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Patrick Terrier, Jean-Michel Charbois, Vincent Devlaminck. Robust estimation of Stokes parameters with a partial liquid-crystal polarimeter under thermal drift. *Applied optics*, 2014, 53 (29), pp.6706-6712. 10.1364/AO.53.006706 . hal-01071835

**HAL Id: hal-01071835**

**<https://hal.science/hal-01071835v1>**

Submitted on 6 Oct 2014

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# Robust estimation of Stokes parameters with a partial liquid-crystal polarimeter under thermal drift

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## 1 - Introduction:

Polarization of light has been used, for a long time, as an important measurement tool for studying phenomena in astronomy, optics, crystallography... and more recently in computer vision. The main difference between these applications is the ability to control the light source illuminating the environment to be characterized. If lighting can be controlled, polarization is generally used to determine the Mueller matrix of the material on which the light wave was reflected. The estimated 4 by 4 Mueller matrix then brings information on observed medium [1,2]. The corresponding optical device is usually called *Mueller polarimeter*.

Conversely, it is frequent to be unable to control lighting, in particular in the field of computer vision when outdoors scenes are observed. In this case it is no longer possible to obtain the Mueller matrix characterizing the material on which there was reflection. However a certain number of information can be obtained on the observed object from the estimate of the polarization state of reflected light wave. These information can be of physical [3,4] or geometrical [5] nature.

The state of polarization of the observed light wave can then be modeled, in the general case, by a four parameters Stokes vector  $\mathcal{S} = [S_0, S_1, S_2, S_3]^T$  and the optical device allowing to obtain these information is usually called *Stokes polarimeter*. Most of these systems are based on a fixed linear polarizer and a rotating retarder. More efficient polarimetric systems use voltage-controlled liquid-crystal devices (LCVR) instead of rotary elements [6, 7]. When using polarimeter based on LCVRs, errors caused by moving parts, unavoidable when using mechanical rotation [8] (inertia with its acceleration and braketimes, misalignments of the signal on the detector,...) are eliminated. Precise rotation stages, stepper motors... are dispensable. However, the main well known disadvantage is that the retardation introduced by LCVR is temperature dependent.

In the field of computer vision, lighting is usually not controlled but may be assumed to be not polarized. A common assumption in computer vision applications [9] states that circular polarization component  $S_3$  of the reflected light vanished. Thus we will only focus in this paper on the evaluation of the first three components of the Stokes vector, i.e., those that represent the linear polarization.

In practice, estimation of  $S_0$ ,  $S_1$  and  $S_2$  requires at least three measurements of light intensity at the output of the device. However, the actual number of measurement may increase significantly in order to have robust estimates. These measurements are made for multiple rotation angles  $\theta_i$  of the retarder or phase retardations  $\delta_i$  of the LCVR. Guarantying

the value of the tuning parameters  $\theta_i$  or  $\delta_i$  over time (errors caused by moving of retarder or temperature dependence of LCVR) is nevertheless difficult.

When a quasi real-time estimation of polarization information is needed, the device based on LCVR components is a usual solution. With this device, the thermal drift is the major drawback that can be addressed in order to built a robust estimation of the polarization information. One solution is to hold constant the LCVR temperature by using a thermostated environment but this is not always possible (power consumption in remote sensing for instance). It was shown [10] that in this case it is necessary to calibrate the LCVR just before carrying out measurement and do this again approximately every ten minutes.

So, in order to avoid these difficulties, this article proposes a method to measure the partially linearly polarized Stokes vector of the reflected light wave without knowing the values of the retardations  $\delta_i$  of a liquid-crystal imaging Stokes polarimeter. A robust and accurate solution to this question is described. It is based on self-calibration principle [11,12]. Unlike methods previously exposed in the literature, our estimate of these polarization parameters is independent of the accurate knowledge of the polarimeter variable retardations values and does not require a calibration process at regular intervals [10].

This article is organized in 5 sections. In section 2, a description of the experimental set-up is presented. A parametric model of the LCVRs used in the device is determined in section 3. Some results are shown in section 4 and summary and some conclusions are presented in section 5.

## 2 - *Materials and methods:*

Figure 1 shows a schematic diagram of the proposed experimental device.

