Effect of lifter shape and operating parameters on the flow of materials in a pilot rotary kiln: Part II. Experimental hold-up and mean residence time modeling

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Effect of Lifters Shape and operating parameters on the Flow of Materials in a Pilot Rotary Kiln : Part II. Experimental Hold-up and Mean Residence Time Modeling

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Abstract

Experiments were carried out on a pilot scale rotary kiln equipped with lifters to investigate the effects of the kiln slope and rotational speed, the mass flow rate of materials and the exit dam height on the hold-up, and the mean residence time (MRT). The MRT was determined from the residence time distribution measurements as detailed in Part I of this work. Two granular solids having different properties were used: sand and broken rice. Furthermore, two shapes of lifters were used to determine the influence of lifters geometry: straight lifters (SL) and rectangular lifters (RL) were compared. A brand new model for the prediction of the MRT is established by means of a dimensional analysis. The correlation not only gave good agreements with the experimental data from the present study, but also demonstrated good predictive performances when applied to published experimental data of other kilns from the literature; the model is applicable for kilns which are inclined, equipped or not with lifters, and fitted or not with a dam at the outlet end.

Keywords: Rotary kiln, Rotary dryer, RTD, MRT, Hold-up, Lifters, Lifting flights

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1. Introduction

Rotary kilns are gas-solid reactors widely used in mineral process applications and other processes applied to specific granular materials. Due to their extensive use in industry, there have been several studies intending to model the solids transport through the kiln cylinder, usually equipped with lifters. Most of these studies attempted to understand and predict key parameters such as the hold-up, the mean residence time (MRT) or, as shown in Part I of this study, the residence time distribution of solid particles.

Most of the substantial earlier body of scientific literature that exists in the field has assumed void axial dispersion through the kiln, such that the time of passage τ was analyzed instead of the mean residence time $\bar{t}$. Other simply confused and supposed the mean residence time to be equal to the time of passage. Indeed the time of passage is very easy to determine; it results from the ratio of the weight of the kiln hold-up to the mass flow rate. However, it should not be forgotten that the time of passage might fail to reflect the flow of solids, especially when there is some axial dispersion, which is increased when using lifters.

One of the earliest equations was developed by Sullivan et al. [1] for the calculation of the time of passage of particles in rotary kilns which do not have lifters, as follows:

$$\tau = \frac{1.77L\sqrt{\theta}}{SDN} \times factor \tag{1}$$

where θ is the angle of repose, S is the kiln slope, N is the rotational speed, L and D respectively the length and internal diameter of the kiln, $factor$ is a parameter accounting for the operating conditions, it is unity for a simple kiln without obstructions or constrictions.

Latter, Chatterjee et al. [2] used a semi-empirical correlation to predict the mean residence time of solid particles. Their model was built on a dimensional analysis taking into account parameters influencing the flow of materials. However, some parameters were missing such as the exit dam height, which does not appear in the list of parameters, though its effect has been studied in their experimental matrix. By applying conditions of dimensional homogeneity, they deduced the following correlation:

$$\bar{t} = k\frac{L^3}{F}\left(\frac{\theta}{S}\right)^a\left(\frac{L^3N}{F}\right)^e\left(\frac{L}{D}\right)^{-c} \tag{2}$$

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where F is the volumetric feed rate, k is a constant equal to 0.1026 and exponents {a,e,c} are evaluated as {1.054,-0.981,-1.1} [3]. It can be noted in this correlation that the effect of the kiln slope and the angle of repose of solid particles are strongly linked.

Models for the mean residence time or time of passage, developed for kilns equipped or not with lifters in literature range from relatively simple empirical equations to fully mechanistic models. However, we will focus in the coming lines on some of the simple empirical models developed for kilns equipped with lifters since the so-called mechanistic models are not suitable for control purposes [4].

Among the earliest models developed for rotary kiln equipped with lifters is the relation of Prutton et al. [5] to predict the time of passage:

$$\tau = \frac{kL}{SDN} + mV_f \tag{3}$$

where V_f is the volume of solids inside a lifter, k is a constant depending on the number and shape of lifters, and m is a constant which depends on the gas flow direction and the materials properties. This model does not consider effect of any changes in solids and/or gas flow rates through the rotary kiln. Moreover the constants k and m must be determined by fitting the model to experimental data from the rotary kiln in use, thus making the model impossible to use for design purposes.

Certainly the most commonly used is the Friedman and Marshall [6] relation which is derived from Sullivan et al. [1] model with an additional term to allow for air drag on the solid particles; the following equation was obtained through the study of hold-up of a range of materials under overloaded conditions:

$$\tau = L\left[\frac{0.3344}{SDN^{0.9}} \pm \frac{0.6085G}{d_p^{0.5}F}\right] \tag{4}$$

where d_p is the average particle diameter, F and G are respectively the solids feed rate and the gas flow rate. The second term in this equation expressed the air drag, thus the negative sign is used for counter-current flow and the positive sign is used for co-current flow. Compared to Sullivan et al. [1] model, this one may account for variations of solids and gas feed rates, however by increasing the solids and gas feed rate by proportional amounts the predicted residence time will remain constant, and the geometric features of lifters are not taken into account in this formulation.

Shahhosseini et al. [4] modified Friedman and Marshall [6] model in order to better represent the dynamic of the system, the main objective being to determine the hold-up through the use of the retention time of sugar in the rotary kiln. An additional term was introduced to account for changes in solids flow rate under zero gas flow rate conditions:

$$\tau = L\left[\frac{8.5}{tan(S)DN^{0.8}} + \frac{0.41(G+0.14)}{d_p^{0.5}F}\right] \tag{5}$$

Even if this model attempted to address some of the shortcomings of Friedman and Marshall [6] model, it still does not consider internal fixtures of the kiln.

Alvarez and Shene [7] presented an empirical relationship for residence time estimation. It was derived from experimental data recovering biological and mineral origin solid particles. This relation requires up to six constants and five other exponents, leading to a total of 11 parameters to determine:

$$\tau = \frac{\alpha d_p^{0.032}\rho_{bulk}^{0.956}}{NF(\beta S+1)} + \frac{\gamma d_p^{-0.065}\rho_{bulk}^{0.002}}{\delta S+1} - \frac{\epsilon G^{0.5}}{\varepsilon S+1} \tag{6}$$

where ρ_{bulk} is the density of the bulk materials. This formulation allows prediction of residence times even when the kiln is horizontal and may apply to a wide range a solid materials. However, due to the large number of model parameters there is a need to fit the model with the system experimental data prior to its utilization.

