



HAL
open science

Identification of anisotropic tensile strength of cortical bone using Brazilian test.

Rachele Allena, Christophe Cluzel

► **To cite this version:**

Rachele Allena, Christophe Cluzel. Identification of anisotropic tensile strength of cortical bone using Brazilian test.. *Journal of the mechanical behavior of biomedical materials*, 2014, 38, pp.134-142. 10.1016/j.jmbbm.2014.06.007 . hal-01068673

HAL Id: hal-01068673

<https://hal.science/hal-01068673>

Submitted on 26 Sep 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Science Arts & Métiers (SAM)

is an open access repository that collects the work of Arts et Métiers ParisTech researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: <http://sam.ensam.eu>
Handle ID: <http://hdl.handle.net/10985/8625>

To cite this version :

Rachele ALLENA, Christophe CLUZEL - Identification of anisotropic tensile strength of cortical bone using Brazilian test. - Journal of the Mechanical Behavior of Biomedical Materials - Vol. 38, p.134-142 - 2014

Any correspondence concerning this service should be sent to the repository

Administrator : archiveouverte@ensam.eu

Identification of anisotropic tensile strength of cortical bone using Brazilian test

Rachele Allena^{a,*}, Christophe Cluzel^{a,b,c}

^a *Arts et Métiers ParisTech, LBM, 151 Boulevard de l'hôpital, 75013, Paris France*

^b *LMT-Cachan, 61 av. du Président Wilson, 94235 Cachan France*

^c *IUT-SGM, rue du PJarlan, 91025 Evry France*

Abstract

For a proper analysis of cortical bone behaviour, it is essential to take into account both the elastic stiffness and the failure criteria. While ultrasound methods allow complete identification of the elastic orthotropic coefficients, tests used to characterise the various failure mechanisms and to identify the brittle tensile strength in all directions are currently inadequate. In the present work we propose the Brazilian test as a complement to conventional tensile tests. In fact, this experimental technique, rarely employed in the biomechanics field, has the potential to provide an accurate description of the anisotropic strength of cortical bone. Additionally, it allows to assess the scale influence on failure behaviour which may be attributed to an intrinsic length in correlation with the cortical bone microstructure. In order to correctly set up the Brazilian test, several aspects such as the machining, the geometrical parameters of the specimen and the loading conditions were determined. The finite element method was used to evaluate the maximal

*Corresponding author. Tel: +33 (0)1 44 24 61 18; Fax: +33 (0)1 44 24 63 66
Email address: rachele.allena@ensam.eu (Rachele Allena)

tensile stress at the centre of a 2D anisotropic elastic specimen as a simple function of the loading. To validate the protocol, the Brazilian test was carried out on 29 cortical bovine cylindrical specimens with diameters ranging from 10 mm to 4 mm.

Keywords: Cortical bone, Anisotropy, Brazilian test, Brittle strength

1 **1. Introduction**

2 *1.1. Bone's structure and behaviour*

3 Bone presents a hierarchical structure (Currey, 2001) (Rho et al., 1998)
4 (Vashishth, 2007) which is organised in different levels as follows: i) the
5 macrostructure: cancellous and cortical bone, ii) the mesostructure (from 10
6 to 500 μm): haversian system, osteons, trabeculae and iii) the microstructure
7 (1-10 μm): the lamellae and the osteocytes iv) the nanostructure (from a few
8 hundred nanometers to 1 μm): fibrillar collagen and embedded mineral v) the
9 sub-nanostructure (below a few hundred nanometers): collagen, molecules
10 and proteins.

11 The complex structure of the bone has been the object of many studies
12 during the last decades in order to decipher the influence of each level on both
13 the mechanical and failure behaviour (Currey, 2001) . At the nanoscale, the
14 orientation of the collagen fibrils and their degree of mineralisation (Turner-
15 Walker and Parry, 1995) may affect the Young's modulus leading to a failure
16 stress in the fibres direction. At the microscale, the stacking of successive
17 lamellae, each composed by collagen fibres oriented in a single direction,
18 provides an isotropic mechanical behaviour in the lamellae plan, while weak
19 properties are observed along the perpendicular direction. At the mesoscale,

20 the osteons structure supplies a transverse isotropy for both the stiffness and
21 the failure stresses (Rho et al., 1998) (Ascenzi et al., 2012). Such a behaviour
22 is maintained at the macroscale due to the main orientation of the osteons
23 along the longitudinal axis of the bone. Finally, at this level the interface
24 between the osteons and the interstitial lamellae (the cement line) brings a
25 further weakness to the failure behaviour.

26 Conventional mechanical tests in traction, compression and torsion on
27 specimens obtained from cortical bone of the femur diaphysis were carried
28 out by Reilly and Burstein (1975). They actually observed that the Young's
29 modulus along the longitudinal direction is double that measured along the
30 circumferential or radial directions. Therefore, the anisotropy of the elastic
31 behaviour is clearly marked and complies with the geometrical organisation
32 of the bone at the mesoscopic scale.

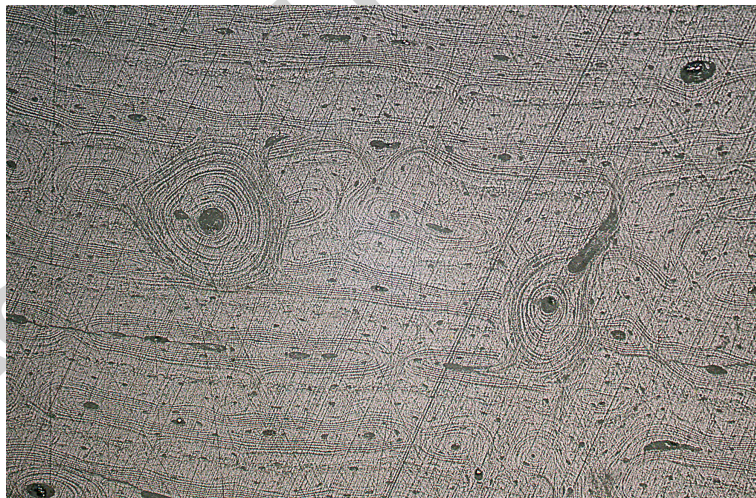


Figure 1: Bovine bone microstructure: sections perpendicular to the longitudinal axis

33 Nevertheless, this anisotropy is not limited to stiffness, it also influences

34 failure behaviour. As has been pointed out in (Norman and Wang, 1997)
35 (O'Brien et al., 2007) (MFeerick et al., 2013), the cement line is a source of
36 weakness that may enhance crack propagation. Similarly, the interface be-
37 tween two lamellae may reduce the failure threshold along their perpendicular
38 direction when several of them are aligned in a circumferential direction as
39 it is shown in Fig. 1 for cortical bone of a young bovine. In contrast, along
40 the longitudinal direction, the lamellae and the osteons are continuous and,
41 for longitudinal loading, rupture occurs with a very high stress. In parallel
42 to the analysis of failure mechanisms, many studies have focused on the fail-
43 ure criterion and have shown that taking into account the failure anisotropy
44 allows better predictive ability (Doblare et al., 2004). Nonetheless, these
45 criteria are very complex to identify experimentally. Additionally, Hashin
46 (1996) and Puck and Schürmann (1998) for fibre reinforced plastic (FRP)
47 composite and Arramon et al. (2000) for bone have pointed out that a mul-
48 ticriterion approach in which each function is related to a specific failure
49 mechanism is more suitable than a quadratic function defining an admissible
50 rupture domain. Therefore, it is essential to identify the failure mechanism
51 in order to determine which stress triggers the rupture.