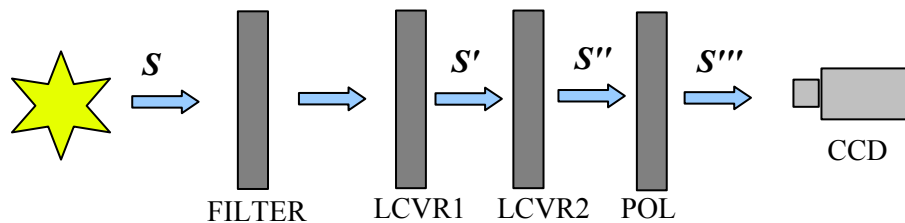


Fig. 1: Measuring device

The partially linearly polarized light  $\mathbf{S}$  reflected by the scene is observed through several optical components. Each of these components modifies the polarization state of the light according to the Mueller formalism. The Stokes vector corresponding to the polarization state of the wave at the output of each optical component is then obtained by multiplication of the Mueller matrix characterizing the component by the Stokes vector of the light wave at the input.

The LCVRs's retardances  $\delta_1$  for LCVR1 and  $\delta_2$  for LCVR2. are the variable parameters of the proposed device (see Fig.1).

These parameters are controlled by the amplitude of a rectangular alternative voltage applied to the LCVR (meadowlark LRC-300 device) via a command interface board (National Instruments PCI-6713).

Since  $\delta$  value is only valid for a monochromatic light, a monochromatic filter is placed at the device input. The monochromatic filter used is centered over a wavelength  $\lambda_0 = 633$  nm.

The theoretical Mueller matrix  $M_{LCVR}(\delta)$  of the LCVR [13] given by

$$M_{LCVR}(\delta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2 \cos \delta & sc(1 - \cos \delta) & -s \sin \delta \\ 0 & sc(1 - \cos \delta) & s^2 + c^2 \cos \delta & c \sin \delta \\ 0 & s \sin \delta & -c \sin \delta & \cos \delta \end{pmatrix} \quad (1)$$

where  $c = \cos 2\theta_{vr}$  and  $s = \sin 2\theta_{vr}$ . The  $\theta_{vr}$  angle is the orientation of the LCVR's fast axis with respect to the horizontal reference Equation (1) transforms the input Stokes vector  $\mathbf{S}$  of the observed partially linearly polarized light into the output Stokes vector  $\mathbf{S}'$  according to

$$\mathbf{S}' = \begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{pmatrix} = \begin{pmatrix} S_0 \\ S_1(c^2 + s^2 \cos \delta) + S_2(sc(1 - \cos \delta)) \\ S_1(sc(1 - \cos \delta)) + S_2(s^2 + c^2 \cos \delta) \\ S_1 s \sin \delta - S_2 c \sin \delta \end{pmatrix} = M_{LCVR1}(\delta) \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ 0 \end{pmatrix} \quad (2)$$

The Mueller matrix of a rotated linear polarizer is given [13] by:

$$M_{POL} = \frac{1}{2} \begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3)$$

As we can see in Eq. (3), the fourth column of the Mueller matrix of a linear polarizer is null, and this is for any values  $\theta$  of the principal axis rotation. The Stokes vector of the light wave at the polarizer output is thus independent of the fourth Stokes parameter at the input. So, in order to keep the fourth independent relation of Eq. (2) ( $S'_3$  parameter), a second LCVR is required. It is placed between LCVR1 and the linear polarizers.

The Stokes vector  $\mathbf{S}'''$  at the output of the polarizer is then given by Eq. (4).

$$\mathbf{S}''' = M_{POL} M_{LCVR2}(\delta_2) M_{LCVR1}(\delta_1) \mathbf{S} = M_{global}(\delta_1, \delta_2) \mathbf{S} \quad (4)$$

A 12 bits CCD sensor (see Fig 1) measures the intensity  $I$  at the polarizer output that corresponds to the first parameter of the Stokes vector  $\mathbf{S}'''$ .

If we make the assumption that the values of the LCVRs retardances  $\delta_1$  and  $\delta_2$  are accurately known (thermostated device), the input Stokes vector  $\mathbf{S}$  is usually computed by the use of 4 measures (and then 4 different values of the couple of retardances  $(\delta_1, \delta_2)$ ) given by Eq. (5).