More recently Thibault et al. [8] proposed two empirical models for the prediction of the mean residence time. The models consist of three factor as follows:

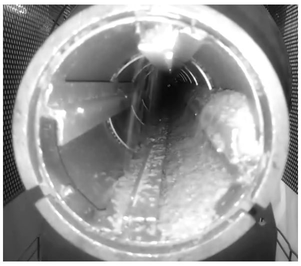
$$\bar{t} = \rho_p \mathrm{f}_1(1/FN)\mathrm{f}_2(S)\mathrm{f}_3(G) \tag{7}$$

The three factors f_1, f_2 and f_3 account for the influence of the product FN, the slope of the rotary kiln, and the impact of the gas flow rate on the mean residence time. Definition of f_2 and f_3 as first order equations in models allows calculation for horizontal kiln with no gas flow rate. The main difference between the two models proposed is the definition of the factor f_1 either as a second-order polynomial with the inverse of FN or as a simple function with F and N each with an exponent.

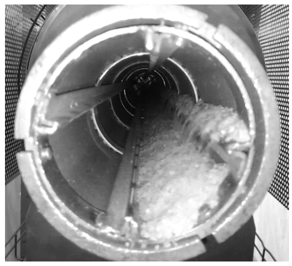
This paper is concerned with the study of the effects of usual operating conditions, shape of lifters, and type of materials on the mean residence time of solids particles as well as on the kiln hold-up. A semi-empirical model obtained from dimensional consideration for the mean residence time of solid particles is also presented. Compared to the other published correlations, this new one should be applicable to a wide range of kilns and operating conditions, and could be also useful for design purposes. Therefore comparisons of the predictions, calculated with this model

Table 1: Details of the circular dams at the kiln outlet end

Exit Dam Height [mm]	Open Diameter [mm]	Diameter Covered [%]	Area Covered [%]
0	101.3	0.00	0.00
13.5	74.3	13.33	46.20
23.5	54.3	23.20	71.27
33.5	34.3	33.07	88.54



(a) Rectangular lifters



(b) Straight lifters

Fig. 1: Experimental Rotary kiln equipped with lifters: a) Rectangular lifters b) Straight lifters. The bulk materials are broken rice.

when using some published experimental conditions, versus the experimental MRT results from the corresponding published data are given.

2. Materials and methods

2.1. Apparatus and materials

The apparatus and materials have been described in Part I of this work. Therefore only their main characteristics are given here. The rotary kiln is a 0.101 m diameter and 1.95 m long tube, which can be tilted to a maximum angle of 5°. Its rotational speed can be adjusted by a driven motor between 2 and 12 rpm. The solids flow rate was adjusted by regulating the rotational speed of a screw feeder. Different dam heights were used as given in Table 1. In order to study the effect of internal flights, 4 straight one-section lifters of 10 mm referred to as straight lifters (SL) and 4 two-section lifters of 10 mm each, with a right angle cross section referred to as rectangular lifters (RL) as shown in Fig.1, were used.

Quartz sand with a narrow distributed size fraction and broken rice were employed as experimental materials. The physical properties of these materials are listed in Table 2.

2.2. Experimental technique

The experimental mean residence time was determined from the residence time distribution (RTD) measurement data. The MRT τ is defined by [9]:

$$\bar{t} = \frac{\int_0^\infty t\mathrm{C(t)}dt}{\int_0^\infty \mathrm{C(t)}dt} \simeq \frac{\sum_i^{N_S} t_i \mathrm{C(t_i)}\Delta t_i}{\sum_i^{N_S} \mathrm{C(t_i)}\Delta t_i} \tag{8}$$

where $\mathrm{C(t_i)}$ represents the tracer concentration at the kiln exit end at the discrete time t_i, i={1, 2, 3,..., N_S}, and Δt_i the associated sampling time.

The kiln hold-up was obtained after the RTD measurement. The feeding system was disabled, and then the solid materials within the kiln were discharged into the recovery tank and weighed. It was also possible to assess the time of passage of particles in the rotary kiln by dividing the weight of solid materials measured by the mean flow rate at steady state.

Part I of this work provides further details on the whole experimental procedure.

Table 2: Physical properties of materials

Materials	Shape	ρ_{bulk} [kg.m^{-3}]	ρ_{tapped} [kg.m^{-3}]	d_p [mm]	θ [°]
Sand	Nodular	1422	1543	0.55	39
Broken rice	Cylindrical	889	934	3.8×1.9	36

Table 3: Experimental results

Operating conditions				RL				SL				NL	
				Rice		Sand		Rice		Sand		Rice	
N [rpm]	S [°]	$\dot{M}$ [kg.h^{-1}]	h [mm]	MRT [min]	HU [kg]	MRT [min]	HU [kg]	MRT [min]	HU [kg]	MRT [min]	HU [kg]	MRT [min]	HU [kg]
3	**2**	**2.5**	**23.5**	43.4	1.953	48.6	1.959	39.2	1.736	44.8	1.886	29.5	1.214
2	2	2.5	23.5	67.4	2.953	65.5	2.808	57.9	2.505	62.7	2.488	-	-
4	2	2.5	23.5	32.3	1.391	39.7	1.536	-	-	35.8	1.508	-	-
6	2	2.5	23.5	25.1	1.033	30.0	1.143	21	0.932	26.7	1.124	18.0	0.724
8	2	2.5	23.5	20.6	0.849	25.5	0.973	-	-	22.2	0.892	-	-
10	2	2.5	23.5	18.2	0.704	22.2	0.831	15.2	0.684	19.3	0.791	13.4	0.532
12	2	2.5	23.5	16.2	0.653	20.5	0.774	-	-	17.1	0.736	-	-
3	**2.5**	2.5	23.5	31.8	1.305	41.9	1.631	-	-	39.8	1.621	-	-
3	**3**	2.5	23.5	26.4	1.137	35.5	1.333	24.9	0.919	30.6	1.198	18.8	0.774
3	**4**	2.5	23.5	20	0.858	24.9	0.937	18.9	0.705	23.2	0.918	-	-
3	**5**	2.5	23.5	14.6	0.625	18.9	0.734	14.5	0.625	16.6	0.693	-	-
3	2	**0.68**	23.5	56.9	0.573	63.9	0.676	54.9	0.570	59.7	0.670	-	-
3	2	**1.3**	23.5	48.3	1.028	54.8	1.105	41.6	0.967	51.7	1.085	34.9	0.758
3	2	**1.9**	23.5	44.5	1.415	52.1	1.541	42.2	1.276	49.1	1.503	-	-
3	2	2.5	**0**	39.6	1.685	41.3	1.646	33.5	1.478	37.0	1.539	-	-
3	2	2.5	**13.5**	39.1	1.713	42.9	1.720	-	-	38.8	1.619	-	-
3	2	2.5	**33.5**	51.1	2167.5	59.6	2.489	48.6	2.122	57.5	2478	42.1	1.742

