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Stochastic modeling of train dynamics under effect of track irregularities and experimental comparisons

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Abstract

This paper aims at modeling all the uncertainties which are encountered when simulating the high-speed train dynamic response for a given track portion. The built model allows us to observe the long-time evolution of the train dynamic response for this track portion. The knowledge of the evolution of such a system is of great concern for the railway industry, in order to maintain a high level of safety and comfort in the trains. A double approach is used to build the model. First, the global stochastic model of track irregularities previously defined is adapted to the given track portion. Then, modeling uncertainties in the computational model of the train are represented by a noise added to the output response of the train dynamics simulation. This additive noise is represented by a polynomial chaos expansion which is identified with measurements. Robust train-dynamics indicators are set up to observe the long-time evolution of the train dynamic response.

1 Introduction

1.1 Objectives

The tracks for the high speed trains are submitted to more and more solicitations, because of the increase of the train traffic, the load and the speed of the trains. These solicitations induce degradations of the track geometry, making evolve track irregularities. Such degradations impact the train dynamic response in return. The knowledge of the effects of the track geometry on the train dynamic response needs to be increased (see [4]). In this framework robust indicators must be defined for the simulated train dynamic response, in order to characterize its evolution.

The goal of this work is to set up robust indicators [3], able to describe the long-time evolution of the train dynamic response, under the influence of the evolution of the track irregularities. The distinction has to be done between the long-time evolution, which will be denoted by τ , and the time of the train dynamics (denoted by t). Actually, the vehicle-track system is a complex system, with high nonlinearities and coupling between inputs (track geometry, track stiffness, track mass) and outputs (train responses). The track geometry is the main source of excitation for the train. Measurements of the track geometry are performed very precisely and frequently, which provides us information on the track geometry in the long

time τ . A global stochastic model of the track geometry has been built by Perrin *et al.* in [5] using a very large experimental data basis concerning the French railway network for high-speed trains. The stochastic modeling is very useful to carry out nonlinear stochastic dynamic analysis of the train excited by the random track geometry. We need now to adapt the stochastic model for a given track portion, in order to be able to start off maintenance operations for this track portion.

1.2 Proposed approach

For the long-time evolution analysis of a given track portion, for which measurements are periodically carried out, the global stochastic model of the track is adapted to this portion introducing a noise which allows measurement errors and variability to be taken into account. Using this adapted stochastic model of the track geometry for such a given portion, the train dynamic response is numerically simulated. The inputs of the simulation are the track design, the track irregularities modeled with the adapted stochastic model, and a model of the train. The model of the train used for the simulation is a multibody dynamical model whose dynamic responses are computed using a commercial software (*Vampire*). The simulation outputs are accelerations in the train and contact forces between the wheels and the rails.

Moreover, the model of the train used for the simulation, as well as the simulation model itself, contain uncertainties. Those uncertainties are due to numerical approximations, wrong values of parameters in the vehicle model, etc. In order to increase the robustness of the chosen dynamic indicators, model uncertainties have to be taken into account in the modeling. Those uncertainties are estimated using comparisons with experimentations, allowing us to identify a noise to be added to the dynamic indicators.

Section 2 will focus on the adapted stochastic modeling of the track irregularities, taking into account the measurements for the studied given track portion. Indicators for the train dynamics will be defined to assess the train behavior on this given portion. In Section 3, model uncertainties will be identified in order to have dynamic indicators more robust.

2 Stochastic modeling of track irregularities

2.1 Track measurements

The track geometry is measured very precisely and very frequently by SNCF company using a measuring train equipped with laser cameras. The track is described by two data sets:

- the initial track design, which corresponds to the theoretical track (as it was planned before the construction), and which is made of straight lines and curves.
- the irregularities of the track, which appear during the track life cycle, and which have to be added to the track design.

