

-The Bank Capital Regulation Model-  
14th Finance, Risk and Accounting Perspective  
Conference, Oxford, hosted by the Oriel college,  
University of Oxford

Presented by Hye-jin CHO  
Economics, University of Paris1-Panthon Sorbonne

September 11, 2014

# Table of contents

Motivation

General Equilibrium with Uncertainty

# Practical need

Cost of Banking Crisis for 1970 to 2007,  
ref: Laeven L, Valencia F (2008) IMF

The average net recapitalization cost to the government was  
**6% of GDP**,

fiscal costs associated with crisis management  
averaged **13% for GDP** (**16% of GDP** if expense recoveries are  
ignored), and economic output losses averaged about  
**20% of GDP** during the first four years of crisis.

Prevention method by the government,  
ref: Slovik, P. (2011) OECD

The estimated medium-term impact of Basel 3 implementation on  
GDP growth would be in the range of **0.05% to 0.15%** per year.

# Practical need

**Macro origins of banking crisis:** Lindgren and al (1996), Dooley and Frankel (2003), Collyns and Kincaid (2003)

**How crisis resolution policies have been used and the tradeoffs involved:** Claessens and al (2003), Hoelscher and Quintyn (2003), Honohan and Laeven (2005)

**Institutional weakness:** Equity and creditor rights

**Counterproductivity of banks and their borrowers:** net worth of banks and tax burdens to finance bank bailouts

**Cross-country analysis:** better institutions, uniformly positively associated for faster recovery

**Timing and sequential causality of crisis:** banking crisis, currency crises and sovereign debt crisis

# Stabilization of Financial Economics

## Incomplete Methodology

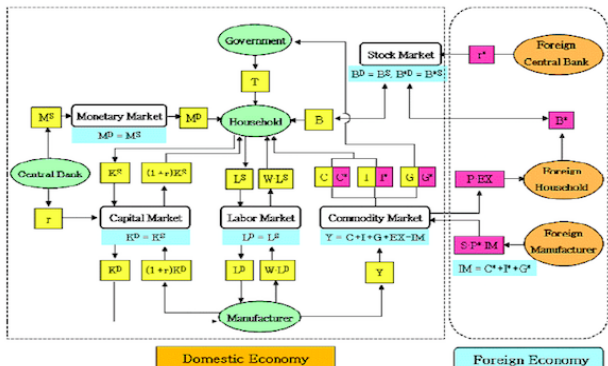
Microeconomics theory of banking does not exist before the foundation of the economics which information were laid in the early 1970s. **Freixas and Rochet (1999)** claim that a general equilibrium approach enables one to explain the specific characteristics of the banking industry. This approach is particularly well developed from **simple general equilibrium models** containing a banking sector under the complete financial market hypothesis (**Arrow, 1953**)

to articulate fundamental concepts to solve problems of a **sudden micro financial shock** in the short run and the **long run macro stabilization** with a balanced perspective.

# SOE(Structure of Equilibrium) of BOK (Bank of Korea)

## 2. Model structure ...

### typical SOE structure



# Vanhoose 2008 - Asset/Liability of banks

**Table 2.1** Assets of U.S. commercial banks

Asset category	\$ Billions	%
Commercial and industrial loans	1,197.9	12.3
Consumer loans	847.4	9.0
Real estate loans	3,573.9	36.7
Interbank loans	364.6	3.6
Other loans	269.0	2.7
<b>Total loans</b>	<b>6,252.8</b>	<b>64.3</b>
Securities	2,017.7	20.7
Cash assets	247.1	2.5
Other assets	1,220.4	12.5
<b>Total assets</b>	<b>9,738.0</b>	<b>100.0</b>

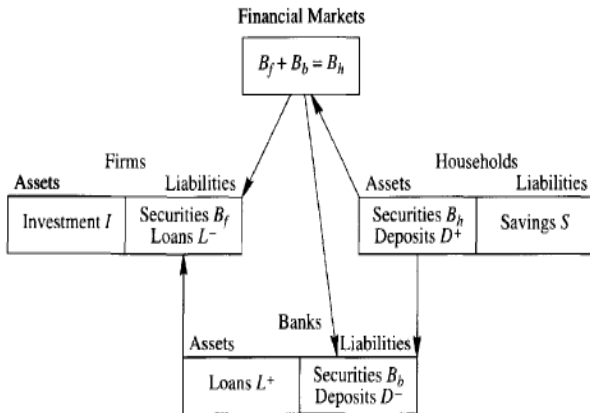
(Source: Board of Governors of the Federal Reserve System, August 2008)