52 1.2. Mechanical tests

53 During mechanical tests on brittle material, two different sets of parame-
54 ters can be measured: i) those describing the elastic behaviour and ii) those
55 describing the failure thresholds for each loading condition.

56 In order to identify the orthotropic elastic coefficients of cortical bone, it
57 is first necessary to perform traction or compression tests in the three main
58 directions as presented in Reilly and Burstein (1975) for a bovine femoral

59 cortical bone. Secondly, the shear elastic behaviour may be assessed through
60 Iosipescu or Arcan tests as described in [Xavier et al. \(2013\)](#), or by torsion
61 tests like those employed by [Reilly and Burstein \(1975\)](#). **Nonetheless, the**
62 **ultrasonic method presented in [Rho et al. \(1998\)](#) on bovine cortical bone**
63 **and the nano indentation used in [Hoc et al. \(2006\)](#) and [Vayron et al. \(2012\)](#)**
64 **may be very useful for a complete identification of the elastic parameters**
65 **and for studying the spatial variations of the modulus, respectively.** Addi-
66 tionally, resonant ultrasonic spectroscopy techniques ([Bernard et al., 2013](#))
67 have been recently employed for both human and bovine cortical bone and
68 have confirmed the previous results with high accuracy. For bovine cortical
69 bone, the values of Young's moduli along the circumferential and transverse
70 directions are of the order of 12.8 GPa, while the Young's modulus along the
71 longitudinal direction is about 20.3 GPa.

72 Several experimental tests may be used to evaluate the strength for a brit-
73 tle and anisotropic material like bone. Tensile testing is one of the classical
74 methods to measure bone's mechanical properties. Nevertheless, specimens
75 must have relatively large dimensions (15-20 mm in length, 4-8 mm in width)
76 and they must be specifically designed to obtain the majority of the strain
77 in the central region ([Reilly and Burnstein, 1974](#)) ([Ashman et al., 1987](#)).
78 If one assumes that the external force is applied without inducing a bend-
79 ing moment, the tensile test provides a good assessment of bone's strength,
80 but is limited in its ability to evaluate the effects of anisotropy due to the
81 constraints on the dimensions of specimens.

82 Bending tests are usually employed for testing the bones of small animals,
83 for which a tensile test is difficult to set up. In such a test, the entire

84 bone is loaded until failure leading to tensile stresses on one side of the
85 bone and compressive stresses on the other side. Additionally, tensile or
86 compressive stresses increase from the neutral axis to the external boundaries
87 of the specimen. Thus, failure commonly occurs on the tensile side since bone
88 is weaker in tension than in compression (Reilly and Burstein, 1975) and
89 may also be highly sensitive to surface defects due to the machining of the
90 specimens for instance. Bending may be applied to the bone through either a
91 three-point or a four-point loading. The former is very simple to set up, but
92 it may cause high shear stress around the middle section of the bone. The
93 latter induces pure bending and ensures zero transverse shear stress between
94 the two upper loading points. Nevertheless, if the specimen is rather small
95 in length and the bending moment is maximum under the loading point, the
96 stress state is not easy to determine. Furthermore, in both three-point and
97 four-point bending tests the total length of the specimen should be about
98 sixteen times the thickness of the specimen to guarantee that 85-90 % of
99 the bone flexion is actually due to bending. Unfortunately, this length-width
100 ratio cannot be acquired in whole bones such as femora or tibias.

101 For compression testing, relatively small specimens (7-10 mm long) can be
102 used and therefore machined along the three directions, but the measurement
103 tends to be less accurate than those for tensile tests because of edge effects. In
104 those regions in fact, the strain is likely to be higher than in the central region,
105 possibly due to the misalignment of the specimen faces or other problems
106 associated with specimen machining. Then, **because of friction between the**
107 **contact surfaces of the bone specimen and the plates of the testing machine,**
108 **one may have a unidirectional strain at the boundaries and a stress static**

109 state in the central region, such that the specimen acquires a barrel-like
110 shape. Although an extensometer is usually employed during tensile tests to
111 determine the axial strain in the specimen, this is not possible in compression
112 due to the small dimensions of the specimen. In this case image correlation
113 represents an alternative method to evaluate the stress-strain relationship.
114 Despite a lower accuracy of the results compared to tensile tests, compressive
115 testing presents some major advantages. First, specimens do not have to be
116 as large as tensile specimens. Second, machining of compressive specimens is
117 easier than for tensile specimens and may be done in different directions to
118 investigate the anisotropic behaviour of the bone. Nevertheless, compression
119 tests do not initiate the same failure modes as tensile tests (for which failure
120 mode and crack shape show a specific brittle mechanism).

121 In recent years, shear tests have been developed to determine the shear
122 modulus of elasticity of the bone. Among them we mention the rail shear test,
123 the torsion tube, cross-beam specimens and tension-compression of notched
124 specimens, including the Iosipescu (ASTM D5379) (Iosipescu, 1967) (Funk
125 and Litsky, 1998) (Sharma et al., 2011) and the Arcan tests (Arcan et al.,
126 1978).

127 Although the previous resistance tests allow partial assessment of the
128 anisotropic characteristics of cortical bone's behaviour and identification of
129 some fracture modes, they fail in evaluating the anisotropy in traction. For
130 this reason, here we propose the Brazilian test as an alternative experimental
131 approach to characterise the bone failure responses along the longitudinal,
132 circumferential and radial axes. Such a test presents interesting features,
133 which appear to be decidedly appropriate to study bone's mechanical be-

134 haviour and to obtain a complete predictive model.

135 *1.3. Brazilian test for brittle materials*

136 The Brazilian test was first introduced by Carneiro (1943) and Akazawa
137 (1943) to determine the tensile strength of brittle materials such as rock,
138 concrete or ceramic, which is difficult to evaluate by performing a direct uni-
139 axial tensile test. It is widely used in the field of civil engineering and has
140 been the object of numerous works for both the calculation of stresses and
141 the identification of material properties (Li et al., 2013). In the biomechanics
142 field, it has been employed to determine the tensile strength of archeologi-
143 cal cortical bone (Turner-Walker and Parry, 1995) and artificially aged bone
144 (Turner-Walker, 2011). Additionally, (Huang et al., 2012) proposed a nu-
145 merical analysis of the Brazilian test of heterogeneous specimens in order to
146 analyse the tensile strength of dental amalgams.

