



HAL
open science

abc-Modeling of Permanent Magnet Machines using N-D Lookup Tables: a Finite Element Validation

Loic Queval, Hiroyuki Ohsaki

► **To cite this version:**

Loic Queval, Hiroyuki Ohsaki. abc-Modeling of Permanent Magnet Machines using N-D Lookup Tables: a Finite Element Validation. Symposium de Génie Électrique 2014, Jul 2014, Cachan, France. hal-01065214

HAL Id: hal-01065214

<https://hal.science/hal-01065214>

Submitted on 18 Sep 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

abc-Modeling of Permanent Magnet Machines using N-D Lookup Tables: a Finite Element Validation

Loïc QUEVAL¹, Hiroyuki OHSAKI²

¹ Lab for Electrical Machines, University of Applied Sciences, Düsseldorf, Germany

² Department of Advanced Energy, Graduate School of Frontier Sciences, The University of Tokyo, Japan

ABSTRACT – The lumped-parameter nonlinear abc-model using $i(\lambda)$ N-dimensional lookup tables can naturally include geometry- and saturation-induced space harmonics. It is as accurate as the finite element model while providing a significant speedup. We validate here this model for permanent magnet synchronous machines by extensive comparison with the circuit-coupled finite element model. Three different types of machines (interior permanent magnet machine, surface mounted permanent magnet machine, flux switching permanent magnet machine) are considered, in both steady and transient state, for balanced and unbalanced operation.

KEYWORDS – abc-model, phase-domain model, space harmonics, saturation, modeling, simulation, validation.

1. INTRODUCTION

For the simulation of electrical machines and their coupling with external circuits, a fast and accurate machine model is always desirable. On the one hand, the classical lumped-parameter dq-model is efficient. However, it is established on a number of approximations that are not always applicable. On the other hand, the finite element (FE) model gives accurate results, but it is computationally expensive. Therefore, the lumped-parameter abc-model (or phase-domain model) has progressively re-gained attention [1–6]. The difficulty of the nonlinear abc-model comes from the fact that its parameters are functions of the rotor angle and the windings currents. To deal with these parameters without making any approximation, the authors proposed a novel implementation using current-flux linkage $i(\lambda)$ and electromagnetic torque N-dimensional lookup tables (N-D LUT) [7]. It is as accurate as the FE model because it includes the space harmonics linked to both the magnetic circuit geometry and the (cross-)saturation. But, it is about two orders of magnitude faster. Besides, its parameters are physical values, its derivation is straightforward, and its implementation is independent of the machine type, geometry or materials. Earlier, we validated this model for an electrically-excited synchronous machine by comparison with the circuit-coupled FE model.

In this article, we extend the validation to permanent magnet synchronous machines (PMSM). This is of interest because the use of permanent magnets and peculiar topologies usually require a special care when building a lumped-parameter model, and calculating its parameters. First, we recall the abc-model hypotheses and equations. This is the occasion to clarify the limitations of inductance-based abc-models for modeling PMSMs. Then we underline the capacity of the $i(\lambda)$ -based abc-model in modeling PMSMs. The model is validated for three different types of PMSM by extensive comparison with the

circuit-coupled FE model. Finally, we discuss some possible applications of the abc-model.

2. ABC-MODEL EQUATIONS

We use matrix notation in motor convention, meaning that positive currents enter the machine terminals. In this article, we consider specifically 3-phase PMSMs. The nonlinear abc-model is,

$$\mathbf{v} = \mathbf{R} \mathbf{i} + \frac{d\boldsymbol{\lambda}(\theta, \mathbf{i})}{dt} \quad (1)$$

$$J \frac{d^2\theta}{dt^2} = T_m + T_e(\theta, \mathbf{i}) + T_{damp} \quad (2)$$

where \mathbf{v} , \mathbf{i} , and $\boldsymbol{\lambda}$ are the instantaneous voltage, current and flux linkage of the stator windings, respectively. θ is the rotor mechanical angle. \mathbf{R} is the resistance matrix of the windings. J is the rotor moment of inertia. T_m , T_e and T_{damp} are the mechanical torque, the electromagnetic torque and the damping torque, respectively. $\boldsymbol{\lambda}$ and T_e are nonlinear functions of rotor position and winding currents.