**Table 2.2** U.S. commercial bank liabilities and equity capital

Category	\$ Billions	%
Transactions deposits	579.1	6.0
Large time deposits	1,016.4	10.4
Savings and Small Time Deposits	4,171.6	42.8
<b>Total deposits</b>	<b>5,767.1</b>	<b>59.2</b>
Borrowings	1,744.8	17.9
Other liabilities	1,051.4	10.8
<b>Total liabilities</b>	<b>8,563.3</b>	<b>87.9</b>
Equity capital	1,174.7	12.1
<b>Total liabilities and equity capital</b>	<b>9,738.0</b>	<b>100.0</b>

(Source: Board of Governors of the Federal Reserve System, August 2008)

## Freixas-Rochet (1999)





# Structure of Equilibrium-Capital Dynamics

## Security circulation, Deposit stability

### Each Market Clearing

$I=S$  (Good Market)

$D_t$  (Firm)- $D_t$  (Firm)+ $D_t$  (Household)- $D_t$  (Household)+ $D_t$  (Bank)- $D_t$  (Bank) (Deposit market)

$L_t$  (Firm)- $L_t$  (Firm)- $L_t$  (Bank)+ $L_t$  (Bank)+ $L_t$  (FR)- $L_t$  (FR) (Credit Market)

$B_t$  (Firm)- $B_t$  (Firm)+ $B_t$  (Household)- $B_t$  (Household)+ $B_t$  (Bank)- $B_t$  (Bank) + $B_t$  (FB)- $B_t$  (FB) (Financial Market)

### Firms

#### Assets

Real Asset  $D_t+B_t$   
(=Investment I)

#### Liabilities

Liabilities to banks  
 $D_t+B_t - L_{tF}$   
Liabilities to central bank  
 $L_{tF}$

Banks: domestically chartered commercial banks, country branches and agencies of foreign banks, Edge Act corporation

#### Assets

Claims to corporate  
 $D_t+B_t - L_{tF}$

#### Liabilities

Deposits  $D_t$   
Borrowing  $B_t - L_{tF}$

### Households

#### Assets

Securities  $B_t$   
Deposits  $D_t$   
Real Asset  $S_t - (B_t + D_t)$

#### Liabilities

Savings  $S_t$

### Federal Reserve Banks

#### Assets

Claims to corporate  $L_{tF}$   
Currency  $C_t$   
Borrowing to banks  
 $B_t - L_{tF}$

#### Liabilities

Securities  
 $B_t$

# Saving preference of Consumers

The two-period model ( $t = 0, 1, 2,$ ) with a unique physical good initially owned by the consumers in the economy in which a continuum of ex-ante identical agents is each endowed with one unit of good at period  $t = 0$ , and this good is to be consumed at periods  $t = 1$  and  $t = 2$ . The consumer chooses her consumption profile  $(C_1, C_2)$ , and the allocation of her savings  $S$  between bank deposits  $D_h$  and securities  $\sum_{s \in \Omega} P_s B_s^h$ , in a way that maximize her utility function  $u$  under her budget constraints:

$$\text{Max } u(C_1, C_2)$$

$$C_1 + \sum_{s \in \Omega} P_s B_s^h + D^h + S_h - \sum_{s \in \Omega} P_s B_s^h - D^h = \omega_1$$

$$C_2 = \Pi_f + \Pi_b + (1 + r) \sum_{s \in \Omega} P_s B_s^h + (1 + r_D) D^h + (1 + r_h) S_h - (1 + r) \sum_{s \in \Omega} P_s B_s^h - (1 + r_D) D^h$$

# Technical finding

## (Usage of Same Account - Saving preference)

Suppose that  $\exists s \in \{G, B\}$  G=Good, B=Bad

**Case1, risk aversion case as below, (resp. risk taking case)**

$\Omega = \{G, B\}$ ,

$$C_1 + \sum_{s \in \Omega} P_{s=B} B_{s=B}^h + D^h + S_h - \sum_{s \in \Omega} P_{s=G} B_{s=G}^h - D^h = \omega_1$$