147 In the Brazilian test, a cylindrical specimen is loaded in compression
148 until failure over a short strip along the specimen length at each end of
149 the vertical diameter. Compression induces tensile stresses normal to the
150 loading direction, which are approximately constant within a region around
151 the centre. Therefore, for a brittle material, a crack appears perpendicular to
152 the maximum traction stress direction, leading to the splitting of the cylinder
153 into two halves.

154 The Brazilian test has some interesting characteristics. Firstly, it greatly
155 simplifies the traction loading of a brittle material. Secondly, it permits
156 reduction of the size of the specimen down to that limited by testing a rep-
157 resentative volume of the material. For the specific case of cortical bone,
158 such a reduction in dimensions (e.g. some millimetres in diameter) leads to

159 three further benefits: i) it decreases the probability of finding very large
160 defects that may induce macroscopic rupture, ii) it provides information on
161 the correlation between specimen size and defect distribution and iii) it en-
162 ables the analysis of the traction fracture along the three main axes of the
163 bone. Therefore, the Brazilian test may be employed to provide an accurate
164 identification of the anisotropic maximal traction stresses in cortical bone.

165 2. Materials and methods

166 2.1. Sample preparation

167 Specimens were obtained from a bovine tibia sourced from a local butcher
168 and conserved at -18° . Once the tibia was defrosted, the internal marrow
169 and spongy bone were removed and the bone was cleaned with water. The
170 three main local axes of the bone were chosen as follows (Fig. 2):

- 171 • the longitudinal axis x_1 corresponds to the main direction of the tibia;
- 172 • the circumferential axis x_2 coincides with the azimuthal direction;
- 173 • the radial axis x_3 is aligned with the outward radius of the bone's
174 section.

175 First, 25 bone cylinders were machined using diamond-tipped tubular
176 drills of internal diameters ϕ 10, 8, 6 and 4 mm. For the sake of convenience,
177 the machining was performed along the x_1 and x_3 directions, which maintains
178 the ability to obtain the three fracture stresses σ_{11}^f , σ_{22}^f and σ_{33}^f ((Fig. 2),
179 the superscript f indicates failure). Second, the cylinders were sectioned
180 perpendicular to the cylinder axis using a diamond disc saw. **Furthermore, for**

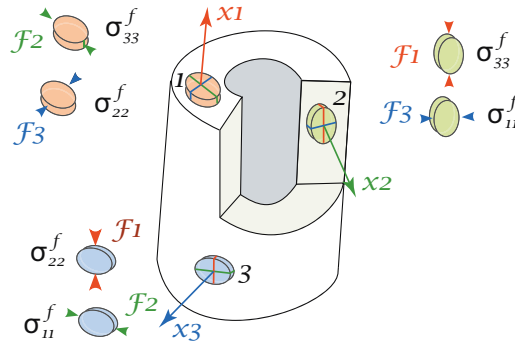


Figure 2: Coordinates system, traction stresses (σ_{ii}^f) and loading directions (F_j).

181 those machined along the x_1 direction, more than one specimen was obtained.
 182 Finally, 29 specimens were acquired. The length L of the samples was set to
 183 6.5 mm, 5.2 mm, 3.9 mm and 2.6 mm respectively for $\phi = 10$ mm, $\phi = 8$ mm,
 184 $\phi = 6$ mm and $\phi = 4$ mm (Fig. 3). Such values provide a minimal average
 185 ratio ϕ/L equal to 1.54. Before sectioning, the three main axes x_1 , x_2 and
 186 x_3 were identified on each specimen which allows to classify the specimens
 187 as follows: $x_i_F_j$, with x_i and F_j indicating the cutting axis and the loading
 188 direction, respectively (Fig. 2). During cutting, water was used in order to
 189 reduce both friction and temperature rise.

190 2.2. Brazilian test for cortical bone

191 The Brazilian tests were performed at room temperature right after the
 192 cutting, using a universal traction-compression machine INSTRON 5500-R
 193 equipped with a 5 KN sensor. We have assumed that the room humidity does
 194 not influence the specimens behaviour. The machine was controlled by fixing
 195 the displacement rate of the upper plate at 0.2 mm/min. The positioning of
 196 the specimen between the two plates of the machine as shown in Fig. 4 must

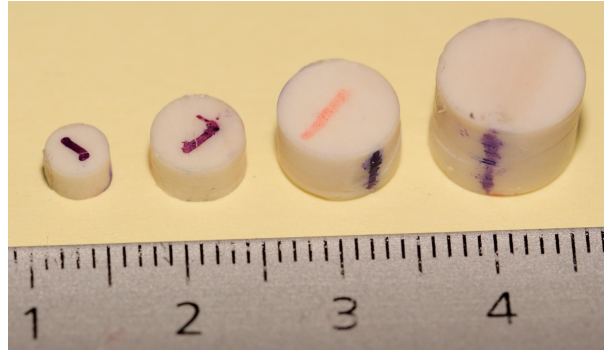


Figure 3: Specimens diameters: 4, 6, 8 and 10 mm.

197 be done very carefully since it was necessary i) to align the cylinder with
 198 respect to the mid-planes of the plates, ii) to orient the cylinder along the
 199 main axis of the machine and iii) to place the cylinder in the central region
 200 of the lower plate. Such conditions may not been verified if, for instance,
 201 there exists a parallelism or a cylindricality defect of the specimen, which
 202 may influence the stress distribution.

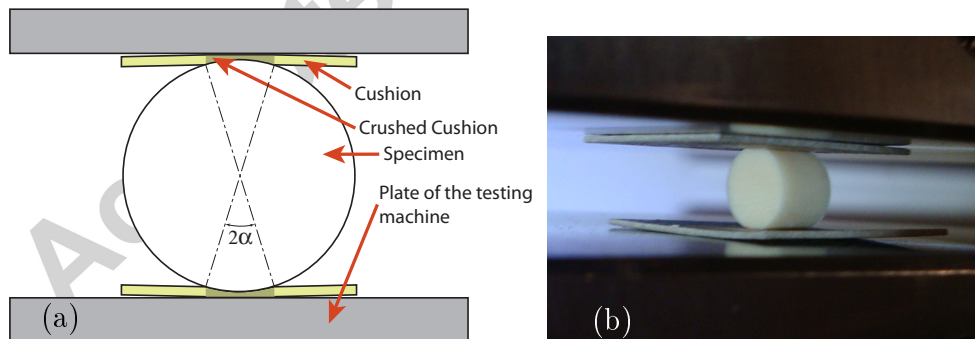


Figure 4: Schematic drawing of the Brazilian test (a) and positioning of a 4 mm diameter specimen (b).