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = M_{gene} \mathbf{S} = \begin{bmatrix} M_{global}(1, \dots)(\delta_{11}, \delta_{21}) \\ M_{global}(1, \dots)(\delta_{12}, \delta_{22}) \\ M_{global}(1, \dots)(\delta_{13}, \delta_{23}) \\ M_{global}(1, \dots)(\delta_{14}, \delta_{24}) \end{bmatrix} \mathbf{S} \quad (5)$$

Where  $M_{gene}$  is a 4x4 matrix formed by the first lines of  $M_{global}$  (termed  $M_{global}(1, \dots)$ ) computed for the four couples of retardances  $(\delta_1, \delta_2)$  of the LCVRs.

In practice all the parameters of the device must be adjusted in order to obtain a well conditioned matrix  $M_{gene}$  with a low condition number [14]. Therefore, in our experimental process these parameters are adjusted at  $\theta_{vr1}=72,4^\circ$ ,  $\theta_{vr2}=27,4^\circ$  and  $\theta=0^\circ$ . The four optimal couples of retardances are  $(\delta_{11}, \delta_{21})=(135^\circ, 135^\circ)$ ,  $(\delta_{12}, \delta_{22})=(135^\circ, 315^\circ)$ ,  $(\delta_{13}, \delta_{23})=(315^\circ, 135^\circ)$  and  $(\delta_{14}, \delta_{24})=(315^\circ, 315^\circ)$ . With these values the condition number of  $M_{gene}$  is equal to  $\sqrt{3}$  [15].

### 3- An experimental parametric model of the LCVR:

The well known drawback of LCVR is that the retardation strongly depends on temperature  $T^\circ$ .

Estimating the Stokes parameters by using the classical measuring device of Fig. 1 and considering the retardances  $\delta_1$  (for LCVR1) and  $\delta_2$  (for LCVR2) introduced by each LCVR as unknown parameters leads nevertheless to an underdetermined equations system whatever can be the number of acquisitions. We obtain N equations for N acquisitions (N different couples of retardance values  $(\delta_1, \delta_2)$ ) but N+3 unknowns ( $S_0, S_1, S_2, (\delta_{1N}, \delta_{2N})$ ).

Furthermore the theoretical model given by Eq. (1) for LCVR Mueller matrix may be just regarded as an approximation [16]. So, instead of using this theoretical model  $M_{LCVR}(\delta)$  of the LCVR component into the derivation of matrix  $M_{gene}$ , the use of an experimental parametric model is preferred.

Under this requirement, we measure, according to [15], the Mueller matrix  $M_{LCVR12} = M_{LCVR2}(\delta_2)M_{LCVR1}(\delta_1)$  for the four driving voltage couples  $(V_1, V_2)$  that correspond to the four couples of retardance  $(\delta_1, \delta_2)=(135^\circ, 135^\circ)$ ,  $(135^\circ, 315^\circ)$ ,  $(315^\circ, 135^\circ)$  and  $(315^\circ, 315^\circ)$  at  $T^\circ=24^\circ\text{C}$ . These couples of voltages are then respectively adjusted at  $(V_1, V_2)=(2.21\text{V}, 2.31\text{V})$ ,  $(2.21\text{V}, 1.38\text{V})$ ,  $(1.39\text{V}, 2.31\text{V})$  and  $(1.39\text{V}, 1.38\text{V})$ .

Experimental measurements of the mueller matrix  $M_{LCVR12}$  are made for different temperatures  $T^\circ$  and for each pixel. So we obtain 16 characteristic curves for each couples of voltages. Given the measurement noise, the Mueller matrices of the block are filtered to be physically admissible. We use filtering matrices algorithm proposed in reference [17].

For example, Fig. 2 shows, the coefficients of the Mueller matrix  $M_{LCVR12}$  versus the temperature  $T^\circ$  measured for an arbitrary chosen pixel and for a driving voltage couple adjusted to (1.39V, 1.38V). As we can see several coefficients ( $a_{2,2}$ ,  $a_{2,3}$ ,  $a_{2,4}$ ,  $a_{3,2}$ ,...) evolve noticeably versus temperature. These measurements confirm that the theoretical matrix  $M_{LCVR}$  (Eq. 1) is just an approximation of the actual behavior of LCVR.