The track irregularities are modeled by a vector-valued random field \mathbf{Y} denoted by

$$s \mapsto \mathbf{Y}(s; \tau) = (Y_1(s; \tau), Y_2(s; \tau), Y_3(s; \tau), Y_4(s; \tau)), \quad (1)$$

indexed by s in $\Omega = [0, S]$, where S is the portion length, and which depends on long-time parameter τ . Long time τ is a discrete parameter that rises between successive measurements of the given track portion,

$$\tau_0 < \tau_1 < \tau_2 < \dots < \tau_{\nu_\tau}, \quad (2)$$

in which τ_0 is the time of the first measurement performed just after a maintenance operation, and where $\tau_1, \tau_2, \dots, \tau_{\nu_\tau}$ correspond to the successive long times for which there are measurements of the track geometry, and τ_{ν_τ} is the time of the last measurement before the next maintenance operation.

2.2 Global stochastic modeling of the track irregularities

A global stochastic model of the track irregularities has been proposed in [5] and detailed in [6, 7, 8, 9, 10]. This model has been built solving an inverse statistical problem using a very large experimental data basis related to the French railway network. It is very robust with respect to measurement errors and has the capability to generate a track irregularity for any given portion belonging to the French railway network.

The track irregularities vector $\mathbf{Y} = (Y_1, Y_2, Y_3, Y_4)$ is modeled by a vector-valued random field, defined on a probability space $(\Theta, \mathcal{F}, \mathcal{P})$, indexed by $\Omega = [0, S]$, with values in \mathbb{R}^4 . It has been proven that random field \mathbf{Y} is neither Gaussian nor stationary (not homogeneous).

Random field \mathbf{Y} is centered,

$$E\{\mathbf{Y}(s)\} = 0 \quad , \quad \forall s \in [0, S], \quad (3)$$

where $E\{\cdot\}$ is the mathematical expectation. The continuous vector-valued random field $\{\mathbf{Y}(s), s \in \Omega\}$, is replaced by its spatial discretization at curvilinear abscissa $s_n = nh$ with h the spatial step and $n = 0, \dots, N_s$, where $S = N_s h$. Keeping the same notation for the continuous random field and its spatial discretization, the following random vector $\mathbf{X} = (\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3, \mathbf{X}^4)$ with values in $\mathbb{R}^{4(N_s+1)}$, is introduced such that

$$\mathbf{X}^k = (Y_k(0), Y_k(h), Y_k(2h), \dots, Y_k(N_s h)) \quad , \quad k = 1, 2, 3, 4, \quad (4)$$

with values in \mathbb{R}^{N_s+1} .

In this construction, the random vector \mathbf{X} is written using a principal components decomposition that is written as:

$$\mathbf{X} \simeq [U] [\lambda]^{1/2} \boldsymbol{\eta}, \quad (5)$$

where $[\lambda]$ gathers the most influencing modes, where the columns of $[U]$ are the associated eigenvectors of the covariance matrix $[C_{\mathbf{X}\mathbf{X}}]$ of \mathbf{X} , and where $\boldsymbol{\eta}$ is the \mathbb{R}^{N_η} -valued random vector of the generalized coordinates of the global stochastic model. Introducing

$$[Q] = [U] [\lambda]^{1/2}, \quad (6)$$

\mathbf{X} can be rewritten as

$$\mathbf{X} \simeq [Q] \boldsymbol{\eta}. \quad (7)$$

Nevertheless, as explained in Section 1, we are interested in constructing a stochastic model adapted to the given track portion. The objective of this adapted stochastic model that has to be constructed is to take into account uncertainties induced by (i) measurement noise associated with local measurements $\mathbf{x}_{\tau_0}^{\text{meas}}, \mathbf{x}_{\tau_1}^{\text{meas}}, \mathbf{x}_{\tau_2}^{\text{meas}}, \dots$, and (ii) the local variability of the given track portion in order to decrease the ‘‘statistical distance’’ between the global stochastic model and the local measurements.

