_j}, r_{N_i N_j}, r_{G_p N_q}\}_{1 \leq i, j, p, q \leq K, i \neq j}$$

with

$$r_{G_i G_j} = f_{G_i} \vee f_{G_j}, \forall i, j \in \{1, \dots, K\}, i > j \quad (2.2)$$

$$r_{G_i N_j} = f_{G_i} \vee f_{N_j}, \forall i, j \in \{1, \dots, K\} \quad (2.3)$$

$$r_{N_i N_j} = f_{N_i} \oplus f_{N_j}, \forall i, j \in \{1, \dots, K\}, i > j \quad (2.4)$$

where \vee and \oplus are boolean *or* and *exclusive or* respectively.

σ_K is the set of permutations of $\{1, \dots, K\}$

I stands for the set of isolable sets of faulty states.

$$I \subset E \quad (2.5)$$

I^c is the complement of I (i.e. the set of non-isolable sets of faulty states)

$C_{l,m}$ is the set of sets of faulty states $e = (f_{G_i}, f_{N_i})_{1 \leq i \leq K}$ such as there are l $f_{G_i} = 1$ and m $f_{N_i} = 1$.

Given a set of faulty states $e \in E$ and two permutations $\sigma, \sigma' \in \sigma_K$, the set of faulty states obtained by permuting variables f_{G_i} (with σ) and variables f_{N_i} (with σ') is denoted by

$$(\sigma, \sigma') \cdot e = (f_{G_{\sigma^{-1}i}}, f_{N_{\sigma'^{-1}i}})_{1 \leq i \leq K} \quad (2.6)$$

where σ^{-1} (resp. σ'^{-1}) stands for the inverse permutation of σ (resp. σ').

$e_{l,m}$ stands for canonical form of $e \in C_{l,m}$. $e_{lm} = (f_{G_i}, f_{N_i})_{1 \leq i \leq K}$ with

$$\begin{cases} f_{G_i} = \begin{pmatrix} 1, \dots, 1, 0, \dots, 0 \\ l \qquad \qquad \qquad K-l \end{pmatrix} \\ f_{M_i} = \begin{pmatrix} 1, \dots, 1, 0, \dots, 0 \\ m \qquad \qquad \qquad K-m \end{pmatrix} \end{cases} \quad (2.7)$$

3 Properties

Proposition 2. *The class $C_{l,m}$ contains all the permutations of the canonical set of faulty states $e_{l,m} : C_{l,m} = \{(\sigma, \sigma') \cdot e_{l,m} | \sigma, \sigma' \in \sigma_K\}$*

Proof. The permutation σ (resp. σ') doesn't change the number of f_{G_i} (resp. f_{N_i}) equals to one. Thus, if $e \in C_{l,m}$ and $\sigma, \sigma' \in \sigma_K$ then $(\sigma, \sigma') \cdot e \in C_{l,m}$. Reciprocally, every set of faulty states $e \in C_{l,m}$ can be obtained by permuting the variables f_{G_i} and f_{N_i} and the proposition follows. \square

Remark 1. $E = \bigcup_{1 \leq l, m \leq K} C_{l,m}$

$C_{l,m}$, $1 \leq l, m \leq K$ is a partition of E .

Let denote by \tilde{E} the disjoint union of $C_{l,m}$ where $(l, m) \neq (K-1, K)$ and $(l, m) \neq (K, K)$

$$\tilde{E} = \bigcup_{1 \leq l, m \leq K} C_{l,m} \mid (l, m) \neq (K-1, K) \text{ and } (l, m) \neq (K, K) \quad (3.1)$$

Proposition 3. *If $e \in E$ and $\sigma, \sigma' \in \sigma_K$ then*

$$e \in I \iff (\sigma, \sigma') \cdot e \in I \quad (3.2)$$

Proof. $e \notin I \implies \exists e' \neq e \mid R(e') = R(e)$. For a pair of permutations $\sigma, \sigma' \in \sigma_K$, then $(\sigma, \sigma') \cdot e \neq (\sigma, \sigma') \cdot e'$ and $R((\sigma, \sigma') \cdot e) = R((\sigma, \sigma') \cdot e')$; then $(\sigma, \sigma') \cdot e \notin I$.