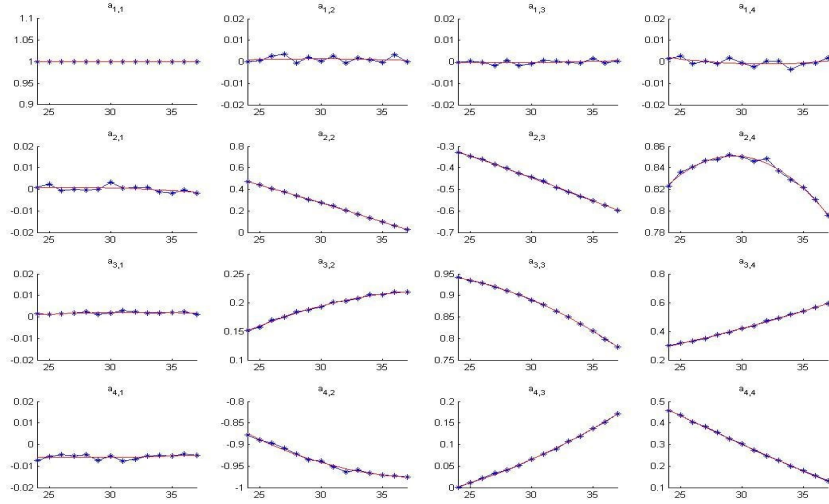


Fig. 2: Coefficients of the Mueller matrix  $M_{LCVR12}$  as function of temperature for a driving voltage couple (1.39V, 1.38V). In order to use a more precise LCVR parameter (Eq. (1)), a polynomial interpolation is therefore applied for each pixel of the image and for each  $M_{LCVR12}$  coefficient and each couple of driving voltage ( $V_1, V_2$ ). A parametric model  $a_{i,j} = AT^2 + BT + C$  (where  $T$  is the temperature in  $^\circ\text{Celsius}$ ) is then obtained for the sixteen entries of  $M_{LCVR12}$ . We find for example the equations given by Eq. (6) for  $a_{i,j}$  coefficients:

$$\begin{cases} a_{2,2} = -0.0002T^2 - 0.0211T + 1.1030 \\ a_{2,3} = -0.0001T^2 - 0.0125T + 0.0548 \\ a_{3,2} = -0.0002T^2 + 0.0224T - 0.2253 \\ a_{3,3} = -0.0005T^2 + 0.0211T + 0.7499 \end{cases} \quad (6)$$

With this parameterization, we obtain a non linear system given by Eq. (7):

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = M_{gene}((V_1, V_2), T) S \quad (7)$$

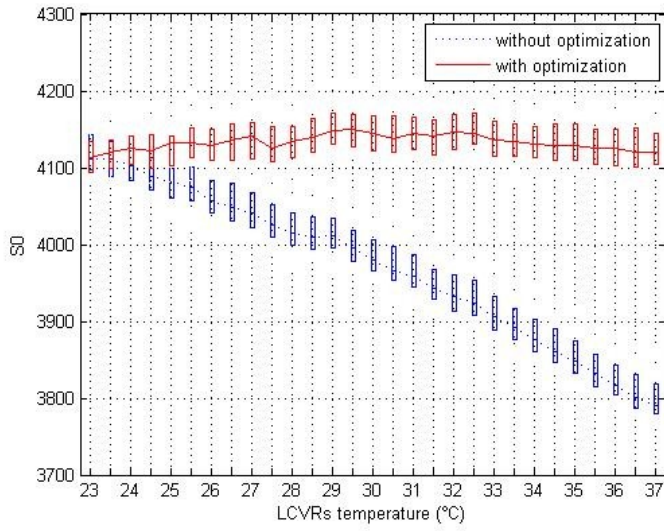
It is important to notice that under this parametric modelization of  $M_{LCVR12}$ ,  $N$  measures ( $N$  different couples of driving voltages) give  $N$  equations (measured intensities) but only 4 unknowns ( $S_0, S_1, S_2, T$ ). The problem is now well defined and can be solved for each pixel of the image by using a classical non linear optimization algorithm.