Regulation - **Portfolio analysis**

**Case2, incompleteness preference**

$\Omega = \{G, B, \text{no choice}\}$  no choice can be selected by the choquet

expectation  $CEv(u(x)) = \min_{\Pi \in \text{core}(v)} E_{\Pi}(u(x))$  where

$\text{core}(v) = \prod \{ \Delta S : \prod(A) \geq v(A) \text{ for all } A \subseteq S \}$

$C_1 + 0 + D^h + S_h - 0 - D^h = \omega_1$  and  $D^h$  offset, hence

$$C_1 + \frac{C_2}{1 + r_h} = \omega_h$$

Regulation - **Initial GDP** caused by partition of initial endowment which is combination of consumption set

# savings containing real assets

## Original form (savings containing real assets)

$$\text{Max } u(C_1, C_2)$$

$$C_1 + \sum_{s \in \Omega} P_s B_s^h + D^h + S_h - \sum_{s \in \Omega} P_s B_s^h - D^h = \omega_1$$

$$C_2 = \Pi_f + \Pi_b + (1+r) \sum_{s \in \Omega} P_s B_s^h + (1+r_D)D^h + (1+r_h)S_h - (1+r) \sum_{s \in \Omega} P_s B_s^h - (1+r_D)D^h$$

## Simplified form for calculation (savings containing real assets)

$$\text{Max } u(C_1, C_2), C_1 = W_1 - S_1, C_2 = R_1 + W_2 - (1+r_1)S_1 - S_2$$

$$\frac{\partial L}{\partial C_1} = u'(C_1, C_2) + \lambda = 0, \frac{\partial L}{\partial C_2} = u'(C_1, C_2) + \lambda = 0$$

households consumes today or tomorrow regardless of banking money flow and it is not affected by the return of initial savings.

$u'(C_1, C_2) = 0$  denotes as time indifference about consumption preference.

## savings without real assets

**Simplified form for calculation (savings without real assets)**

$$\text{Max } u(C_1, C_2)$$

$$C_1 + \sum P_1 B_1 + D_1 = W_1$$

$$R_1 + C_2 + (1 + r_1) \sum P_1 B_1 + (1 + r_1) D_1 - \sum P_2 B_2 - D_2 = W_2$$

$$\text{also, } C_1 = (W_1 - S_1) + (S_1 - \sum P_1 B_1 + D_1)$$

**The first-order conditions**

$$\frac{\partial L}{\partial C_1} = u'(C_1, C_2) + \lambda = 0, \quad \frac{\partial L}{\partial C_2} = u'(C_1, C_2) + \lambda = 0$$

$$\frac{\partial L}{\partial D_1} \lambda = 0, \quad \frac{\partial L}{\partial D_1}, -\lambda(1 + r_1) = 0, \quad \frac{\partial L}{\partial D_2}, \lambda = 0 \quad u'(C_1, C_2) = 0,$$

$$\frac{\partial L}{\partial \sum P_1 B_1}, -\lambda(1 + r_1) = 0, \quad \frac{\partial L}{\partial \sum P_1 B_1}, \lambda = 0,$$

$$\frac{\partial L}{\partial r_1} = \lambda(\sum P_1 B_1 + D_1) = 0$$

## savings without real assets

In conclusion,  $u'(C_1, C_2) = 0$  (time indifference about consumption preference) is the same condition regardless of real assets whether is contained in savings or not.

Consumption choice is not affected by the interest rate  $r_1$  for initial savings (same condition regardless of real assets whether is contained in savings or not) and summation of securities and deposits  $\sum P_1 B_1 + D_1$ . Evidently, this condition appears in this banking model when we ignore the real asset which is most stable in the household economy and can be interpreted as the big portion expense and intangible asset producing future benefits. Therefore, with the condition which real assets is contained in savings, we can explain the house economy affected by the proportion of securities and deposits.