203 During a regular test, the crack was generally initiated at the centre of

204 the cylinder along the vertical axis (Fig. 5). Nevertheless, abnormal split-
 205 ting might be observed due to i) shear stress (Fig. 6a), ii) crushing issues
 206 (Fig. 6b) or iii) a non centred crack. Defects such as those presented in Fig.
 207 6a were mainly found during a preliminary series of tests with specimens
 208 having a ratio $\phi/L < 1.54$. To limit the crushing of the contact surface
 209 (Fig. 6b), a cushion can be inserted between the specimen and each load
 210 plate as described in the standard for Brazilian tests applied to rocks (ISRM
 211 1978, ASTM 2008). In our case, a 0.52 mm thick square of cardboard was
 212 used (Fig. 4). The imprint was measured after each test to estimate the
 213 contact area and we found that it can actually be defined independently of
 214 the specimen diameter ϕ through the angle α (Fig. 4a) as described in Wang
 215 et al. (2004).

216 2.3. Structural analysis of Brazilian test for anisotropic elastic behaviour

217 Through a structural analysis, we may be able to evaluate the maximal
 218 tensile stress $\sigma_{xx,max}$ along the x direction at the centre of each specimen.
 219 For isotropic materials, an analytical solution was proposed by Peltier (1954)
 220 giving the tensile stress in the centre of the disc as follows

$$\sigma_{xx,max} = \frac{2F}{\phi L \pi} \quad (1)$$

221 where F is the applied load.

222 To account for the effect of a soft cushion between the specimen and the
 223 loading plates, a factor of correction k was introduced by Hondros (1959)
 224 and Wang et al. (2004) as a function of the angle α (Fig. 7). Thus, Eq. [1]
 225 becomes :



Figure 5: Appearance of a vertical crack at the centre of a specimen $x_1_F_3$ with a 8.12 mm diameter.

$$\sigma_{xx,max} = k(\alpha) \frac{2F}{\phi L \pi} \quad (2)$$

226 Nonetheless, the previous relation is no longer valid for an anisotropic
 227 elastic behaviour as for the cortical bone. In [Exadaktylos and Kaklis \(2001\)](#),
 228 the authors propose an analytical approach in the form of a sum of Fourier
 229 series, which is validated for the isotropic case by comparing it with the
 230 results of [Hondros \(1959\)](#). In the present work, in order to have an extensive

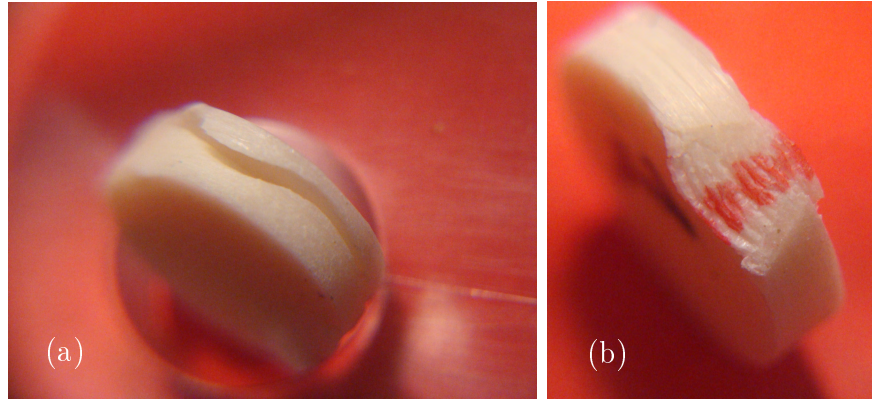


Figure 6: Examples of abnormal splitting due to shear stress (a) and matting (b).

231 overview of anisotropy effects, the definition of the maximum tensile stresses
 232 (Fig. 2) is similar to that proposed in Eq. [2], but the coefficient of correction
 233 is now expressed as a function of both the direction of the failure stress and
 234 of the loading. Thus, we have:

$$\sigma_{ii,max} = \beta_{ii-j} \frac{2F_j}{\phi L \pi} \quad (3)$$

235 where β_{ii-j} is the correction factor and F_j is the applied vertical load.
 236 The subscripts ii and j indicate the principal stresses and loading direction,
 237 respectively. The main objective of the structural analysis is to find the
 238 coefficient β_{ii-j} for the different directions independently of the specimen
 239 diameter ϕ .

240 In the present study, the analysis was performed using the finite elements
 241 (FE) method, which provides a better validation and simplify the manage-
 242 ment of various input and output data. The FE software COMSOL 3.5a
 243 was used to run two dimensional (2D) simulations and to evaluate the linear

244 elastic stress field within the samples along x_1 , x_2 and x_3 with two loading
 245 directions each. The cylindrical specimens were represented as circles with an
 246 anisotropic elastic behaviour. The elastic material parameters were deduced
 247 from [Bernard et al. \(2013\)](#) (Table 1).

Modulus	Value	Reference or formula
E_1	20.3 GPa	(Bernard et al., 2013)
E_2	12.8 GPa	(Bernard et al., 2013)
E_3	12.8 GPa	(Bernard et al., 2013)
G_{12}	6.38 GPa	(Bernard et al., 2013)
G_{13}	6.32 GPa	(Bernard et al., 2013)
G_{23}	6.38 GPa	(Bernard et al., 2013)
ν_{12}	0.421	(Bernard et al., 2013)
ν_{13}	0.434	(Bernard et al., 2013)
ν_{23}	0.348	(Bernard et al., 2013)
ν_{21}	0.265	$\frac{\nu_{12}E_2}{E_1}$
ν_{31}	0.273	$\frac{\nu_{13}E_3}{E_1}$
ν_{32}	0.348	$\frac{\nu_{23}E_3}{E_2}$

Table 1: Elastic parameters sourced and deduced from [Bernard et al. \(2013\)](#).

248 The problem was solved using the plane stress hypothesis. The displace-
 249 ment of the point A was constrained along the x direction while the point B
 250 was totally constrained to prevent rigid body motion (Fig. 7). Finally, the
 251 vertical load was applied along the upper and lower boundaries l (blue lines
 252 in Fig. 7), which, as previously explained (Sec. 2.2), have been calculated

253 using the angle $\alpha = 14^\circ$. Thus, l is equal to 2.4 mm, 1.9 mm, 1.4 mm and
 254 1 mm for $\phi = 10$ mm, $\phi = 8$ mm $\phi = 6$ mm and $\phi = 4$ mm, respectively.

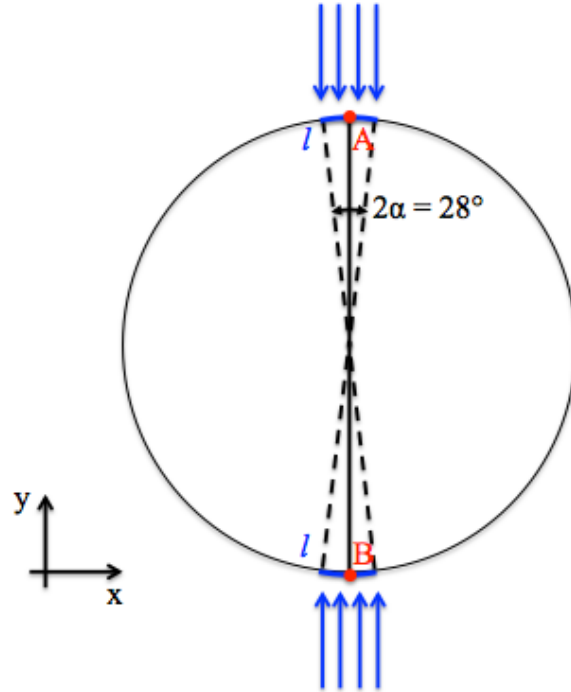


Figure 7: Boundary conditions for the simulation of the Bazilian test in COMSOL 3.5a.