We use a Levenberg-Marquardt least squares algorithm [18] implemented with MATLAB<sup>®</sup> to solve the non linear system of Eq. (7). Results are reported in the following paragraph. These results are obtained for four couples of driving voltage  $(V_1, V_2) = (2.21V, 2.31V)$ ,  $(2.21V, 1.38V)$ ,  $(1.39V, 2.31V)$  and  $(1.39V, 1.38V)$ .

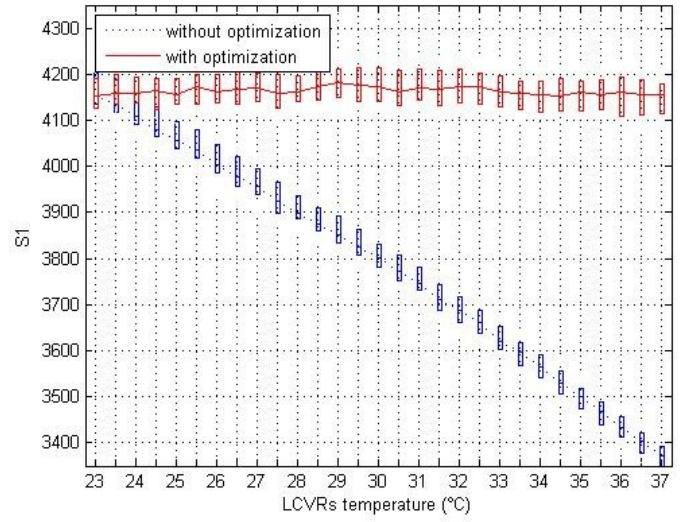
#### **4 - Experimental results:**

One of the main advantages of the proposed measurement approach is that the optimal  $M_{LCVR12}$  parametric model is evaluated independently for each pixel, no temperature sensor or thermal control is required. Assuming a homogeneous temperature distribution over the LCVR is thus absolutely not necessary.

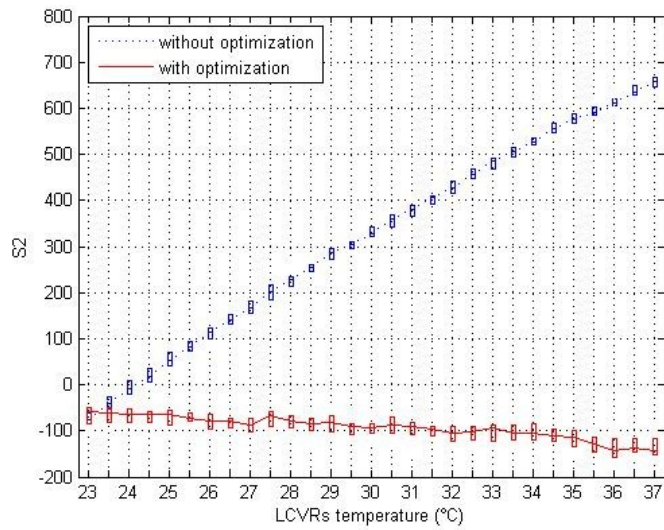
Fig. 3 shows the estimation of the Stokes parameters  $S_0$ ,  $S_1$ ,  $S_2$  and temperature  $T$  versus temperature fluctuation, for an arbitrary 10 by 10 set of chosen pixels. The observed light is the output of a linear polarizer oriented at  $0^\circ$ . The normalized Stokes vector to estimated is  $\mathcal{S} = [1, 1, 0, 0]^T$ . This vector simulates for example the polarization state of an unpolarized light reflected by a dielectric surface at the Brewster angle (polarization degree equal to 1).



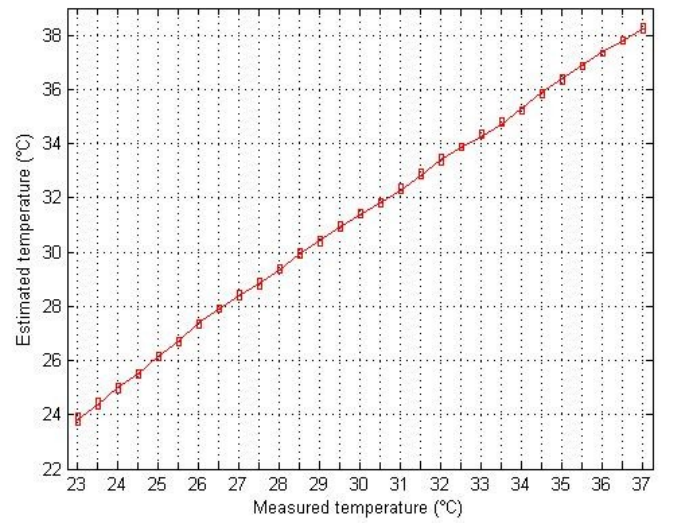
(a)