2.3 Local stochastic modeling

The local stochastic modeling aims at constructing an adapted stochastic model of the track irregularities related to a given track portion. The method proposed to construct this adapted stochastic model consists in introducing a random field noise for which the spatial properties are driven by the global stochastic model and whose intensity of its statistical fluctuations is identified at long time τ_0 using measurement $\mathbf{x}_{\tau_0}^{\text{meas}}$. For $k = 1, \dots, 4$, \mathbf{X}^k is the random vector of dimension $N_s + 1$ defined (using Eq. (7)) as

$$\mathbf{X}^k = [Q^k] \boldsymbol{\eta}, \quad (8)$$

in which the $((N_s + 1) \times N_\eta)$ real matrix $[Q^k]$ is extracted from matrix $[Q]$. The proposed adapted stochastic model is written as

$$\tilde{\mathbf{X}}^k(\delta_k) = [Q^k] \left(\boldsymbol{\eta} + \delta_k \mathbf{G}^k \right) \quad , \quad k = 1, 2, 3, 4, \quad (9)$$

in which $\boldsymbol{\delta} = (\delta_1, \delta_2, \delta_3, \delta_4)$ is the vector-valued hyperparameter allowing the uncertainty level to be controlled, and which has to be identified for each track portion using experimental data. For fixed k , \mathbf{G}^k is a \mathbb{R}^{N_η} -valued random noise. In the model proposed, $\mathbf{G} = (\mathbf{G}^1, \mathbf{G}^2, \mathbf{G}^3, \mathbf{G}^4)$ is chosen as a \mathbb{R}^{4N_η} -valued Gaussian second-order centered random variable, defined on the probability space $(\Theta, \mathcal{F}', \mathcal{P}')$, for which its covariance matrix is the unity matrix. From Eq. (9), the adapted stochastic model can be rewritten as

$$\tilde{\mathbf{X}}^k(\delta_k) = \mathbf{X}^k + \mathbf{B}^k(\delta_k), \quad (10)$$

in which the random vector $\mathbf{B}^k(\delta_k)$ that depends on δ_k is such that

$$\mathbf{B}^k(\delta_k) = \delta_k [Q^k] \mathbf{G}^k. \quad (11)$$

The optimal value $\boldsymbol{\delta}^{\text{opt}}$ of hyperparameter $\boldsymbol{\delta}$ is estimated by using the maximum log-likelihood method with experimental data applied to the observation random vector $\mathbf{W}(\boldsymbol{\delta}) = (W_1(\delta_1), W_2(\delta_2), W_3(\delta_3), W_4(\delta_4))$, in which

$$W_k(\delta_k) = \frac{\|\tilde{\mathbf{X}}^k(\delta_k)\|}{E\{\|\mathbf{X}^k\|\}}, \quad (12)$$

where $\|\mathbf{X}^k\| = \|\tilde{\mathbf{X}}^k(0)\|$ is the Euclidean norm of the global stochastic model \mathbf{X}^k , as explained in [3]. The experimental observation vector $\mathbf{w}^{\text{meas}} = (w_1^{\text{meas}}, w_2^{\text{meas}}, w_3^{\text{meas}}, w_4^{\text{meas}})$, that corresponds to experimental measurements is such that

$$w_k^{\text{meas}} = \frac{\|\mathbf{x}^{k,\text{meas}}\|}{E\{\|\mathbf{X}^k\|\}}, \quad 1 \leq k \leq 4. \quad (13)$$

Let $\mathcal{L}_{\mathbf{W}}(\mathbf{w}^{\text{meas}}; \boldsymbol{\delta}) = \log p_{\mathbf{W}}(\mathbf{w}^{\text{meas}}; \boldsymbol{\delta})$ be the log-likelihood in which $p_{\mathbf{W}}(\mathbf{w}^{\text{meas}}; \boldsymbol{\delta})$ is the value of the probability density function $\mathbf{w} \mapsto p_{\mathbf{W}}(\mathbf{w}; \boldsymbol{\delta})$ of random vector \mathbf{W} for $\mathbf{w} = \mathbf{w}^{\text{meas}}$. The optimal value $\boldsymbol{\delta}^{\text{opt}}$ is then identified solving the following optimization problem,

$$\boldsymbol{\delta}^{\text{opt}} = \arg \max_{\boldsymbol{\delta}} \{\mathcal{L}_{\mathbf{W}}(\mathbf{w}^{\text{meas}}; \boldsymbol{\delta})\}. \quad (14)$$