# Empirical balance when the capital of a household economy is concerned

Ref: European Central Bank, the 4th quarter in 2013

	(billion euros, growth rate)
Loans for house purchase (3858.1, 0.7)	insurance corporations and pension funds (653.2, -5.3)
other loans (796.7, -1.6)	other financial intermediaries (1854.1, -3.1)
consumer credit (576.1, -3.0)	non-financial corporations are (1870.7, 6.7)
	government (440.8, -1.8)
	non-euro area residents (2522.9, -11.2)
7341.7	7752.2

# Borrowing composition of Firms

The firm chooses its investment level  $I$  and its financing (through real asset  $D_h + \sum_{s \in \Omega} P_s B_h$ , liabilities to bank  $D_h + \sum_{s \in \Omega} P_s B_h - L_{fr}$  (or Liabilities to central bank  $L_{fr}$ ) in a way that maximizes its profit:

$$\text{Max } \Pi_f (P_f)$$

$$\Pi_f = f(I) + r_f (D_h + \sum_{s \in \Omega} P_s B_s^h) - r_{L_{Bank}} (D_h + \sum_{s \in \Omega} P_s B_s^h - L_{fr}) - r_{L_{fr}} L_{fr}$$

$$I = S_h = D_h + \sum_{s \in \Omega} P_s B_s^h$$

Where  $r_f$  is the premium of firm real asset.  $r_{L_{Bank}}$ ,  $r_{L_{fr}}$  is the interest rate on bank loans and federal reserve bank loan.  $D_h$  denotes for bank deposits.  $B_h$  denotes for securities. Especially  $B_{fr}$  denotes for securities of federal reserve banks.  $L_{fr}$  is loan claimed by the firm to the federal reserve bank.



# Borrowing composition of Firms

*Conservative Business Cycle* assumption  $\exists$  multiplier  $\mu$ ,  
 $\mu(D_h + B_h) > D_h + B_h - L^{fr} > L^{fr}$

Each Market Clearing

$I=S$  (Good Market)

$D_{fr}(Firm) - D_{fr}(Firm) + D_{fr}(Household) - D_{fr}(Household) + D_{fr}(Bank) - D_{fr}(Bank)$  (Deposit market)

$L_{fr}(Firm) - L_{fr}(Firm) - L_{fr}(Bank) + L_{fr}(Bank) + L_{fr}(FR) - L_{fr}(FR)$  (Credit Market)

$B_{fr}(Firm) - B_{fr}(Firm) + B_{fr}(Household) - B_{fr}(Household) + B_{fr}(Bank) - B_{fr}(Bank) + B_{fr}(FB) - B_{fr}(FB)$  (Financial Market)

Firms

Assets

Real Asset  $D_{fr} + B_{fr}$   
 (=Investment I)

Liabilities

Liabilities to banks

$D_{fr} + B_{fr} - L_{fr}$   
 Liabilities to central bank  
 $L_{fr}$

Banks: domestically chartered commercial banks,  
 country branches and agencies of foreign banks,  
 Edge Act corporation

Assets

Claims to corporate  
 $D_{fr} + B_{fr} - L_{fr}$

Liabilities

Deposits  $D_{fr}$   
 Borrowing  $B_{fr} - L_{fr}$

Households

Assets

Securities  $B_{fr}$   
 Deposits  $D_{fr}$   
 Real Asset  $S_{fr} - (B_{fr} + D_{fr})$

Liabilities

Savings  $S_{fr}$

Federal Reserve Banks

Assets

Claims to corporate  $L_{fr}$   
 Currency  $C_r$   
 Borrowing to banks  
 $B_{fr} - L_{fr}$

Liabilities

Securities  
 $B_{fr}$

# The portfolio composition effected by the minimum equity capital regulation

In the model of Kahane (1977), the minimum capital requirement causes an unintended result: it worsened, rather than improved the intermediary's condition and increases its probability of ruin. He check this calculation with the ruin constraint and given standard deviation of rate of return at the portfolio composition of liability, stock and bonds.