(b)



(c)



(d)

Fig. 3: Estimation of the temperature and the Stokes parameters of a linearly polarized light wave as function of temperature: (a)  $S_0$ , (b)  $S_1$ , (c)  $S_2$ , (d)  $T$ .

For each temperature we use a statistical representation of the results compute for the 10 by 10 set of pixels for which the Stokes parameters are assumed to be equal. On each box, the central mark is the median and the edges of the box are the 25th and 75th percentiles.

The dotted curves correspond to the estimation of Stokes parameters by conventional polarimeter [19] where the LCVRs mueller matrices are assumed to be temperature independent for four couples of retardance. These parameters are adjusted respectively to  $(\delta_1, \delta_2) = (135^\circ, 135^\circ)$ ,  $(135^\circ, 315^\circ)$ ,  $(315^\circ, 135^\circ)$ . and  $(315^\circ, 315^\circ)$  at  $T^\circ = 24^\circ\text{C}$ . The solid curves show the estimation by the proposed approach. In order to reduce noise effect each intensity  $I$  of Eq. (7) is obtained by averaging 32 frames grabbing.

As we can see, estimation of the Stokes vector is robust with the proposed approach under thermal drift. The estimated Stokes parameters are still constant when the temperature drifts.

Fig. 4 and 5 show respectively the corresponding polarization angle  $\varphi$  and degree  $P$  computed according to



$$\phi = \frac{1}{2} \arctan\left(\frac{S_2}{S_1}\right) \quad (8)$$

$$P = \frac{\sqrt{(S_1^2 + S_2^2)}}{S_0} \quad 0 \leq P \leq 1 \quad (9)$$

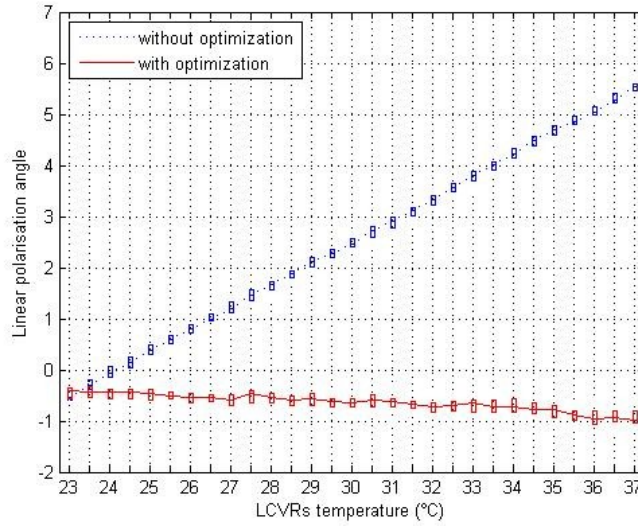


Fig. 4: Estimation of the polarization angle of a linearly polarized light wave as function of temperature