Reciprocally, if $(\sigma^{-1}, \sigma'^{-1}) \cdot e \notin I$, then there exists $(\sigma^{-1}, \sigma'^{-1}) \cdot e' \in E$ such as $(\sigma^{-1}, \sigma'^{-1}) \cdot e' \neq (\sigma^{-1}, \sigma'^{-1}) \cdot e$ and $R((\sigma^{-1}, \sigma'^{-1}) \cdot e') = R((\sigma^{-1}, \sigma'^{-1}) \cdot e)$. One can apply the same permutation (σ, σ') to those sets of faulty states.

$$\begin{aligned} & (\sigma, \sigma') \cdot (\sigma^{-1}, \sigma'^{-1}) \cdot e' \neq (\sigma, \sigma') \cdot (\sigma^{-1}, \sigma'^{-1}) \cdot e \\ \text{and } & R((\sigma, \sigma') \cdot (\sigma^{-1}, \sigma'^{-1}) \cdot e') = R((\sigma, \sigma') \cdot (\sigma^{-1}, \sigma'^{-1}) \cdot e) \\ & \implies e' \neq e \text{ and } R(e') = R(e) \\ & \implies e \notin I \end{aligned}$$

\square

If a set of faulty states e of E is isolable, then every permutation of e is isolable. More precisely, the following proposition holds :

Corollary 1. *For all $e \in C_{l,m}$, $e \in I \iff C_{l,m} \in I$*

This is true in particular for the canonical set of faulty states $e_{l,m}$ of class $C_{l,m}$. The canonical set of faulty states describes it entire class in terms of isolability.

4 New problem statement

According to the previous developments, studying isolability of a set of faulty states is equivalent to evaluating the isolability of the canonical set of faulty states of every class. The residuals are calculated using boolean operations between f_G and f_N variables. As stated in Section 2, the *OR* operator is used for $f_G f_G$ and $f_G f_N$ pairs combination and *Exclusive OR* is used for $f_N f_N$ combination. The canonical sets of faulty states of class $C_{l,m}$ are represented in the Tables 5.2a, 5.2b and 5.2c. For f_G (resp. f_N), the l (resp. m) ones are written first and the $K - l$ (resp. $K - m$) zeros are written then. The the result of the boolean operation is written in the table which forms the residual.

These tables offer the advantage of showing clearly the consequence of the parameters l and m on the residual of a set of faulty states. It has been shown previously that the isolability of a canonical set of faulty states is the same as the class it belongs to. The isolability study of a class is made by looking at the number of zeros in the tables with respect to l and m .

4.1 If $l, m < K$

If $l < K$ and $m < K$, there is a rectangle of zeros in Table 5.2c of size $n_c = (K - l) \cdot (K - m) > 0$. Assume that $e \in E$ is another set of faulty states with the same residual as $e_{l,m}$. Looking at Table 5.2c we see that the $K - l$ last variables f_{G_i} and the $K - m$ last variables N_i are necessarily 0. Then, because of the two sub-tables made of 1, we see that all the others variables are 1. In other words, $e = e_{l,m}$. As a consequence, we have:

$$C_{l,m} \subset I, \forall l, m \in \{1, \dots, K - 1\} \quad (4.1)$$

4.2 If $l = K$

If $l = K$, Tables 5.2a and 5.2c are full of ones for all m and Table 5.2b is the only one which can make a difference in the residual. Moreover, $N_i \oplus N_j = \overline{N_i} \oplus \overline{N_j} \forall i \neq j$. Thus, for all $m \in \{1, \dots, K\}$ and $e = (f_{G_i}, f_{N_i}) \in C_{l,m}$, replacing N_i by $\overline{N_i}$ doesn't change the residual of e . Hence:

$$C_{K,m} \subset I^c, \forall m \in \{1, \dots, K\} \quad (4.2)$$

4.3 If $m = K$

If $m = K$ and for all l , the Table 5.2b is full of zeros and the Table 5.2c is full of ones. Then the most significant table is Table 5.2a.