The quantity $p_{\mathbf{W}}(\mathbf{w}^{\text{meas}}; \boldsymbol{\delta})$ is computed using independent realizations of \mathbf{W} generated with the adapted stochastic model, and fitted by using the multivariate Gaussian kernel method (see for instance [2, 1]). As an illustration, and for $k = 1$, Fig. 1 displays the variation of the marginal probability density function (PDF) $w_k \mapsto p_{W_k(\delta_k)}(w_k; \delta_k)$ of random variable $W_k(\delta_k)$ as a function of δ_k .

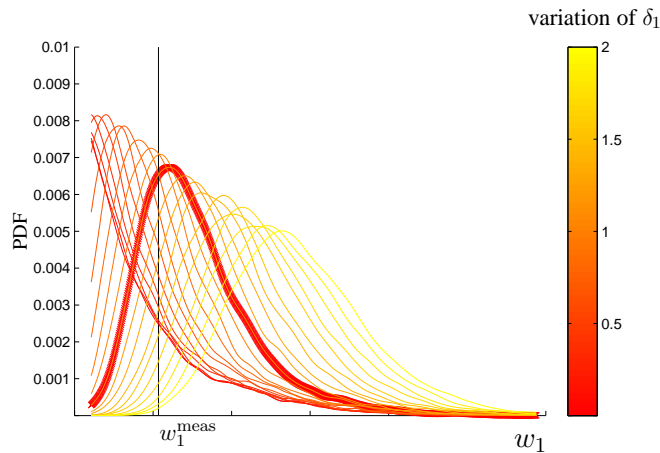


Figure 1: Graphs the PDF, $w_1 \mapsto p_{W_1(\delta_1)}(w_1; \delta_1)$, in function of δ_1 (the bold line is obtained for $\delta_1 = \delta_1^{\text{opt}}$).

2.4 Adapted stochastic modeling of the track irregularities

For the given track portion, the optimal value δ^{opt} of δ is identified using the measurement $\mathbf{x}_{\tau_0}^{\text{meas}} = (\mathbf{x}_{\tau_0}^{1,\text{meas}}, \mathbf{x}_{\tau_0}^{2,\text{meas}}, \mathbf{x}_{\tau_0}^{3,\text{meas}}, \mathbf{x}_{\tau_0}^{4,\text{meas}})$ of the portion at time τ_0 (Eq. (14) for the first time τ_0).

$$\delta^{\text{opt}} = \arg \max_{\delta} \{\mathcal{L}_{\mathbf{w}}(\mathbf{w}_{\tau_0}^{\text{meas}}, \delta)\}. \quad (15)$$

It is then assumed that this optimal value is representative of the level of uncertainties (noise and variability) for all the values of the long time of this given track portion. The adapted stochastic modeling of the long-time evolution for this given track portion is constructed as follows. At long time τ , the measurement of the track portion is $\mathbf{x}_{\tau}^{\text{meas}} = (\mathbf{x}_{\tau}^{1,\text{meas}}, \mathbf{x}_{\tau}^{2,\text{meas}}, \mathbf{x}_{\tau}^{3,\text{meas}}, \mathbf{x}_{\tau}^{4,\text{meas}})$. We then have to calculate the realization $\boldsymbol{\eta}_{\tau}^{\text{meas}}$ of random vector $\boldsymbol{\eta}$ of the generalized coordinates of the global stochastic model introduced in Eqs. (5) and (6). This realization is calculated as the projection of the measurement on the global stochastic model, which yields

$$\boldsymbol{\eta}_{\tau}^{\text{meas}} = [\lambda]^{-1} [Q]^T \mathbf{x}_{\tau}^{\text{meas}}. \quad (16)$$

At long time τ , the adapted stochastic model is then defined as

$$\tilde{\mathbf{X}}_{\tau}^k(\delta_k^{\text{opt}}) = [Q^k] \left(\boldsymbol{\eta}_{\tau}^{\text{meas}} + \delta_k^{\text{opt}} \mathbf{G}^k \right), \quad k = 1, 2, 3, 4. \quad (17)$$

As an illustration, $\mathbf{x}_{\tau_0}^{1,\text{meas}}$ and the confidence region at 95% of $\tilde{\mathbf{X}}_{\tau_0}^1(\delta_1^{\text{opt}})$ are compared in Fig. 2. It can be noticed that the geometrical and physical properties of the irregularities are preserved with the identified adapted stochastic modeling.

