

# Risk Management Behavior of a Forest Owner to Address Growth Risk

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## Appendix

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## Appendix

### Proofs of the Comparative Static Results for the Financial Savings Model

This appendix derives the comparative static results for the financial savings model reported in Table 1.

The expected utility maximization problem is reproduced here as:

$$\text{Max}_{x_1, x_2, s} V = u(c_1) + mk_1 + \delta[u(c_2) + mE[\tilde{k}_2(\tilde{\theta})]]$$

subject to:

$$c_1 = Y_1 + \pi(x_1) - s$$

$$c_2 = Y_2 + Rs + \pi(x_2)$$

$$k_1 = Q - x_1$$

$$\tilde{k}_2(\tilde{\theta}) = \tilde{\theta}g(Q - x_1) - x_2$$

For interior solutions, the first-order conditions are:

$$V_{x_1} = u'(c_1)\pi'(x_1) - m - \delta mE[\tilde{\theta}g'(k_1)] = 0$$

$$V_{x_2} = \delta[u'(c_2)\pi'(x_2) - m] = 0$$

$$V_s = -u'(c_1) + R\delta u'(c_2) = 0$$

The second-order conditions hold due to the concavity of the utility function and the growth function:

$$V_{x_1x_1} = u''(c_1)(\pi'(x_1))^2 + \delta mE[\tilde{\theta}g''(k_1)] < 0$$

$$V_{x_2x_2} = \delta[u''(c_2)(\pi'(x_2))^2] < 0$$

$$V_{ss} = u''(c_1) + R^2\delta u''(c_2) < 0$$

The matrix of second-order conditions is:

$$\begin{bmatrix} V_{x_1x_1} & V_{x_1x_2} & V_{x_1s} \\ V_{x_2x_1} & V_{x_2x_2} & V_{x_2s} \\ V_{sx_1} & V_{sx_2} & V_{ss} \end{bmatrix} = \begin{bmatrix} u''(c_1)(\pi'(x_1))^2 + \delta mE[\tilde{\theta}g''(k_1)] & 0 & -u''(c_1)\pi'(x_1) \\ 0 & \delta u''(c_2)(\pi'(x_2))^2 & \delta R u''(c_2)\pi'(x_2) \\ -\pi'(x_1)u''(c_1) & R\delta u''(c_2)\pi'(x_2) & u''(c_1) + R^2\delta u''(c_2) \end{bmatrix}$$

The determinant of this matrix is:  $\Delta = \delta mE[\tilde{\theta}g''(k_1)]\delta u''(c_2)(\pi'(x_2))^2 u''(c_1) < 0$

To find how current and future harvesting and savings vary as a parameter changes, we use the Cramer's rule. Then, we have:

$$\begin{bmatrix} V_{x_1x_1} & V_{x_1x_2} & V_{x_1s} \\ V_{x_2x_1} & V_{x_2x_2} & V_{x_2s} \\ V_{sx_1} & V_{sx_2} & V_{ss} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ ds \end{bmatrix} = - \begin{bmatrix} V_{x_1Y_1} & V_{x_1Y_2} & V_{x_1Q} & V_{x_1m} & V_{x_1p_1} & V_{x_1p_2} & V_{x_1R} \\ V_{x_2Y_1} & V_{x_2Y_2} & V_{x_2Q} & V_{x_2m} & V_{x_2p_1} & V_{x_2p_2} & V_{x_2R} \\ V_{sY_1} & V_{sY_2} & V_{sQ} & V_{sm} & V_{sp_1} & V_{sp_2} & V_{sR} \end{bmatrix} \begin{bmatrix} dY_1 \\ dY_2 \\ dQ \\ dm \\ dp_1 \\ dp_2 \\ dR \end{bmatrix}$$

Impacts of  $Y_1$ :

$$dx_1/dY_1 = 0$$

$$dx_2/dY_1 = -\frac{R}{\pi'(x_2)} < 0$$

$$ds/dY_1 = 1$$

Impacts of Q:

$$dx_1/dQ = 1$$

$$dx_2/dQ = \pi'(x_1) \frac{dx_2}{dY_1} < 0$$

$$ds/dQ = \pi'(x_1) > 0$$

Impacts of  $Y_2$ :

$$dx_1/dY_2 = 0$$

$$dx_2/dY_2 = -\frac{1}{\pi'(x_2)} < 0$$

$$ds/dY_2 = 0$$

Impacts of m:

$$dx_1/dm = 0$$

$$dx_2/dm = \frac{u''(c_1) + R^2 \delta u''(c_2)}{u''(c_2)(\pi'(x_2))^2 u''(c_1)} < 0$$