3. Results and discussion

Table 3 gives the results of the mean residence time and the hold-up for a number of experiments where sand and broken rice were operated, with either straight, rectangular or no lifters. The details of these runs are also given. It must be pointed out that the average mass flow rate measured was within a margin of ± 0.05 kg.h^{-1} of the reported values. Repeated experiments are not reported. In this part of the study will be discussed the effects of kiln rotational speed, kiln slope, mass flow rate, exit dam height, lifters shape and bulk materials properties on the fractional volumetric hold-up and the mean residence time. And then a model for the prediction of the MRT will be presented.

3.1. Hold-up, volume fraction

As shown in Table 2, the particulate solids used as bulk materials have varying properties. It was convenient to use the fractional volumetric hold-up (or elsewhere the degree of filling) for the analysis of the results. This latter is defined as the ratio of the volume of the working content to the volume of the kiln cylinder. The effects of operating parameters on the volumetric hold-up are illustrated in Fig.2. At given operational conditions, experiments processing broken rice filled a higher volume than those with sand, even though the values of the hold-up weight were very close. This is a direct consequence of the materials differing bulk density properties. Another remark is about the effect of the lifters: the experimental results showed that the presence of lifters significantly increases the hold-up (see Table 2). It was also observed that the use of rectangular lifters slightly increases the hold-up within the kiln compared to the use of straight lifters. This may be related to the lifters hold-up capacity, which is higher in the case of rectangular lifters. Hence the loading of a kiln will be partly dependent on the presence, size and shape of the lifters as previously suggested by Prutton et al. [5] and Afacan and Masliyah [10].

The classical operational parameters also significantly influence the volumetric hold-up. The fractional volumetric hold-up sharply decreases at smaller rotational speed values before slowing down at higher speed values, as if a bottom was to be reached (see Fig.2a). Indeed the hold-up must be at the lowest when the motion is starting centrifuging. This usually happened at very high speed. The fractional volumetric hold-up gradually goes down as the kiln slope increases (see Fig.2b). On the contrary, the volumetric hold-up steadily increases with the feed rate (see Fig.2c). This is not obvious but still can be understand if one extrapolates the work of Saeman [11] followed by Descoins et al. [12] on the bed depth profiles. These studies showed that in a kiln without lifters any increase of the feed rate will result in an increase in the bed depth and so an increase in the fractional volumetric hold-up. Even if their models are not directly applicable to kilns equipped with lifters, the results remain the same in regard to the effect of the flow rate on the volumetric hold-up. Eventually, it has been observed that the volumetric hold-up increases slightly with increasing exit dam height (see Fig.2d). The higher the dam height, the higher the kilning bed depth, which explains the obtained results.

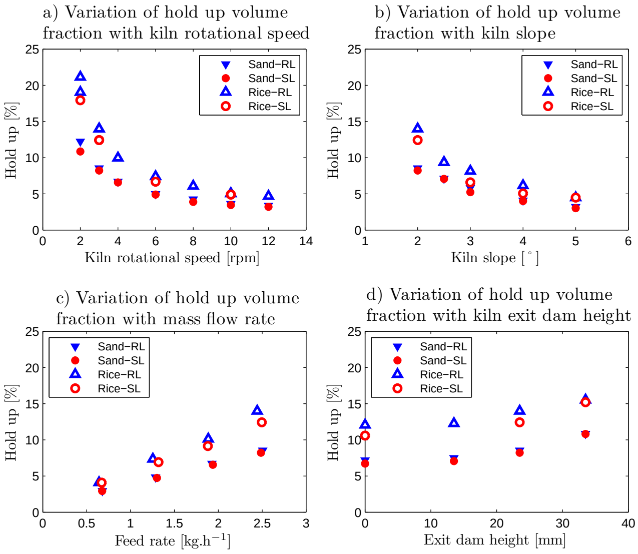


Fig. 2: Influence of operating parameters (N, S, $\dot{M}$ and exit dam height) on the hold-up volume fraction for the flow of sand and broken rice, when the kiln is equipped with either straight or rectangular lifters.

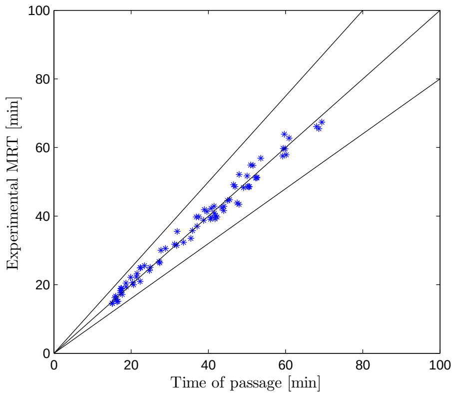


Fig. 3: Comparison of experimental MRT and time of passage of solid particles. Solid lines are ±20% margins.

3.1.1. Time of passage

The hold-up measurements allow the calculation of the time of passage. This latter is calculated as follows:

$$\tau = \frac{HU}{\dot{M}} \tag{9}$$

Most of the previous studies focus on this simple relation to investigate the solids transport through a rotary kiln, sometimes on behalf of the MRT, as previously stated. Therefore a comparison is made between the time of passage and the MRT results over the whole experimental campaign as shown in Fig.3. It must be said that, not surprisingly there are quite good agreements between the MRT and corresponding time of passage in general. This is because the dispersion coefficients observed in the system are very low, as showed in Part I of this work. However some deviations are still observable because contrary to the MRT, this method does not offer an insight in the dispersion related issues which are linked to the solids residence time distribution. The time of passage estimates the mean residence time only with a certain degree of accuracy.