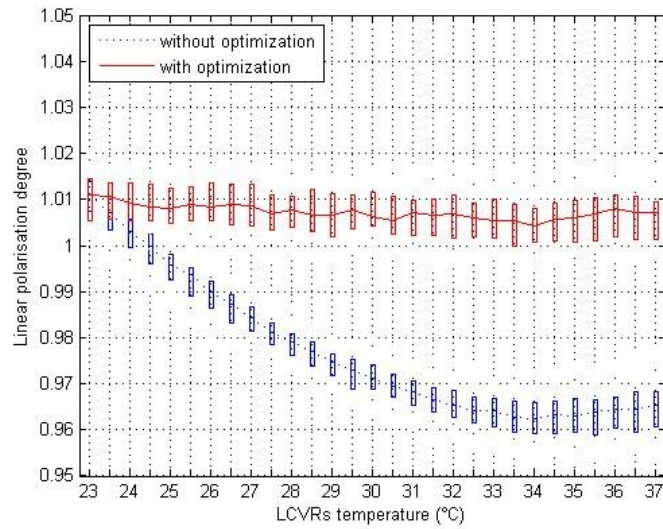
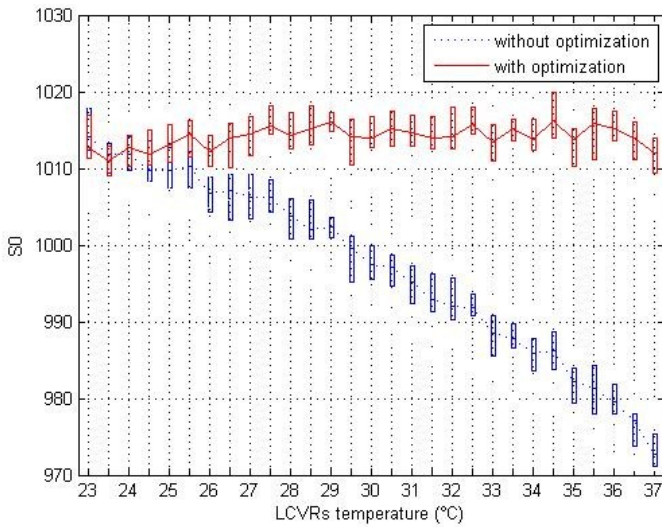


Fig. 5: Estimation of the polarization degree of a linearly polarized light wave as function of temperature

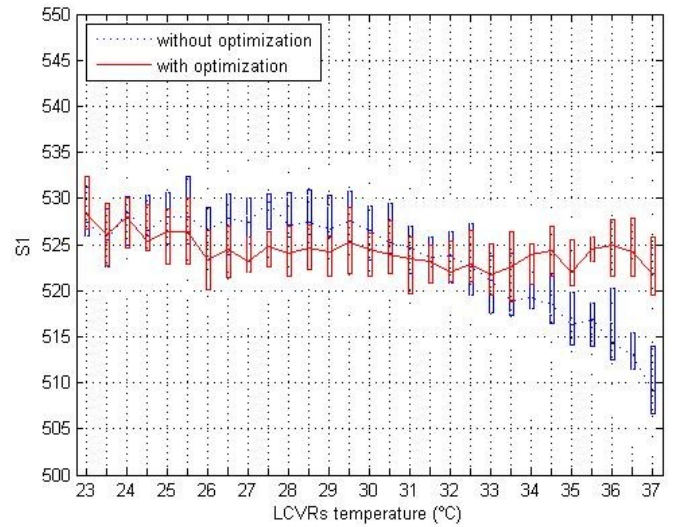
We can see that the result of the estimation of these parameters, using the proposed measuring approach, is independent of temperature.

The second experimentation is shown in Fig. 6. It corresponds to the estimation of a partially linearly polarized light. This light is obtained by the reflection of an unpolarized light reflected by a glass (refractive index equal 1.5) oriented at  $25^\circ$  with an observation angle at  $45^\circ$ .

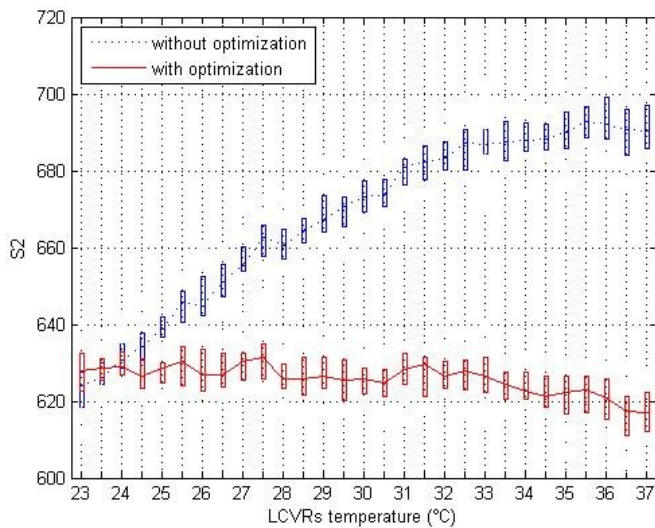
Under this experimental condition, the theoretical linear polarization degree is 0.83. As we can see Fig. 6(d) the proposed measuring device estimates this parameter equal to 0.8 and this estimation is not affected by temperature contrary to the estimation using the conventional polarimeter approach. The polarization angle Fig. 6(e) is also more robust versus temperature as compared to conventional polarimeter and is close to the surface orientation.