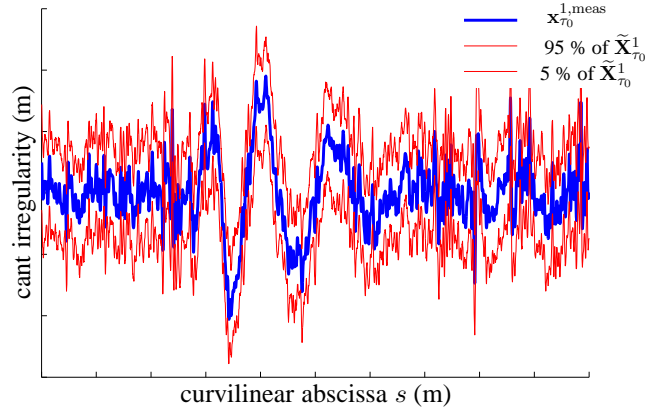


Figure 2: Irregularity $\mathbf{x}_{\tau_0}^{1,\text{meas}}$ and confidence region at 95% of $\tilde{\mathbf{X}}_{\tau_0}^1(\delta_1^{\text{opt}})$ for the given track portion.

2.5 Dynamic response of the train

For fixed long time τ , the stochastic model of the track irregularities is given by $\tilde{\mathbf{X}}_{\tau}(\boldsymbol{\delta}^{\text{opt}}) = (\tilde{\mathbf{X}}_{\tau}^1(\delta_1^{\text{opt}}), \tilde{\mathbf{X}}_{\tau}^2(\delta_2^{\text{opt}}), \tilde{\mathbf{X}}_{\tau}^3(\delta_3^{\text{opt}}), \tilde{\mathbf{X}}_{\tau}^4(\delta_4^{\text{opt}}))$, defined by Eq. (17). The stochastic response of the train is then computed using the Monte-Carlo method. For each realization $\tilde{\mathbf{X}}_{\tau}(\boldsymbol{\delta}^{\text{opt}}; \theta')$, with θ' in Θ' , the deterministic realization of the train response is computed. The set of these realizations allows statistical estimators to be constructed for analyzing the stochastic responses through dynamic indicators. The chosen dynamic indicators are based on criteria described in norm UIC 518 [13] for the homologation of railway vehicles. For $j = 1, \dots, N_C$, the indicators simulated with the computational model are denoted by $C_j^{\text{sim}}(\tau)$ at long time τ , and are function

of the dynamic outputs, denoted by $\{A_j^{\text{sim}}(s, \tau), s \in \Omega\}$, which are forces and accelerations in the train. Indicators $C_j^{\text{sim}}(\tau)$ are defined by

$$C_j^{\text{sim}}(\tau) = \max_{s \in \Omega} |A_j^{\text{sim}}(s, \tau)| \quad , \quad j = 1, \dots, N_C. \quad (18)$$

For given j and for given long time τ , the probability density function (PDF) of random variable $C_j^{\text{sim}}(\tau)$ is plotted in Fig. 3.