3.2. Mean residence time

3.2.1. Influence of operating conditions

The mean residence times of sand particles were found to be higher than those of the broken rice particles mainly due to the effect of gravity and their differing angle of repose. For a given type of materials and fixed operating parameters, the MRT obtained while using rectangular lifters was higher than that obtained while using straight lifters, due to the differing hold-up capacity of the lifters. These trends as shown in Fig.4 were previously discussed in detail by Bongo Njeng et al. [13].

The MRT significantly decreases by about 69% as the rotational speed is increased 6 times, as shown in Fig.4a. These results also agreed well with Abouzeid et al. [14] findings while varying the rotational speed over a wide range

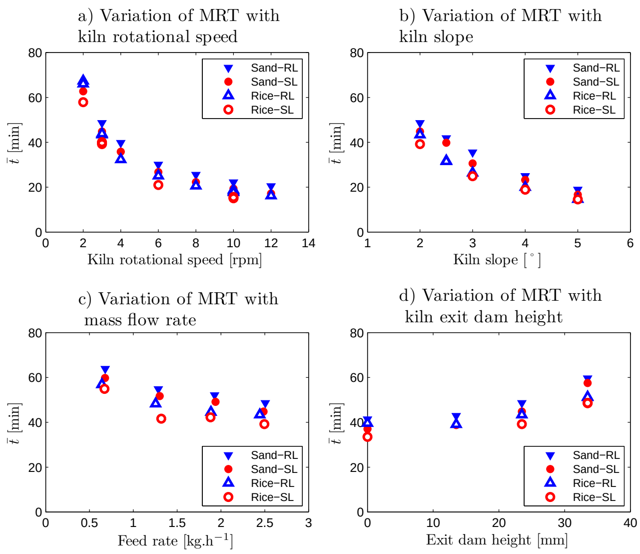


Fig. 4: Influence of operating parameters (N, S, $\dot{M}$ and exit dam height) on the MRT for the flow of sand and broken rice, when the kiln is equipped with either straight or rectangular lifters.

of values. He found that the MRT declines with increasing speed and then reaches a bottom at approximately 20 rpm, from which it follows a plateau before rising. Fig.4b and Fig.4c show that the MRT also decreases with the kiln slope and the mass flow rate. But it must be further specified in the case of the inclination that there is a sharp decrease between 2 and 4 degrees followed by a slow decrease for higher slope, whereas the MRT slowly decreases when the mass flow rate increases from 0.68 to 2.5 kg.h^{-1} (see Fig.4c). This tendency is similar to the one observed by some other researchers [2, 3, 14, 15]. When increasing the exit dam height the MRT slightly increases (see Fig.4d) as observed previously by Chatterjee et al. [2]. However, in our experiments, a maximum increase of only 5% was observed on the MRT by using a 13.5 mm exit dam height compared to the case where no exit dam was fitted at the kiln outlet end. It has been observed that these two experimental conditions, all in all, led to similar bed profiles. The only difference which can be mentioned is that the bed depth was kept constant till the outlet end with the use of the dam, while it started displaying a slope assumed to be close to the angle of repose approximately 20 mm to the exit with no dam. Hence the design of a kiln and its exit dam should give careful consideration to the singular aspects pointed out here to be optimal with regard to the production objectives. For a dam to have a significant impact on the materials transport characteristics, its height must be higher than the bed depth observed when the kiln has a clear exit.

3.2.2. Model

The main purpose of this model is to come up with a prediction of the MRT of solid particles defined as a function of operating parameters, namely the kiln rotational speed, the kiln slope, the mass flow rate and the exit dam height, as well as the kiln design and the bulk materials properties. For the purpose of application to other kiln design and operating conditions, and thus to meet scaling-up issues, the model for the prediction of the MRT is based on a dimensional analysis as described below.

Dimensional analysis. In dimensional analysis, a certain functional form is assumed, to relate the varying parameters. All the possible parameters are commonly present with their own exponent. In this study the main operating, geometrical, and physical parameters which can affect the MRT are listed below along with their dimensions:

N	rotational speed	T^{-1}
$\dot{M}$	mass flow rate	MT^{-1}
S	kiln slope	-
D_{open}	effective exit open diameter	L
S_{lift}	cross section of materials in lifters	L^2
L	length of the kiln	L
D	internal diameter of the kiln	L
ρ_{bulk}	bulk density of materials	ML^{-3}
ρ_{tapped}	tapped bulk density of materials	ML^{-3}
θ	angle of repose of the bulk materials	-
g	gravitational acceleration	LT^{-2}

$$\bar{t} = k\frac{\rho_{bulk}LD^2}{\dot{M}}\left(\frac{N^2D}{g}\right)^{\alpha}\left(\frac{D_{open}}{D}\right)^{\beta}(\theta)^{\gamma}(S)^{\delta}\left(\frac{\dot{M}}{\rho_{bulk}NLD^2}\right)^{\epsilon}\left(\frac{4S_{lift}}{\pi D^2}\right)^{\varepsilon}\left(\frac{\rho_{bulk}}{\rho_{tapped}}\right)^{\zeta}\left(\frac{L}{D}\right)^{\eta} \tag{13}$$

Table 4: Calculated values of exponents and constant k for the MRT dimensional correlation

Exp. data	k	α	β	γ	δ	ϵ	ε	ζ	η [3]
Sand	0.1363	0.0508	-0.4008	0.8749	-0.9814	0.8115	-4.5285	0.7723	1.1
Rice	0.0792	-0.0218	-0.3387	0.8749	-1.2277	0.8184	-8.0175	0.7723	1.1
Sand & Rice	0.2611	0.0842	-0.3649	0.8749	-1.1243	0.8350	-5.5283	0.7723	1.1