It must be noticed first that, $l = K$ and $l = K - 1$ makes Table 5.2a be full of ones. $C_{K,K}$ and $C_{K-1,K}$ have therefore the same residual. Then:

$$C_{K,K} \subset I^c \quad (4.3)$$

$$C_{K-1,K} \subset I^c \quad (4.4)$$

Secondly, if $l \leq K - 2$, there are $(K - l)^2 - (K - l) > 0$ zeros in Table 5.2a. Assume that e is a set of faulty states with the same residual as $e_{l,K}$. For the same reasons as in the case 4.1, this implies that the l first variables f_{G_i} are 1 and the others are 0: the variables f_{G_i} of e are the same as f_{G_i} of $e_{l,K}$. Moreover, in view of Table 5.2c, we see that all the variables f_{N_i} of e are 1 or else a 0 would appear in Table 5.2a corresponding to the residual of e . So, $e = e_{l,K}$ and we have shown that $e_{l,K}$ is isolable. This is sufficient to obtain the following inclusion:

$$C_{l,K} \subset I, \forall l \in \{1, \dots, K - 2\} \quad (4.5)$$

5 Conclusion

It was demonstrated that:

$$I^c = C_{K-1,K} \cup \bigcup_{l=0}^K C_{K,l} \quad (5.1)$$

In other words, a set of faulty states is not isolable if and only if, it complies to one of the following rules:

- $N_j = 1, \forall j \in \{1, \dots, K\}$ and $\exists! i \in \{1, \dots, K\}$ such as $G_i = 0$
- $G_i = 1, \forall i \in \{1, \dots, K\}$

Table 5.1: Residual generation

$f_G \vee$	f_G	$\overbrace{1 \ 1 \ \dots \ 1}^l$	$\overbrace{0 \ 0 \ \dots \ 0}^{K-l}$
f_G			
l	$\begin{cases} 1 \\ 1 \\ \vdots \\ 1 \end{cases}$	$\begin{matrix} \text{n.a.} & 1 & \dots & 1 \\ 1 & \text{n.a.} & \dots & 1 \\ \vdots & \ddots & \ddots & \vdots \\ 1 & 1 & \dots & \text{n.a.} \end{matrix}$	$\begin{matrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{matrix}$
$K-l$	$\begin{cases} 0 \\ 0 \\ \vdots \\ 0 \end{cases}$	$\begin{matrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{matrix}$	$\begin{matrix} \text{n.a.} & 0 & \dots & 0 \\ 0 & \text{n.a.} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \text{n.a.} \end{matrix}$

(a) $f_G f_G$ residual generation

$f_N \oplus$	f_N	$\overbrace{1 \ 1 \ \dots \ 1}^m$	$\overbrace{0 \ 0 \ \dots \ 0}^{K-m}$
f_N			
m	$\begin{cases} 1 \\ 1 \\ \vdots \\ 1 \end{cases}$	$\begin{matrix} \text{n.a.} & 0 & \dots & 0 \\ 0 & \text{n.a.} & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \text{n.a.} \end{matrix}$	$\begin{matrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{matrix}$
$K-m$	$\begin{cases} 0 \\ 0 \\ \vdots \\ 0 \end{cases}$	$\begin{matrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{matrix}$	$\begin{matrix} \text{n.a.} & 0 & \dots & 0 \\ 0 & \text{n.a.} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \text{n.a.} \end{matrix}$

(b) $f_N f_N$ residual generation

$f_G \vee$	f_N	$\overbrace{1 \ \dots \ 1}^m$	$\overbrace{0 \ \dots \ 0}^{K-m}$
f_G			
l	$\begin{cases} 1 \\ \vdots \\ 1 \end{cases}$	$\begin{matrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{matrix}$	$\begin{matrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{matrix}$
$K-l$	$\begin{cases} 0 \\ \vdots \\ 0 \end{cases}$	$\begin{matrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{matrix}$	$\begin{matrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{matrix}$

(c) $f_G f_N$ residual generation