255 2.4. Sensitivity analysis

256 The correction factor β_{ii-j} may change with respect to the elasticity co-
 257 efficients. Therefore, a sensitivity analysis was performed for each specimen
 258 by varying the Young's moduli and Poisson's ratios by $\pm 10\%$ relative to the
 259 'benchmark' values.

260 **3. Results**

261 *3.1. Stress state in the loaded specimen*

262 In this section we present the numerical results and in particular we dis-
 263 cuss the stress field inside the specimen. As it is possible to observe in Fig. 8a
 264 and 8b, for a load per length unit $F = 1400$ N/mm (which is the same for
 265 each tested diameter), the compressive (σ_{yy}) and tensile (σ_{xx}) stresses are
 266 heterogeneously distributed. Their pattern is very similar to that of the
 267 isotropic case as reported in Wang et al. (2004) and specifically σ_{yy} and σ_{xx}
 268 are maximal along the loading surfaces and at the centre, respectively.

269 Actually, there exists a relationship between such stresses and the failure
 270 mechanism. In fact, as shown in Fig. 5, the crack is distinctly open at the
 271 centre of the disc ($(x,y) = (0,0)$) where the stress state is plane and given by

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (4)$$

272 with $\sigma_{xx} = 55$ MPa and $\sigma_{yy} = -147$ MPa (blue line in Fig. 8d and 8b,
 273 respectively).

274 Let \underline{n} and \underline{t} be respectively the normal and the tangent vectors to the failure
 275 plane defined as

$$\underline{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \quad \text{and} \quad \underline{t} = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} \quad (5)$$

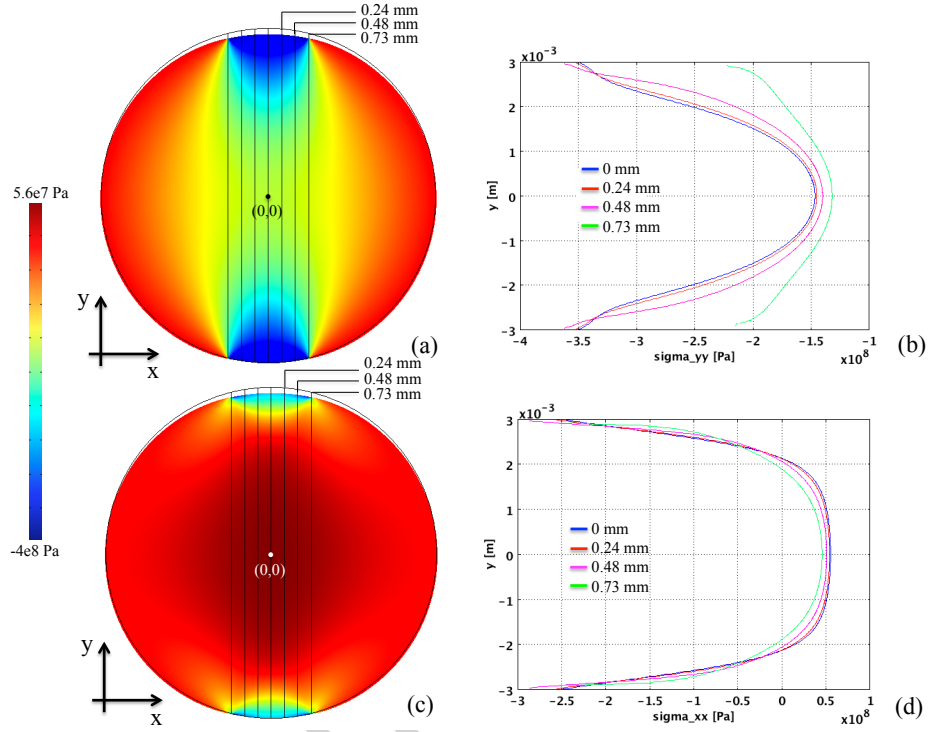


Figure 8: (a, c) Plot of σ_{yy} and σ_{xx} , respectively for a $x_1_F_2$ specimen of diameter 6mm. (b, d) Outline of σ_{yy} and σ_{xx} respectively along the vertical diameter (blue line, $x = 0$) and along vertical lines placed at $x = 0.24$ mm (red line), $x = 0.48$ mm (purple line) and $x = 0.73$ mm (green line).

276

277 where θ is the angle between \underline{n} and the \underline{x} axis. Then, the normal (σ_n) and
 278 the shear (τ) stresses read

$$\sigma_n = \underline{n}^{tr} \underline{\underline{\sigma}} \underline{n} \quad (6)$$

$$\tau = \underline{t}^{tr} \underline{\underline{\sigma}} \underline{n} \quad (7)$$

279 with \underline{n}^{tr} the transposition of \underline{n} .

280 It is interesting to evaluate the evolution of σ_n and τ for i) θ varying between
 281 0° and 180° and ii) the axial coordinate x of the point of interest (x,y) varying
 282 between ± 0.73 mm from the centre of the disc (Fig. 9).

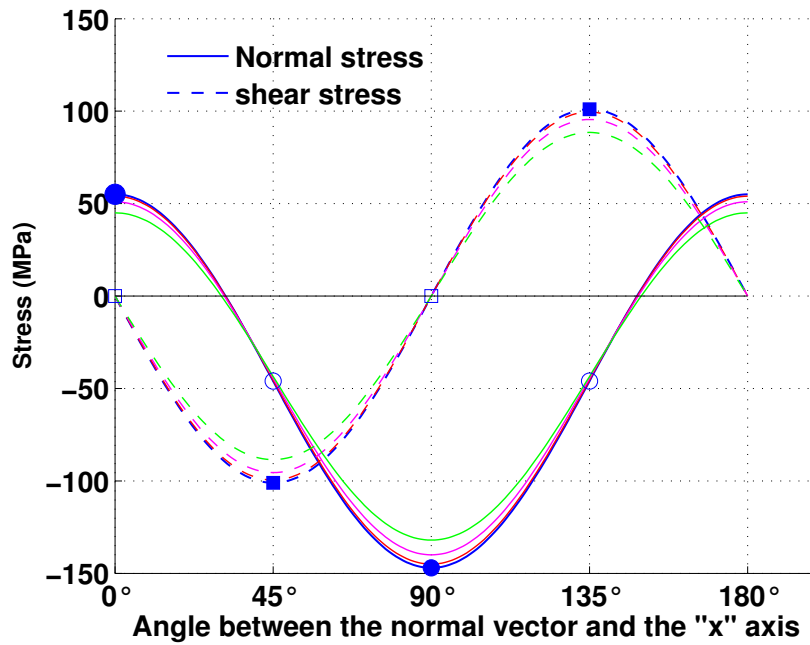


Figure 9: Normal and shear stress distribution at the centre and at 0.24 mm (red lines), 0.48 mm (purple lines) and 0.73 mm (green lines) from the centre along the x axis.