The analysis involves the fundamental dimensions MLT: mass, length and time. It follows from the above that the relation being sought can be summarized as:

$$F(\bar{t},\, N,\, \dot{M},\, S,\, D_{open},\, S_{lift},\, L,\, D,\, \rho_{bulk},\, \rho_{tapped},\, \theta,\, g) \tag{10}$$

It is convenient to rewrite Eq.10 so that the dependent variable $\bar{t}$ appears directly. Whatever the function F may be, Eq.10 can be written in the form:

$$F(N,\, \dot{M},\, S,\, D_{open},\, S_{lift},\, L,\, D,\, \rho_{bulk},\, \rho_{tapped},\, \theta,\, g)\cdot\bar{t} = 1 \tag{11}$$

Assuming that Eq.11 is dimensionally homogeneous, parameters of the function F must be subject to the power dependence, as previously stated. According to the Buckingham's theorem, the formula connecting the set of parameters can be expressed as a function of dimensionless arguments. And each of these arguments is a dimensionless product of a combination of the original parameters raised to integer power. Having in Eq.11 a number of variables r=12 and a number of fundamental units n=3, the number of dimensionless groups p is, according to Buckingham's second theorem:

$$p = r - n = 12 - 3 = 9$$

A collection of 9 dimensionless groupings formed from the parameters in Eq.11 were found as follows:

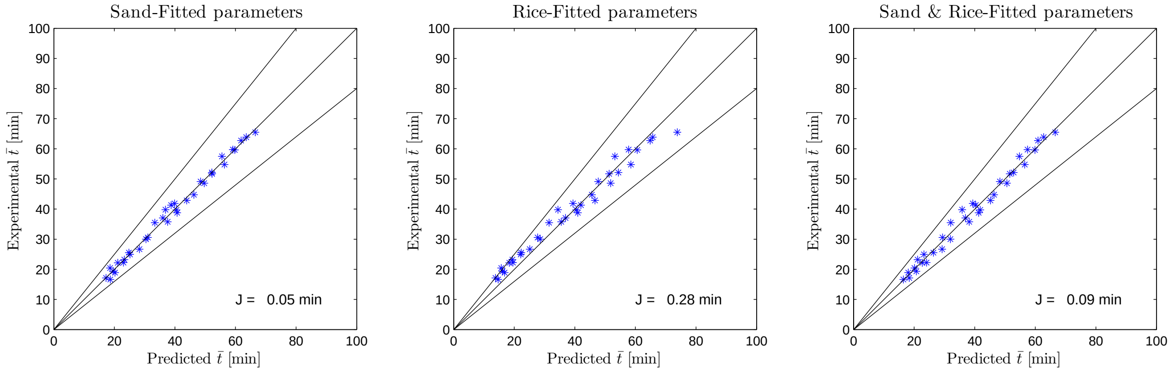
$$\frac{\bar{t}\dot{M}}{\rho_{bulk}LD^2} = F\left[\frac{N^2D}{g},\, \frac{D_{open}}{D},\, \theta,\, S,\, \frac{\dot{M}}{\rho_{bulk}NLD^2},\, \frac{4S_{lift}}{\pi D^2},\, \frac{\rho_{bulk}}{\rho_{tapped}},\, \frac{L}{D}\right] \tag{12}$$

Some of these dimensionless expressions are already famous such as the Froude number, the length-to-diameter ratio, or the (inverse of) Hausner ratio. $\frac{D_{open}}{D}$ is the fractional open diameter at the kiln outlet end, $\frac{4S_{lift}}{\pi D^2}$ estimates the fraction of solids held in lifters, where $S_{lift} = \frac{\pi D^2}{4} - \frac{n_{lift}-1}{2}S_{horlift}$ with n_{lift} the total number of lifters and $S_{horlift}$ the section of materials in a lifter at horizontal position. The two dimensionless groupings $\frac{\bar{t}\dot{M}}{\rho_{bulk}LD^2}$ and $\frac{\dot{M}}{\rho_{bulk}NLD^2}$ are very similar to those found by Chatterjee et al. [2] in their correlation. However, in this work, contrary to what Chatterjee et al. [2] proposed, the bulk materials angle of repose and the slope of the kiln are taken as separate factors rather than gathered as an angular ratio (see Eq.2). Eq.12 can be written as specified by Buckingham's theorem so that an expression of the MRT is obtained from the defined dimensionless numbers as given in Eq.13.

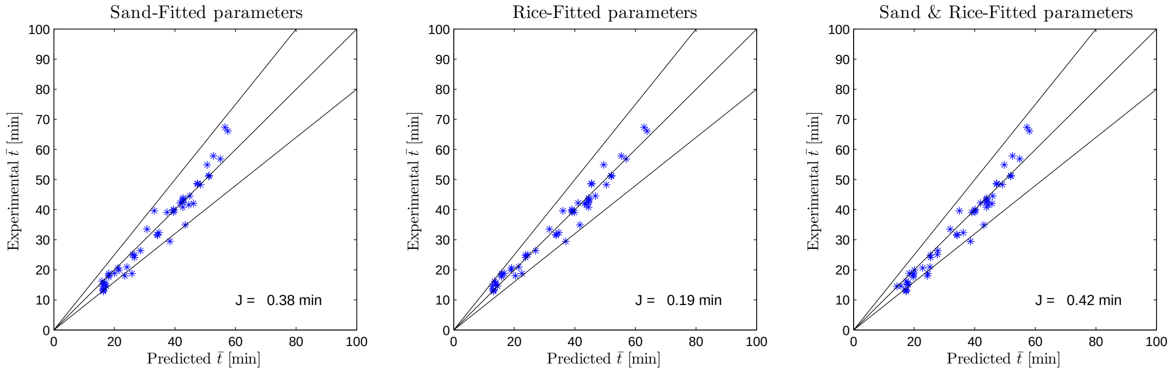
The variables in Eq.13 are expressed in the International System of units, therefore the mean residence time obtained is in seconds. The exponents $\alpha, \beta, ..., \zeta$ and the constant k are to be determined with the use of experimental data. The length-to-diameter ratio has not been varied in our experimental matrix, therefore the exponent η was fixed to the value of 1.1 which has been determined by Chatterjee et al. [3] when varying the dimensionless group $\frac{L}{D}$ from 3.3 to 16. To determine the other exponents and the constant k, a MatLab script was written. More specifically the program sought to determine the parameters by minimizing the sum of square error between the experimental and predicted values of the MRT. The program was first run with the whole experimental data matrix, i.e. resulting from the flow of both sand and broken rice, then it was run with sand and broken rice experimental data separately. The angle of repose and the (inverse of) Hausner ratio are strongly dependent on the materials properties; for this reason, when determining the parameters with either sand or broken rice experimental data, γ and ζ were fixed to the values obtained from the whole experimental data matrix . Thus, three sets of parameters fitting the best the experimental data using the presented model are obtained as given in Table 4.