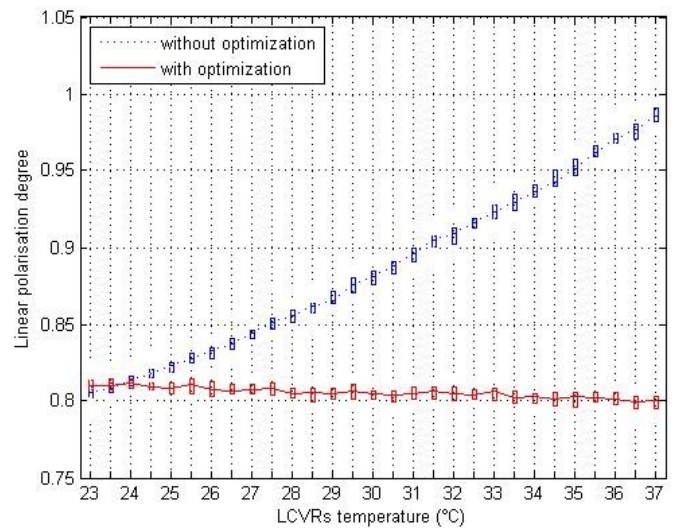
(a)



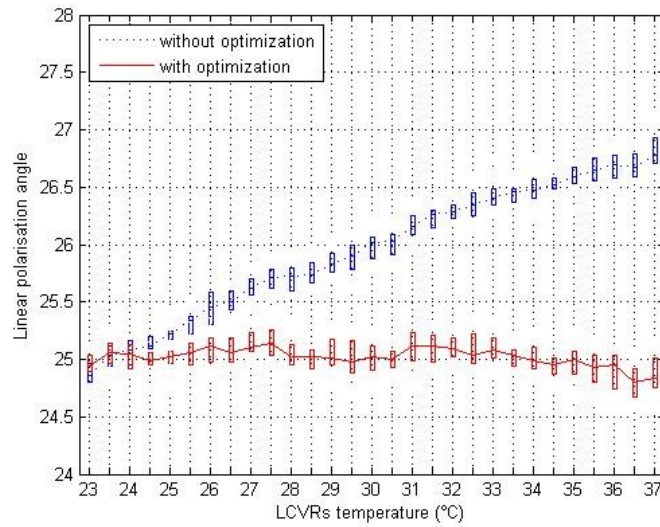
(b)



(c)



(d)



(e)

Fig. 6: Estimation of the polarization parameters of a partially linearly polarized light wave as function of temperature: (a)  $S_0$ , (b)  $S_1$ , (c)  $S_2$ , (d) polarization degree, (e) polarization angle

## 5- Discussion and summary

Using LCVR components when designing a polarimeter allows to estimate polarization information in quasi real-time. However the main well known disadvantage is that the retardation introduced by LCVR depends on temperature. So it is recommended to calibrate the device every ten minutes or to use a thermostated environment.

In the presented work we experimentally model the 16 LCVR coefficients as functions of the temperature and derive a parametric model in order to characterize the behavior of the two LCVRs under thermal drift.

We proposed a new approach where the estimation of the observed Stokes vector is independent of the thermal drift. The classical solution needs to recalibrate the device during time of use or to control temperature. These drawbacks disappear with the proposed approach. An offline calibration of the LCVRs is only required. Sixteen characteristic curves are obtained from this calibration process for each LCVRs couple of driving voltage. So if  $N$  couples of voltages are used, then  $16N$  characteristic curves are needed. During the online measurement process, the operating point of the LCVRs and the Stokes vector are jointly estimated.

In the case of an imaging polarimeter, the estimations of the operating point and Stokes parameters are achieved independently for each pixel of the image. So no temperature sensor and thermal homogeneity of the optical component are required.

The presented solution deals only with the partially linearly polarized light. Therefore, the circular polarization component  $S_3$  is supposed to have a zero value that is a usual assumption in computer vision applications. We are currently investigating an other approach based on two branches measuring device in order to estimate the complete Stokes vector.

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