283 For $\theta = 0^\circ$, we find $\sigma_n = 55$ MPa and no shear stress, while τ is maximal
 284 (± 100 MPa) for $\theta = 45^\circ$ and 135° . Finally, for $\theta = 90^\circ$, σ_n is equal to -147
 285 MPa showing a compressive stress state (Fig. 9). It can be noticed that for

286 all these stresses, the maximal values are found at the centre of the disc (blue
 287 line in Fig. 8b, 8d, 9). For a brittle material, the failure plane is a useful
 288 parameter to evaluate the cracking mechanism and the corresponding stress.
 289 Here, failure is not activated at $\theta = 90^\circ$ nor at $\theta = 45^\circ$. On the contrary, the
 290 traction stress σ_{xx} is assumed to be responsible for the failure each time the
 291 crack occurs parallel to the loading axis.

292 The main objective of the numerical simulations was to evaluate the cor-
 293 rection factor β_{ii-j} defined in Eq. [3], which is independent of the diameter
 294 ϕ of the disc. For an isotropic material, we found that such a coefficient is
 295 equal to 1 in the case of a concentrated load F_j and to 0.912 in the case of
 296 a distributed load as described in Sec. 2.3, which is very close to 0.92, the
 297 coefficient analytically calculated from Wang et al. (2004).

298 In order to use β_{ii-j} as a consistent indicator, the variation of the stress
 299 state must be low with respect to the cracking position. In Fig. 8b and 8d
 300 σ_{yy} and σ_{xx} respectively are plotted for a plane placed at $x = 0, 0.24, 0.48$
 301 and 0.73 mm for a disc with a diameter of 6mm. We notice that if the crack
 302 occurs between ± 0.4 mm from the vertical axis of the disc, the maximum
 303 stress only varies by about ± 5.5 %. To keep such a low variability, the
 304 corresponding spatial tolerances for $\phi = 4, 8$ and 10 mm are $\pm 0.27, \pm 0.4$ and
 305 ± 0.67 mm, respectively. As an example, in Fig. 5, the diameter ϕ of the
 306 specimen is equal to 6 mm and the position of the crack is at 0.16 mm from
 307 the centre with an error of -1 % for the coefficient β_{ii-j} .

308 Finally, as mentioned in Sec. 2.4, a source of uncertainty for the correction
 309 factor β_{ii-j} is related to the variations of the elastic coefficients. According
 310 to the sensitivity analysis that has been carried out, the results for the four

311 loading cases are reported in Table 2.

Specimen	σ_{11}^f	σ_{22}^f	σ_{33}^f
$x_1_F_2$	-	-	$\beta_{33-2} = 1.007 \pm 4\%$
$x_1_F_3$	-	$\beta_{22-3} = 1.007 \pm 4\%$	-
$x_2_F_1$	-	-	$\beta_{33-1} = 0.802 \pm 5\%$
$x_2_F_3$	$\beta_{11-3} = 1.044 \pm 3.5\%$	-	-
$x_3_F_1$	-	$\beta_{22-1} = 0.802 \pm 5\%$	-
$x_3_F_2$	$\beta_{11-2} = 1.045 \pm 3.5\%$	-	-

Table 2: Results of the sensitivity analysis and values of correction factors β_{ii-j} for the four loading cases.

312 3.2. Experimental data

313 The experimental tests were exploited to assess the failure force as well
 314 as the crack direction, which must be vertical, and shape, which must be
 315 sharp-cut. Furthermore, by using the correction coefficients β_{ii-j} derived
 316 from the numerical analysis (Table 2), the values of the tensile failure stress
 317 were determined depending on the specimen diameter ϕ for each direction of
 318 failure stress tested σ_{11}^f , σ_{22}^f and σ_{33}^f (Fig. 10). Among the 29 tests carried out,
 319 4 were stopped due to a crushing problem on the loading area (Sec. 2.2) and
 320 4 presented a cracking mechanism outside the admissible region (Sec. 3.1)
 321 (hollow arrows in Fig. 10). For these specific cases, stress leading to failure
 322 was not usable as a value to rupture, but as an underestimation of the failure
 323 stress.

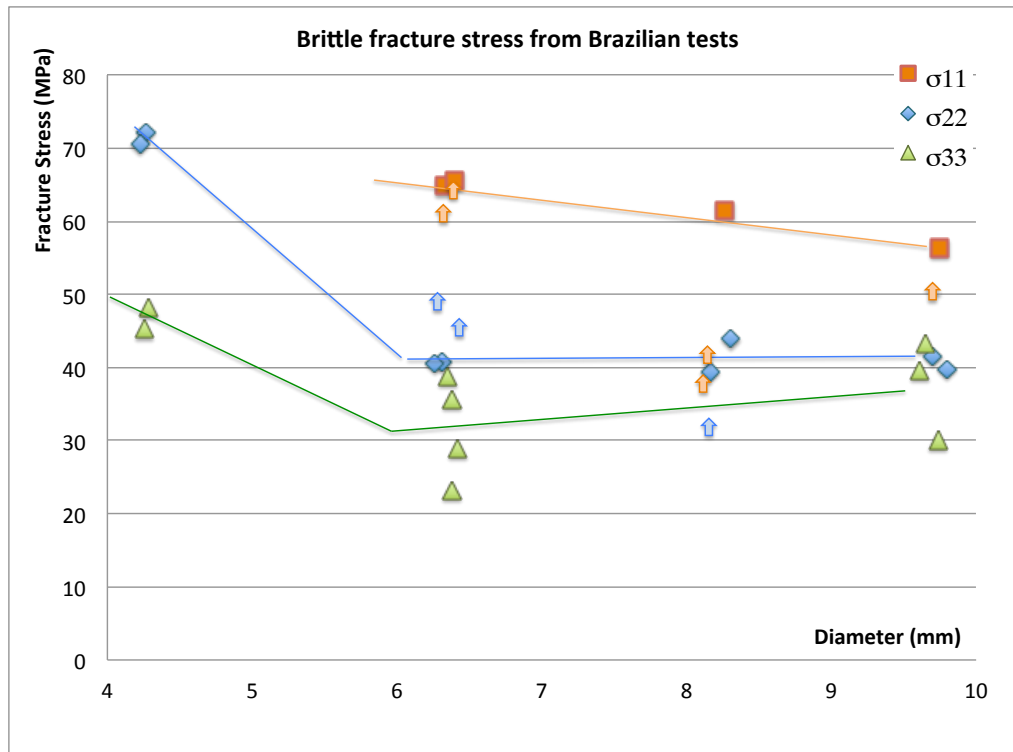


Figure 10: Maximum tensile stress versus diameter for three directions of loading.