$$ds/dm = -\frac{\delta R}{\pi'(x_2) u''(c_1)} > 0$$

Impacts of  $p_1$ :

$$dx_1/dp_1 = -\frac{u'(c_1)}{\delta m E[\tilde{\theta} g''(k_1)]} > 0$$

$$dx_2/dp_1 = \frac{u'(c_1) R \pi'(x_1)}{\delta m E[\tilde{\theta} g''(k_1)] \pi'(x_2)} + x_1 \frac{dx_2}{dY_1} < 0$$

$$ds/dp_1 = -\frac{u'(c_1) \pi'(x_1)}{\delta m E[\tilde{\theta} g''(k_1)]} + x_1 \frac{ds}{dY_1} > 0$$

Impacts of  $p_2$ :

$$dx_1/dp_2 = \frac{u'(c_2) \pi'(x_1)}{m E[\tilde{\theta} g''(k_1)] \pi'(x_2)} < 0$$

$$dx_2/dp_2 = -\frac{1}{\Delta} \left[ \left( (\pi'(x_1))^2 u''(c_1) + \delta m E[\tilde{\theta} g''(k_1)] \right) \delta u'(c_2) R^2 \delta u''(c_2) + \delta m E[\tilde{\theta} g''(k_1)] u''(c_1) \delta u'(c_2) \right] + x_2 \frac{dx_2}{dY_2}$$

The total effect consists of two opposite effects, a positive substitution effect and a negative income effect. After rewriting, we have:

$$dx_2/dp_2 = -\frac{(\pi'(x_1))^2 R^2 u'(c_2)}{m E[\tilde{\theta} g''(k_1)] (\pi'(x_2))^2} - \frac{\delta u'(c_2) R^2}{(\pi'(x_2))^2 u''(c_1)} - \left[ \frac{u'(c_2)(1 - A_p)}{u''(c_2)(\pi'(x_2))^2} \right]$$

with  $A_p = -\frac{x_2 \pi'(x_2) u''(c_2)}{u'(c_2)}$  the partial risk aversion coefficient. Consequently, if  $A_p < 1$ , then  $dx_2/dp_2 > 0$ .

$$ds/dp_2 = \frac{(\pi'(x_1))^2 R u'(c_2)}{m E[\tilde{\theta} g''(k_1)] \pi'(x_2)} + \frac{R \delta \pi'(x_2) u'(c_2)}{\pi'(x_2) u''(c_1)} < 0$$

Impacts of R:

$$dx_1/dR = -\frac{\pi'(x_1) u'(c_2)}{m E[\tilde{\theta} g''(k_1)]} > 0$$

$$dx_2/dR = \frac{(\pi'(x_1))^2 R \delta u'(c_2)}{m E[\tilde{\theta} g''(k_1)] \pi'(x_2)} - \frac{s}{\pi'(x_2)} + \frac{R \delta u'(c_2)}{\pi'(x_2) u''(c_2)} < 0$$

$$ds/dR = -\frac{(\pi'(x_1))^2 u'(c_2)}{m E[\tilde{\theta} g''(k_1)]} - \frac{u'(c_2)}{u''(c_1)} > 0$$

**Table 1. Comparative Static Results**

	Financial Savings			Physical Savings		
	$x_1^*$	$x_2^*$	$s^*$	$x_1^*$	$x_2^*$	$q^*$
Initial wealth, period 1: $Y_1$	0	–	+	0	0	+
Initial wealth, period 2: $Y_2$	0	–	0	0	–	0
Initial forest stock: $Q$	+	–	+	+	0	+
Timber price, period 1: $p_1$	+	–	+	+	0	+
Timber price, period 2: $p_2$	–	A	–	0	A	0
Marginal utility of amenities: $m$	0	–	+	0	–	+
Rate of return on financial savings: $R$	+	–	+			
Marginal cost of physical savings: $e$				–	0	–

A: Ambiguous result without assumption. Assuming that the partial risk aversion coefficient<sup>1</sup> is less than 1 leads to positive result.

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<sup>1</sup> The partial risk aversion coefficient is as follows:  $A_p = -\frac{w_0''}{w_0} \frac{u''(w_0)}{u'(w_0)}$ , where  $w_0 = w_0' + w_0''$  is the global wealth.

This global wealth is composed of two elements,  $w_0'$ , which is certain, and  $w_0''$ , which is exposed to a multiplicative risk that is assumed to be actuarially neutral (Eeckhoudt and Gollier 1992). A reasonable assumption is to consider that the partial risk aversion coefficient is inferior to one (Cayatte 2004, Reynaud et al. 2010).