# Sunspot caused by the portfolio composition effected by the minimum equity capital regulation

Sunspot: States do not affect fundamentals of the economy.  
Effects of increasing the equity at the portfolio composition

Period	Required Equity, Liability	Portfolio composition (risky portfolio, stable portfolio)
1	12 (12%), 88(88%)	(-61.6%, 161.6%)
2	13 (13%), 88(87.12%)	(-61 %, 161 %)
3	14 (14%), 88(86.72%)	(-60.4 %, 160.4 %)

# The portfolio composition effected by the minimum equity capital regulation

Thus, in order to obtain a mean return of 0.84%, 0.90%, 0.96%, the bank must invest -61.6%, -61%, -60.4 of total funds in the risky portfolio and 161.6%, 161%, 160.4% in stable portfolio.

Standard deviation which implies the probability to get mean return, is also increasing.

Standard Deviation

$$0.12 \times 0.27 = 0.0324$$

$$0.13 \times 0.27 = 0.0351$$

$$0.14 \times 0.27 = 0.0378$$

whenever only minimum equity capital regulation makes portfolios of banks riskier.

# Federal Reserve Banks and general equilibrium

The Federal Reserve Banks chooses its investment level  $I$  and its financing (through real asset  $D_h + \sum_{s \in \Omega} P_s B_s^h$ , liabilities to bank  $D_h + \sum_{s \in \Omega} P_s B_s^h - L_{fr}$  (or Liabilities to central bank  $L_{fr}$ ) in a way that maximizes its profit:

$$\text{Max } \Pi_f (P_f)$$

$$\Pi_f = f(I) + r_f (D_h + \sum_{s \in \Omega} P_s B_s^h) - r_{L_{Bank}} (D_h + \sum_{s \in \Omega} P_s B_s^h - L_{fr}) - r_{L_{fr}} L_{fr}$$

$$I = S_h$$

## description about the BCR model

(result) Arrow (1953) If firms and households have unrestricted access to perfect financial markets, then at the competitive equilibrium

(result) Cho (2014) If the sum accumulated variables is not negative, for example, the components Investment  $I$ , Savings  $S_h$ ,  $L_{fr}$  are not negative, there is the equilibrium in the economy and the existence of each factors like firms, Households, Banks, Federal Reserve Banks is fulfilled. **The size of banks is affecting on each agent because equity capitals depend on previous deposits.** Depending the change of bank size influencing on total deposit  $D_h$ , the liability of firms is affected by liabilities to banks  $D_h + \sum_{s \in \Omega} P_s B_s^h - L_{fr}$ , deposit of household  $D_h$  and real asset of household and firms.

# Deposit affects optimized equity capital

$n$	Deposits	Borrowings	OptimizedEquityCapital
$n = 0$	$D_0 = 1$	-	-
$n = 1$	$D_1 = (1 - \beta - K)$	$B_1 = (1 - \beta)$	$OEC_1 = K$
$n = 2$	$D_2 = (1 - \beta - K)^2$	$B_2 = (1 - \beta)(1 - \beta - K)$	$OEC_2 = K(1 - \beta - K)$
$n = 3$	$D_3 = (1 - \beta - K)^3$	$B_3 = (1 - \beta)(1 - \beta - K)^2$	$OEC_3 = K(1 - \beta - K)^2$
...	...	...	...
$n = k$	$D_k = (1 - \beta - K)^k$	$B_k = (1 - \beta)(1 - \beta - K)^{k-1}$	$OEC_k = K(1 - \beta - K)^{k-1}$
...	...	...	...
$n \rightarrow \infty$	$D_\infty = 0$	$B_\infty = 0$	$OEC_\infty = 0$
	total deposits	total borrowings	total optimized equity capital
	$D = \frac{1}{K + \beta}$	$B = \frac{1 - \beta}{K + \beta}$	$OEC = \frac{K}{K + \beta}$

# Deposit affects optimized equity capital

[ $\beta$  = restriction of borrowing] Then, Borrowings can be executed between Deposit 1 and Restriction  $\beta$

[balance sheet equality constraint]  $D_n = B_n - OEC_n$

Hart and Jaffee(1974) analyzed the properties of the feasible and efficient set with the assumption that the initial equity capital is zero (i.e.  $K=0$ ) in the substantial degrees of leverage. In the paper, following the KAHANE (1977), we assume the equity is positive ( $K > 0$ ) so that the opportunity set does not pass through the origin (i.e. the vector of Deposit  $D$ , Borrowing  $B$ , Optimized Equity Capital = 0 give an infeasible solution).