324 The brittle strength is anisotropic for all the tested diameters and signif-
 325 icantly higher along the axial direction. According to Fig. 10, the size of the
 326 specimen may influence the failure stress. For instance, for specimens with a
 327 diameter of 4 mm we observe an increase of the failure stresses. However, for
 328 $\phi = 6, 8$ and 10 mm, failure stresses are in the same order of magnitude along
 329 each direction and the average values are respectively equal to $\sigma_{11}^f = 62$ MPa,
 330 $\sigma_{22}^f = 41$ MPa and $\sigma_{33}^f = 34$ MPa.

331 As the traction stress σ_{xx} is not homogeneous within the sample (Fig. 8d),
 332 it may be of interest to identify a failure region for each specimen diameter

333 rather than simply determining the relationship between the failure stress and
 334 the sample dimensions. Thus, a rectangular area $S_{failure}$ of height $h_{failure}$
 335 and width $e_{failure}$ can be defined for each diameter ϕ such that $0.9\sigma_{xx,max} <$
 336 $\sigma_{xx} < \sigma_{xx,max}$. We notice that the dimensions and consequently the area of
 337 the failure region decrease with the specimen diameter (Table 3).

Specimen diameter ϕ (mm)	4	6	8	10
Failure region height $h_{failure}$ (mm)	1.5	2.2	2.9	3.7
Failure region width $e_{failure}$ (mm)	0.8	1.2	1.6	2
Failure region area $S_{failure} = h \cdot e$ (mm ²)	1.2	2.64	4.64	7.4

Table 3: Values of the failure region area according to the specimen diameter ϕ .

338 4. Discussion

339 The Brazilian test is suitable for brittle materials only, but the experi-
 340 mental validation of the failure mechanism is very easy to achieve because
 341 the crack must be unique and in a vertical plane as described in [Tavallali and](#)
 342 [Vervoort \(2010\)](#). Additionally, if the ϕ/L ratio is controlled and optimised,
 343 the rare faulty tests may be attributed to machining or positioning defects. In
 344 the present work, although the bovine cortical bone we tested seemed rather
 345 young with a marked microstructure, the experimental dispersion was quite
 346 reasonable and the anisotropy of brittle fracture clearly appeared leading to
 347 a ratio $\sigma_{ii,max}/\sigma_{ii,min}$ of the order of 2.

348 For elastic isotropic materials, the fairly simple geometry of the specimen
 349 used for the Brazilian test allows the existence of analytical descriptions

350 of the stress field either for a concentrated or a distributed load. In this
351 case, the analytical solution and our numerical simulation were in very good
352 agreement. Specifically, for a concentrated load, the correction coefficient
353 β_{ii-j} defined in Eq. [3] is exactly equal to 1, while for a distributed load as
354 described in Sec. 2.3, β_{ii-j} is equal to 0.912.

355 For an anisotropic material such as cortical bone, the elastic coefficients
356 deduced from Bernard et al. (2013) were used to run the numerical simula-
357 tions for specimens of different diameters. We were able to determine the
358 correction factors β_{ii-j} associated to each failure stress and we found that all
359 the coefficients are between 0.802 and 1.05 or in a range of $0.92 \pm 14\%$. This
360 results in a variation of the maximum stress of the order of 14%. Further-
361 more, according to the sensitivity analysis we performed, the uncertainties
362 on β_{ii-j} due to the variation of the elastic parameters are not higher than 5
363 %, which is quite low. Therefore, the coefficients can be directly used or, for
364 better accuracy, recalculated after verification of the rigidity by, for example,
365 an ultrasonic method.

366 The Brazilian test also allowed us to assess the scale influence on failure
367 mechanism. The areas of the failure regions for the different specimens re-
368 ported in Table 3 are very small for a tensile test on a brittle material, which
369 results in failure stresses for specimens with a diameter of 4 mm higher than
370 those for larger samples (Fig. 10). Previous works have focused on this spe-
371 cific aspect and have used either a Weibull distribution of the defect size
372 (Fok et al., 2001) or a cohesive crack model (Guinea et al., 2000) to describe
373 such a behaviour. In both cases, the size effect is attributed to an intrinsic
374 length in correlation with the microstructure of the material, below which

375 the failure stress increases. This might also be the case for cortical bone. In
376 fact, we can see that as the specimen diameter ϕ decreases, the dimensions
377 $h_{failure}$ and $e_{failure}$ of the failure region decrease too (Table 3) and approach
378 the dimensions of a portion of the cement line (Sec. 1.1), which may consti-
379 tute a weakness for failure behaviour as mentioned in (Norman and Wang,
380 1997) (O'Brien et al., 2007) (MFeerick et al., 2013).

381 According to the previous remarks, it would be interesting to perform
382 the Brazilian test on a large number of specimens within a range of small
383 dimensions. In fact, this would allow to consistently investigate the scale
384 influence and the statistical dispersion and to characterise a suitable nonlocal
385 model to be adopted for numerical simulations.

386 5. Conclusion

387 In this paper we have proposed the Brazilian test as an alternative tech-
388 nique to investigate both the anisotropic strength and failure mechanism
389 of cortical bone. In fact, although this test has rarely been employed in the
390 field of bone biomechanics (Turner-Walker and Parry, 1995) (Turner-Walker,
391 2011) (Huang et al., 2012), it presents some interesting features. Firstly, it
392 allows testing of brittle materials in traction through the use of a compressive
393 load. Secondly, it allows to reduce the specimen dimensions down to those of
394 the representative volume of the material. Then, for specific case of cortical
395 bone it has been possible to assess the tensile failure along its three main
396 axes and its anisotropy.

397 6. Bibliography

- 398 Akazawa, T., 1943. A new test method for evaluating internal stress due to
399 compression of concrete: the splitting tension test. *J. Japan. Soc. Civil.*
400 *Eng.* 19, 777–787. [8](#)
- 401 Arcan, M., Hashin, Z., Voloshin, A., 1978. A method to produce uniform
402 plane-stress states with applications to fiber-reinforced materials. *Exp.*
403 *Mech.* 18 (141-146). [7](#)
- 404 Arramon, Y. P., Mehrabadi, M. M., Martin, D. W., Cowin, S. C., 2000.
405 A multidimensional anisotropic strength criterion based on a multidimen-
406 sional anisotropic strength criterion based on kelvin modes. *International*
407 *Journal of Solids and Structures* 37, 2915±2935. [4](#)
- 408 Ascenzi, M.-G., Kavas, N. P., Lutz, A., Kardas, D., Nackenhorst, U., Keyak,
409 J. H., 2012. Individual-specific multi-scale finite element simulation of cor-
410 tical bone of human proximal femur. *Journal of Computational Physics.*
411 [3](#)
- 412 Ashman, R., Cowin, S., Buskirk, W. V., Rice, J., 1987. Elastic properties of
413 cancellous bone: Measurement by an ultrasonic technique. *J. Biomech.* 17,
414 349–361. [5](#)
- 415 Bernard, S., Grimal, Q., Laugier, P., 2013. Accurate measurement of cortical
416 bone elasticity tensor with resonant ultrasound spectroscopy. *Journal of*
417 *the Mechanical behavior of biomedical materials* 18, 12–19. [5](#), [15](#), [24](#)
- 418 Carneiro, 1943. A new method to determine the tensile strength of concrete.
419 In: *Proceedings of the 5th meeting of the Brazilian Association for Tech-*
420 *nical Rules*, 3d. section. [8](#)