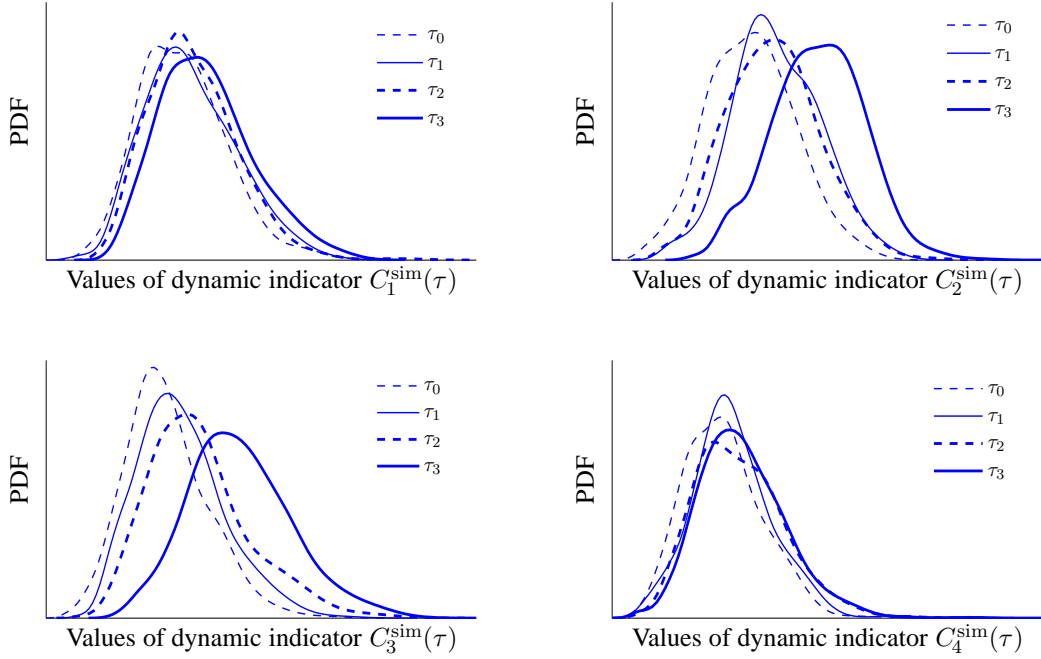


Figure 3: Evolution of the PDF of the dynamic indicators $C_1^{\text{sim}}(\tau)$, $C_2^{\text{sim}}(\tau)$, $C_3^{\text{sim}}(\tau)$ and $C_4^{\text{sim}}(\tau)$ as a function of time τ .

3 Taking into account modeling errors in the train dynamics computational model

The goal of this section is to take into account the uncertainties on the train response output, induced by both the system parameters uncertainties and the modeling errors in the train dynamics computational model. The stochastic model of these uncertainties consists in adding an output noise to the dynamic indicators $C_j^{\text{sim}}(\tau)$, $j = 1, \dots, N_C$, defined in Section 2.5. The stochastic model $C_j^{\text{mod}}(\tau)$ of the dynamic indicators is defined as

$$C_j^{\text{mod}}(\tau) = C_j^{\text{sim}}(\tau) (1 + \tilde{B}_j) \quad , \quad j = 1, \dots, N_C. \quad (19)$$

The noise $\tilde{\mathbf{B}} = (\tilde{B}_1, \dots, \tilde{B}_{N_C})$ is a \mathbb{R}^{N_C} -valued non-Gaussian second-order random vector to be identified comparing simulated dynamic indicators $C_j^{\text{sim}}(\tau)$, $j = 1, \dots, N_C$ with the corresponding dynamic indicators $C_j^{\text{exp}}(\tau)$, $j = 1, \dots, N_C$ that are experimentally measured. The non-Gaussian random vector $\tilde{\mathbf{B}}$ is represented by a polynomial chaos expansion with a uniform germ [2, 11], and the coefficients are estimated using the maximum likelihood with the experiments [7, 10].

4 Long-time evolution of the stochastic model of the dynamic indicators

The long-time evolution of the dynamic indicators are analyzed using the complete stochastic model including the adapted stochastic model of the track irregularities and the output noise. For the given track portion used in Section 2, for each long time τ , 2,000 realizations $\tilde{\mathbf{X}}(\tau; \theta'_1), \dots, \tilde{\mathbf{X}}(\tau; \theta'_{2,000})$ of the track geometry are generated using the adapted stochastic model. The train stochastic dynamic response is simulated by the Monte-Carlo method. The random simulated dynamic indicators $\mathbf{C}^{\text{sim}}(\tau; \theta')$, with θ' in Θ' , are computed using Eq. (18). Besides, at each long time τ , 2,000 independent realizations of the output noise $\tilde{\mathbf{B}}(\tau)$ are generated by the polynomial chaos expansion. The realizations of $C_j^{\text{mod}}(\tau)$ are computed using Eq. (19).