# Deposit affects optimized equity capital

Then the theoretical superior limit for deposits is defined by the following:

$$Deposits = \sum_{n=0}^{\infty} [(1 - K - \beta)] = \frac{1}{K + \beta}$$

Theoretically, superior limit for the equity capital by the firm is defined by the following:

$$OptimizedEquityCapital = K \times Deposits = \frac{K}{K + \beta}$$

and the theoretical superior limit for total borrowings in banks is defined by the following:

$$Borrowings = (1 - \beta) \times Deposits = \frac{1 - \beta}{K + \beta}$$

## Deposit affects optimized equity capital

Borrowings at stage  $k$  are a function of the deposits at the precedent stage:  $B_k = (1 - \beta - K) \times D_{k-1}$

Optimized Equity Capital at stage  $k$  is a function of the deposits at the precedent stage:  $OEC_k = K \times D_{k-1}$

Hence, if the optimized equity capital depends on the initial deposit and assume the terminal condition of bank is liquidation of bank deposits,

(result) Hence, Optimized Equity Capital depends on the previous deposit. In addition, deposit insurance cost also increases because deposit insurance depends on the number of household.

# $k$ index for the indicator of risk taking

Define the equity capital ratio with respect to total liabilities and equity capital,  $\frac{\text{EquityCapital}}{D_h + B_{fr} - L_{fr}}$ ,  $K \in (0, 1)$ , the borrowing (from the federal banks) ratio  $\frac{B_{fr} - L_{fr}}{D_h + B_{fr} - L_{fr}}$ ,  $\beta \in (0, 1)$ ; suppose the demand for funds is unlimited;

By summing up two quantities, the theoretical equity capital multiplier is defined as

$$k = \frac{\text{Deposits} + \text{OptimizedEquityCapital}}{\text{Borrowings} + \text{OptimizedEquityCapital}} = \frac{1 + K}{K + \beta}$$

## $k$ index for the indicator of risk taking

$k$  is the index to decline to increase the risk at the portfolio of commercial banks. The deposit is fixed at total 1 and borrowings have the constraint can not be negative value beyond the minimum borrowings  $\beta$ . For example, if deposit=1, the minimum of required equity = 10%, borrowings = 0.3

$$\frac{1 + 0.1}{0.3 + 0.1} = \frac{1.1}{0.4} = 2.75$$

If the minimum of required equity is raised from 10%, to 15%,  $k$  index was downed as below.

$$\frac{1 + 0.15}{0.3 + 0.15} = \frac{1.15}{0.45} = 2.55$$

To increase the  $k$  index, the bank should increase the deposit beyond the initial deposit level (1 in this simulation) or allocate the borrowing portfolio.

# Conclusion

The combination of portfolio composition test, deposit-equity optimization and k index enables bounding the bank capital regulation problems.

The **minimum capital requirement** is a necessary condition for banking sector stability to raise the quality, consistency and transparency of the capital base. However, it has **friction with the portfolio management**. By using effects of increasing the equity at the portfolio composition, *reducing procyclicality (to the financial shocks) and promoting the countercyclical buffer* are pursued.

# Conclusion

In the **Basel 3** system, The **risk coverage** framework intends to capture all material risks by using **counterparty credit risk formula** weighted on the external rating of the counter party. Exposure measures contain on-balance sheet, repurchase agreements and securities finance, derivatives and off-balance sheet (OBS) items. *In the paper, rather than enlarging the risk contagion, related factors and risk affection scope are detected without overstatement by using the general equilibrium model and deposit affection to the optimized equity capital.*

# Conclusion

Basel 3 introduced a minimum leverage ratio. the **leverage ratio** was calculated by **dividing Tier1 capital by the bank's average total consolidated assets**. In the paper, **k index** is suggested as the **indicator of risk taking**. Within the liability, three major fractions like deposits, borrowings and optimized equity capital are considered as the complementary of minimum capital requirement. Assets of commercial banks are mainly consisted with loans and securities. Because the optimized equity capital grows and deposits is restricted by change, *borrowings which is the difference between asset and deposit+equity capital should be checked whether borrowings can cover the optimized equity by k index.*

## Further concerns

Stress testing with the k index  
social welfare theorem  
Thank you.