- 421 Currey, J. D., 2001. Bones: structure and mechanics. Princeton University
422 Press. 2
- 423 Doblare, M., Garcia, J., Gomez, M., 2004. Modelling bone tissue fracture
424 and healing: a review. *Engineering Fracture Mechanics* 71, 1809–1840. 4
- 425 Exadaktylos, G., Kaklis, K., 2001. Applications of an explicit solution for the
426 transversely isotropic circular disc compressed diametrically. *International*
427 *Journal of Rock Mechanics and Mining Sciences* 38, 227–243. 13
- 428 Fok, S., Mitchell, B., Smart, J., Marsden, B., 2001. A numerical study on the
429 application of the weibull theory to brittle materials. *Engineering Fracture*
430 *Mechanics* 68, 1171–1179. 24
- 431 Funk, M., Litsky, A., 1998. Effect of cement modulus on the shear properties
432 of the bone-cement interface. *Biomaterials* 19, 1561–1567. 7
- 433 Guinea, G., Elices, M., Planas, J., 2000. Assessment of the tensile strength
434 through size effect curves. *Engineering Fracture Mechanics* 65, 189–207.
435 24
- 436 Hashin, Z., 1996. Finite thermoelastic fracture criterion with application to
437 laminate racking analysis. *J. Mech. Phys. Solids* 44 (7), 1129–1145. 4
- 438 Hoc, T., Henry, L., Verdier, M., Aubry, D., Sedel, L., meunier, A., 2006.
439 Effect of microstructure on the mechanical properties of haversian cortical
440 bone. *Bone* 38, 466–474. 5
- 441 Hondros, G., 1959. The evaluation of poisson's ratio and the modulus of
442 materials of a low tensile resistance by the brazilian (indirect tensile) test

- 443 with particular reference to concrete. *Journal of applied science* 10 (3),
444 243–268. [12](#), [13](#)
- 445 Huang, S., L.S. Lin, A. F., Lin, C., 2012. Diametral compression test with
446 composite disk for dentin bond strength measurement – finite element.
447 *Dent. Mater.* 28, 1098–1104. [8](#), [25](#)
- 448 Iosipescu, N., 1967. New accurate procedure for single shear testing of metals.
449 *J. Mater.* 2, 537–566. [7](#)
- 450 Li, S., Demirci, E., Silberschmidt, V. V., 2013. Variability and anisotropy of
451 mechanical behavior of cortical bone in tension and compression. *Journal*
452 *of the Mechanical behavior of biomedical materials* 21, 109–120. [8](#)
- 453 MFeerick, E., Liu, X. C., McGarry, P., 2013. Anisotropic mode-dependent
454 damage of cortical bone using the extended finite element method (xfem).
455 *Journal of the Mechanical behavior of biomedical materials* 20, 77–89. [4](#),
456 [25](#)
- 457 Norman, T. L., Wang, Z., 1997. Microdamage of human cortical bone: Inci-
458 dence and morphology in long bones. *Bone* 20 (4), 375–379. [4](#), [25](#)
- 459 O'Brien, F. J., Taylor, D., Lee, T. C., 2007. Bone as a composite material:
460 The role of osteons as barriers to crack growth in compact bone. *Interna-*
461 *tional journal of fatigue* 29, 1051–1056. [4](#), [25](#)
- 462 Peltier, R., 1954. Theoretical investigation of the brazilian test. *Rilem Bull*
463 19, 26–69. [12](#)

- 464 Puck, A., Schürmann, H., 1998. Failure analysis of FRP laminates by means
465 of physically based phenomenological models. *Composites Science and*
466 *Technology* 58, 1045–1067. [4](#)
- 467 Reilly, D., Burnstein, A., 1974. The elastic modulus for bone. *J. Biomech.* 7,
468 271–275. [5](#)
- 469 Reilly, D., Burstein, A., 1975. The elastic and ultimate properties of compact
470 bone tissue. *J. Biomech.* 8, 393–405. [3](#), [4](#), [5](#), [6](#)
- 471 Rho, J.-Y., Kuhn-Spearing, L., Zioupos, P., 1998. Mechanical properties and
472 the hierarchical structure of bone. *Medical Engineering and Physics* 20,
473 92–102. [2](#), [3](#), [5](#)
- 474 Sharma, N., Sehgal, D., Pandey, R., 2011. Studies on locational variation of
475 shear properties in cortical bone with iosipescu shear test. *Applied Me-*
476 *chanics and Materials* 148-149 (276-281). [7](#)
- 477 Tavallali, A., Vervoort, A., 2010. Behaviour of layered sandstone under brazil-
478 ian test conditions: Layer orientation and shape effects. *International Jour-*
479 *nal of Rock Mechanics and Mining Sciences* 47, 313–322. [23](#)
- 480 Turner-Walker, G., 2011. The mechanical properties of artificially aged bone:
481 Probing the nature of the collagen–mineral bond. *Palaeogeography, Palaeo-*
482 *climatology, Palaeoecology* 310, 17–22. [8](#), [25](#)
- 483 Turner-Walker, G., Parry, T., 1995. The tensile strength of archaeological
484 bone. *Journal of Archaeological Science* 22, 185–191. [2](#), [8](#), [25](#)

- 485 Vashishth, D., 2007. Hierarchy of bone microdamage at multiple length
486 scales. *International journal of fatigue* 29 (6), 1024–1033. [2](#)
- 487 Vayron, R., Barthel, E., mathieu, V., Soffer, E., Anagnostou, F., Haiat,
488 G., 2012. Nanoindentation measurements of biomechanical properties in
489 mature and newly formed bone tissue surrounding an implant. *J. Biomech.*
490 *Eng.* 134, 021007. [5](#)
- 491 Wang, Q., Jia, X., Kou, S., Zhang, Z., Lindqvist, P.-A., 2004. The flattened
492 brazilian disc specimen used for testing elastic modulus, tensile strength
493 and fracture toughness of brittle rocks: analytical and numerical results.
494 *International Journal of Rock Mechanics and Mining Sciences* 41, 245–253.
495 [12](#), [17](#), [20](#)
- 496 Xavier, J., Diaquino, B., Morais, J., Pereira, F., 2013. Characterisation of
497 shear behaviour of bovine cortical bone by coupling the arcan test with
498 digital image correlation. *Journal of the Mechanical behavior of biomedical*
499 *materials*. [5](#)