It should be noted that the aforementionned correlation for the MRT is applicable in the case of inclined rotary kilns, equipped or not with lifters, and fitted or not with dam at the outlet end.

Predictive performance of the model. The performance of the model was assessed using the three sets of optimal parameters separately on the experimental results. For reasons of clarity, a criterion in performance assessments J was defined as follows:



(a) Experiments with sand



(b) Experiments with broken rice

Fig. 5: Comparison of the experimental MRT while using a) a bulk of sand and b) a bulk of broken rice with predicted values from Eq.13 using the 3 sets of parameters as given in Table 4. Solid lines are ±20% margins.

$$J = \frac{1}{N_T} \sum_{i=1}^{i=N_T} \frac{\left(\bar{t}_{i_{exp}} - \bar{t}_{i_{calc}}\right)^2}{\bar{t}_{i_{exp}}} \tag{14}$$

This criterion can be defined as the sum of the relative square error between predicted and experimentally determined values of the MRT divided by the number of experiments. It will be used to determine which one among the three sets of optimal parameters gives the best predictions. The lower is the value of this criterion the better are the predictions.

The performance of the proposed model was first tested on the experimental data from this study as given in Fig.5. There are very good agreements between the experimental MRT obtained while processing sand and the predictions in general, however as one may expect the best predictions are obtained with the "sand-fitted parameters" which displayed the lowest value of the criterion J as shown in Fig.5a. Similar remarks can be made when looking at the experiments processing broken rice. There as well, good agreements are found between the experimental and predicted values of the MRT, and the "rice-fitted parameters" give the least deviations from experimental values according to the criterion value (see Fig.5b). In both cases the "sand & rice-parameters" give some good results, with values of the criterion sometimes very close to the minimum observed.

The predictive capacity of the proposed model was then tested on some experimental data from the literature. Kilns of different geometries and processing a range of materials at varying conditions were chosen. Only few researchers investigating the residence time of particles through rotary kiln have published their experimental data along with sufficient details on the bulk materials properties. This is even more true for kilns equipped with lifters. The results from the work of Debacq [16] are used in this case. The results from kilns equipped with dams and no lifters were also tested. These latter are from the work of Sai et al. [18] and Chatterjee et al. [2]. The results from the kiln used by Colin et al. [17], which was not equipped with lifters nor exit dam, are also used in this study. The experimental results of the mentioned kilns were used to evaluate the predictive performance of the model for kilns of varying geometries with differing types of materials properties and operating conditions. Details about the kilns geometry and materials properties are specified in AppendixA. However it should be pointed out that the mass flow rates achieved in most of these experiments extracted from the literature are (between 2 to 300 times) higher than those in the present study.

Fig.6 shows a comparison of Debacq [16] experimental MRT with corresponding predictions. The model predictions are very good and the values of the criterion are very close for the three sets of parameters, but the "rice-fitted

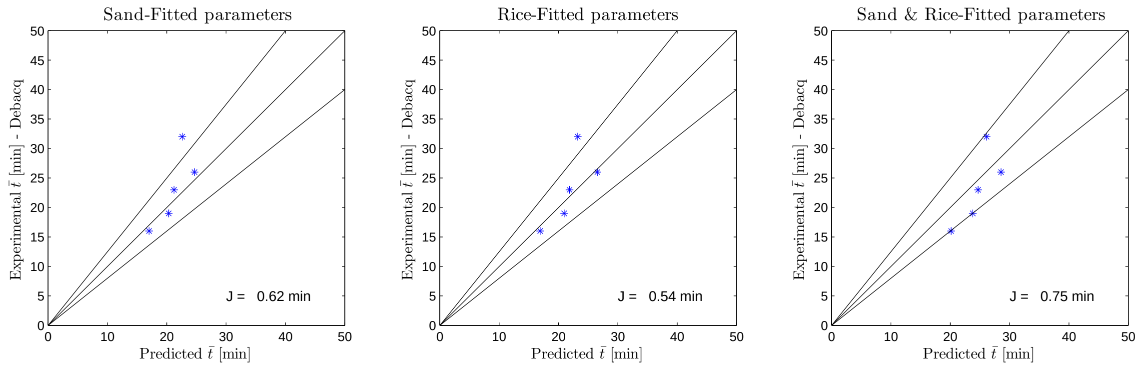


Fig. 6: Comparison of experimental MRT from Debacq [16] with predicted values from Eq.13 using the 3 sets of parameters as given in Table 4. Solid lines are ±20% margins.