For given j and for long time τ , the PDF of random dynamic indicator $C_j^{\text{mod}}(\tau)$ is plotted in Fig. 4.

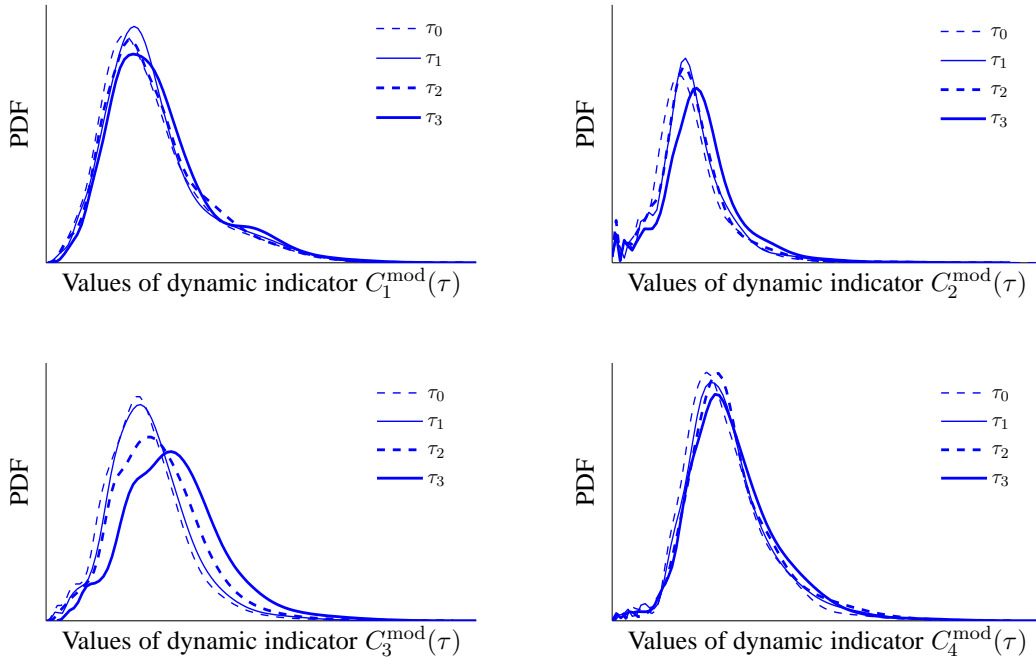


Figure 4: Evolution of the PDFs of the dynamic indicators $C_1^{\text{mod}}(\tau)$, $C_2^{\text{mod}}(\tau)$, $C_3^{\text{mod}}(\tau)$ and $C_4^{\text{mod}}(\tau)$ as a function of time τ .

The constructed indicators are robust enough, because the uncertainties induced by the modeling errors in the computational model of the train dynamics have been removed in the construction of the stochastic representation of the output (this phenomenon can be viewed comparing Fig.4 and Fig. 3). Fig. 4 allows us to observe the PDF of the random indicators as a function of the long time.

5 Conclusion and prospects

This work provides a reliable frame to analyze the evolution of the train dynamic response for a given track portion, knowing the evolution of the track geometry irregularities measured at several long times. The stochastic model that is proposed has been constructed using a very large experimental data basis related to all the French railway network for the high speed trains, and consequently, can be considered as robust. It

takes into account both the track geometry uncertainties and the uncertainties on the train response output induced by the modeling errors in the computational model of the train dynamics. In future works, this model will be used to characterize the long-time evolution of the train dynamic response, in order to identify a long-time evolution model.

Acknowledgements

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