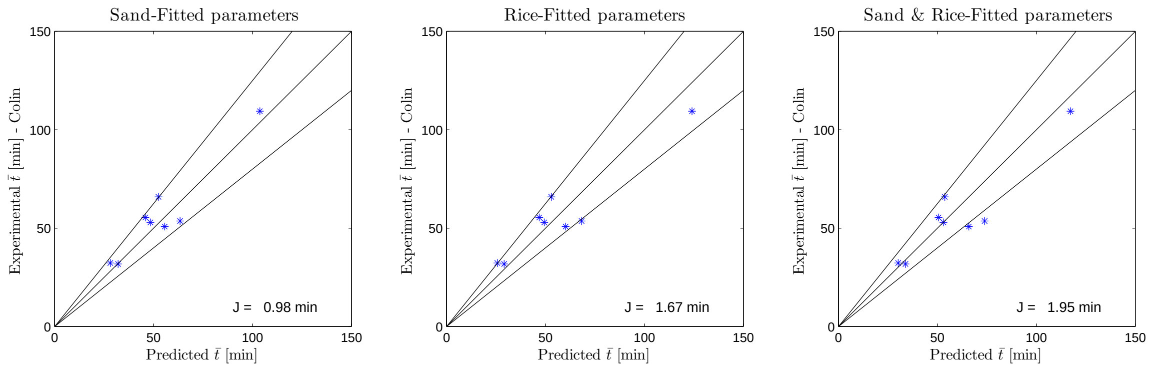
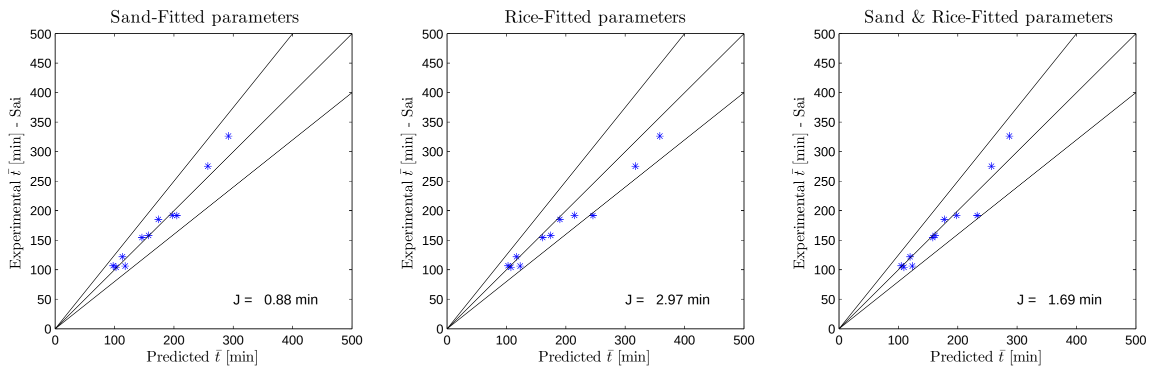
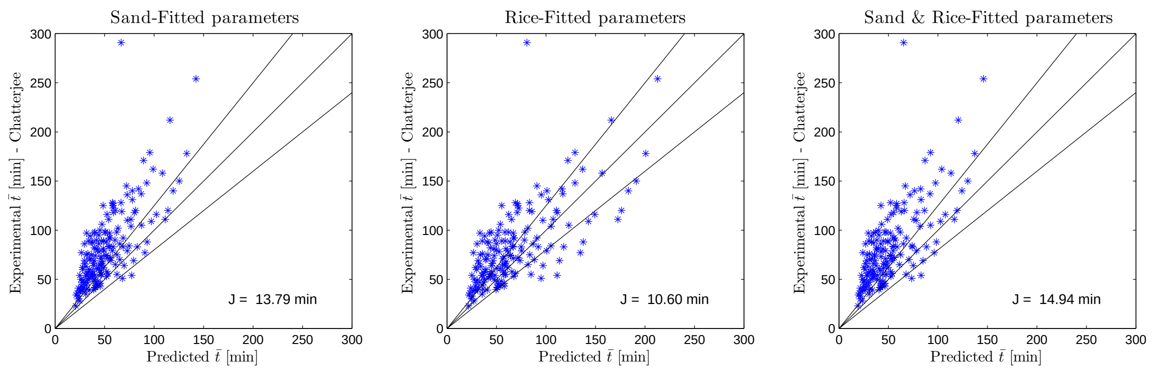


Fig. 7: Comparison of experimental MRT from Colin et al. [17] with predicted values from Eq.13 using the 3 sets of parameters as given in Table 4. Solid lines are ±20% margins.



(a) Sai et al. [18]



(b) Chatterjee et al. [2]

Fig. 8: Comparison of experimental MRT from a) Sai et al. [18] and b) Chatterjee et al. [2] with predicted values from Eq.13 using the 3 sets of parameters as given in Table 4. Solid lines are ±20% margins.

parameters" are revealed to be the best set of parameters with the lowest value of the criterion. The results are even better knowing that this industrial kiln was equipped with six rectangular lifters and processed high mass flow rate of cohesive particles .

The correlation was then used to calculate the MRT of beech particles flowing through the kiln used by Colin et al. [17], and operating without any internal fixtures or exit dam. It should be mentioned that this kiln is twice as big than the one use in the present study, so that these kilns have similar length-to-diameter ratio. The comparisons of predicted and experimental values of the MRT are given in Fig.7. Results show good agreements between the experimental and calculated MRT regardless of the set of parameters used. However, it should be noted that results while using the "sand-fitted parameters" give the lowest value of the performance criterion followed by the "rice-fitted parameters".

Other tests were carried out with data from rotary kilns equipped with dams of different height at their outlet end. These rotary kilns were those used by Sai et al. [18] and Chatterjee et al. [2] as previously mentioned. Fig.8 shows the results of the comparison of the experimental and predicted values of the MRT. Looking at the results obtained with the data from Sai et al. [18], the correlation gives good predictive performance when using either the "rice- or sand & rice-fitted parameters", but revealed an even better agreement with the experimental MRT with the "sand-fitted parameters", which gives a significant low criterion compared with the other results. Chatterjee et al. [2] achieved a huge experimental matrix including 5 types of dam, but their correlation for the MRT was not adapted for kilns fitted with a dam (see Eq.2). The comparisons of the predicted and experimental values of the MRT show some deviations. In this case the predictive performance of the model seems to be somehow diminished, as suggested by the elevated values obtained for the criterion J. But still about 30 to 50% of the results are within reasonable margins of the MRT experimentally measured. These latter poor results of the model can be attributed to (i) the difficulties in extrapolating to kilns of rotational speed lower than 2 rpm, leading most of time to small Froude numbers characteristic of beds in slipping motion [19], or perhaps (ii) the wide range of variation of the effective exit open diameter achieved by Chatterjee et al. [2] even if the results obtained in the case of Sai et al. [18] experiments were quite acceptable.

It appears, in accordance with the results of the present study, that the correlation proposed in this work for the MRT is also valid when applied to kilns with materials and operating conditions different from those used in the study. This confirms the excellent predictive capacity of Eq.13 compared to similar semi-empirical models which are often limited to a small range of conditions and operating variables. Regardless of the set of parameters, the predictions of the correlation are usually within reasonable margins of the experimental MRT. For that reason the authors recommend the use of the "sand & rice-fitted parameters".

4. Conclusions

The effects of operating parameters, namely, the rotational speed, the kiln slope, the mass flow rate, the exit dam height, the type of materials and shape of lifters, on the hold-up and mean residence time have been investigated. As far as these parameters are concerned, in this work it was observed that:

- Increasing either the rotational speed or the kiln slope increased the kiln hold-up and the MRT of the charge.
- Increasing the feed rate tended to increase the hold-up while the MRT of particles decreased. With an increase in the exit dam height these two latter gently increased.
- The use of rectangular lifters slightly increased the kiln hold-up and the MRT of particulate solids compared with straight or no lifters.
- Sand and broken rice bulks have close Hausner ratio though they have different bulk and tapped density and different particle size. Their MRT were relatively close but the sand MRT were always higher whereas the rice hold-ups were usually higher.

A semi-empirical model was developed in order to predict the mean residence time of solid materials in inclined rotating kiln equipped or not with lifters and exit dam under steady state conditions, as follows:

$$\begin{aligned}\bar{t} \;=\; & k\frac{\rho_{bulk}LD^2}{\dot{M}}\left(\frac{N^2D}{g}\right)^{\alpha}\left(\frac{D_{open}}{D}\right)^{\beta}(\theta)^{\gamma}(S)^{\delta} \\ & \left(\frac{\dot{M}}{\rho_{bulk}NLD^2}\right)^{\epsilon}\left(\frac{4S_{lift}}{\pi D^2}\right)^{\varepsilon}\left(\frac{\rho_{bulk}}{\rho_{tapped}}\right)^{\zeta}\left(\frac{L}{D}\right)^{\eta}\end{aligned}$$

The correlation consists of dimensionless factors accounting for (i) the kiln basic operational parameters, (ii) the bulk solids properties, (iii) the kiln geometry, (iv) the number, shape and presence of lifters, (v) the height of the exit dam. The parameters, namely k, $\alpha, \beta, \ldots, \zeta$ have been determined from experimental data. Authors recommend the use of the "sand & rice-fitted parameters" as given in Table 4, which give good agreements with experimental data from this study and published data as well. However, the sets of parameters presented here are only applicable to beds in cascading (tumbling) motion as defined by Mellmann [19]. Therefore, possible directions for further work could be (i) to extend the validity of the model to other types of bed motion with adequate set of parameters, (ii) to enrich the experimental data matrix with results from the use of new types of solid materials for a better confidence in the set of parameters determined.

List of symbols

C	Tracer concentration, sample conductivity	g/g, μS.cm^{-1}
D_{open}	Effective exit open diameter	m
d_p	Average particle size	mm
D	Kiln internal diameter	m
F, f_1,f_2,f_3	Function	-
F	Solids feed rate	kg.s^{-1}
$factor$	Fitting parameters	-
G	Gas flow rate	kg.s^{-1}
g	Gravitational acceleration	m.s^{-2}
HU	Hold-up	kg
k	Fitting parameters	-
J	Performance criterion	min
L	Kiln Length	m
Ls	Area occupied by solid particles contained in loading lifters	m^2
$\dot{M}$	Mass flow rate	kg.h^{-1}
MRT	Mean residence time	min
N	Kiln rotational speed	rpm
NL	No lifters	-
n_{lift}	Total number of lifters	-
N_S	Total number of samples	-
N_T	Total number of experiments	-
RL	Rectangular lifters	-
rpm	Rotation per minute	-
RTD	Residence Time Distribution	-
S	Kiln slope	degrees
SL	Straight lifters	-
S_{lift}	Area covered by solid particles in a lifter at horizontal position	m^2
$S_{horlift}$	Area covered by solid particles in a lifter at horizontal position	m^2
t	Time	min
$\bar{t}$	MRT	min
V_f	Volume of solids inside a lifter	m^3
X	hold-up volume fraction	-

Greek letters

Table A.5: Characteristics of differing kilns with their operating parameters range of variation as given in literature.

	L	D	L/D	Materials	Size	θ	ρ_{bulk}	ρ_{tapped}	N	S	$\dot{M}$	h	S_{lift}
	[m]	[m]	[-]	[-]	[mm]	[°]	$\left[kg.m^{-3}\right]$	$\left[kg.m^{-3}\right]$	[rpm]	[°]	$\left[kg.h^{-1}\right]$	[mm]	$\left[m^2\right]$
Bongo Njeng et al. [13]	1.95	0.10	19.50	Sand	0.4-0.8	39	1422	1543	2-12	2-5	0.69-2.5	0-33.5	0-0.008
				Rice	1.8-4.4	36	889	934					
Debacq [16]	6.85	0.75	9.13	UO_2F_2	0.02-0.5	42	400	500	1.6-2.1	1.43	508-895	0	0.438
				U_3O_8	0.01-0.05	33	1000	1300					
Colin et al. [17]	4.2	0.21	20.00	Beech	5-15	42	260	284	2-4	1-2	4-8	0	0
Sai et al. [18]	5.9	0.15	39.33	Ilmenite	1.1	27.4	4200	4410*	1-3	0.78-1.37	6-36	15-28	0
Chatterjee et al. [2]	2	0.3	6.67	Iron ore	3-6	35	2500	2625*	0.3-0.7	1-3	15-60	0-75	0

α,β,γ, δ,ϵ,ε, ζ,η	Fitting parameters	-
Δt_i	Sampling times	s
θ	Angle of repose	degrees
ρ_{bulk}	Bulk density	$kg.m^{-3}$
ρ_p	Particle density	$kg.m^{-3}$
ρ_{true}	Tapped density	$kg.m^{-3}$
τ	Time of passage	min

AppendixA. Geometrical characteristics and operating conditions of some kilns

Table A.5 gives the characteristics of different kilns from the literature, along with the range of their operating conditions and characteristics of the materials used. For some materials the tapped density was missing in the published papers (those marked with * in Table 2). For these cases a literature review was done to determine the ability of the solid particles to flow. The flow ability is influenced by the particle size; fine particles ($<$ 0.1 mm) tend to be more cohesive and thus less free-flowing, whereas larger, denser particles tend to be free-flowing. It was decided to used a Hausner ratio of 1.05, characteristic of a free flowing bulk, for both Ilmenite and Iron ore.

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