2.1. Inductance-based abc-model

Implementations of the abc-model is generally realized by extending the inductance concept to the nonlinear case. Mohamed *et al.* [2] derived a nonlinear abc-model using the current \mathbf{i} as state variable. By separating the flux linkage contributed by the stator winding $\boldsymbol{\lambda}_s$ from the one contributed by the permanent magnets $\boldsymbol{\lambda}_m$, they obtained,

$$\frac{d\mathbf{i}}{dt} = \mathbf{L}_s^d(\theta, \mathbf{i})^{-1} \left[\mathbf{v} - \mathbf{R} \mathbf{i} - \omega \frac{\partial \mathbf{L}_s^a(\theta, \mathbf{i})}{\partial \theta} \mathbf{i} - \mathbf{e}_s(\theta, \mathbf{i}) \right] \quad (3)$$

where $L_{s,jk}^d = \frac{\partial \lambda_{s,j}}{\partial i_k}$, $L_{s,jk}^a = \frac{\lambda_{s,j}}{i_k}$, ω is the rotor mechanical speed and \mathbf{e}_s is called "back-emf". Poltschak *et al.* [5] derived a different nonlinear abc-model using the current \mathbf{i} as state variable but without resorting to flux linkage separation,

$$\frac{d\mathbf{i}}{dt} = \mathbf{L}^d(\theta, \mathbf{i})^{-1} [\mathbf{v} - \mathbf{R} \mathbf{i} - \mathbf{e}(\theta, \mathbf{i})] \quad (4)$$

where $L_{jk}^d = \frac{\partial \lambda_j}{\partial i_k}$ and \mathbf{e} is called "generalized back-emf". Another possibility is to use the flux linkage $\boldsymbol{\lambda}$ as state variable. Extending the derivation of [8] to PMSMs, one obtains,

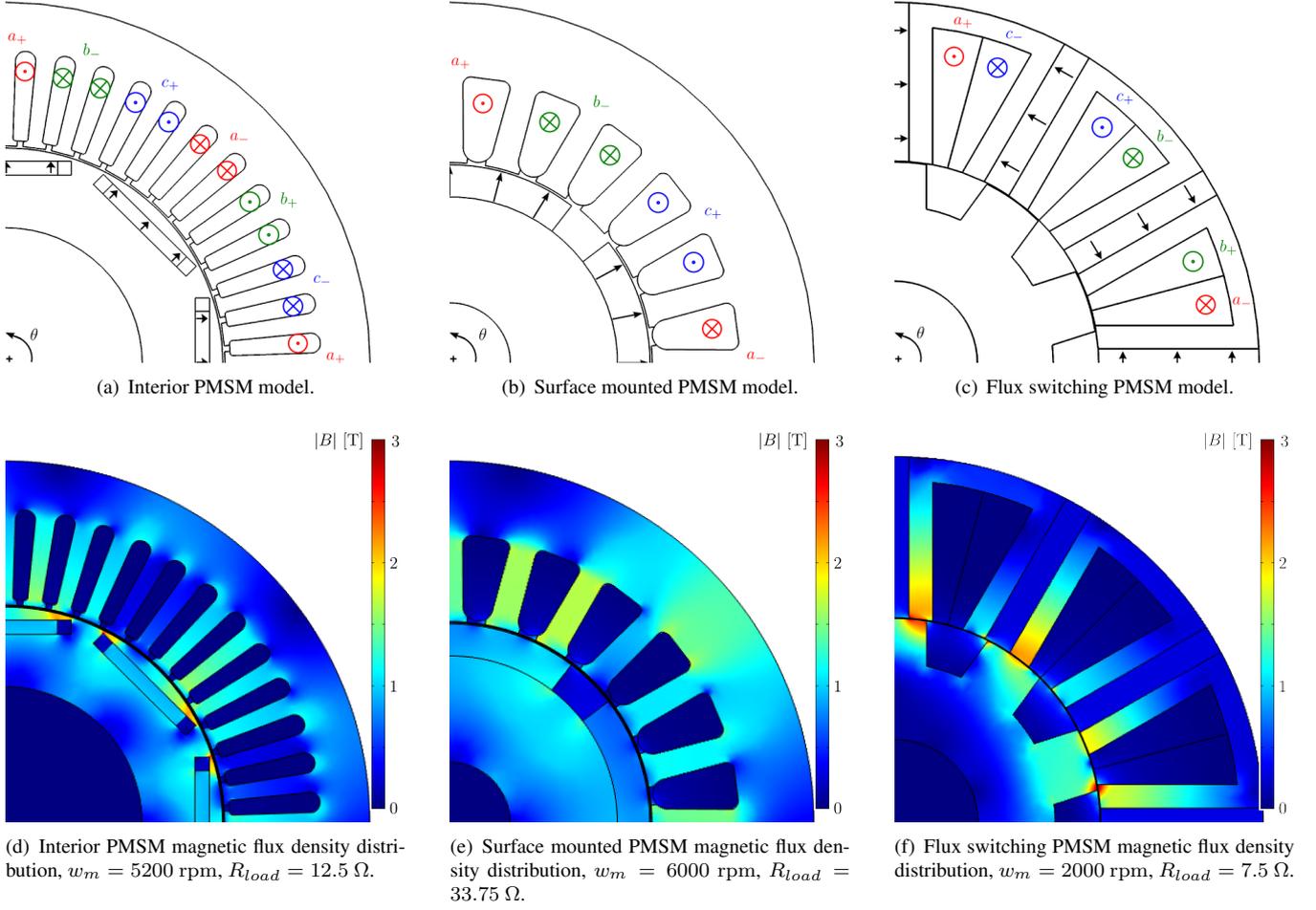


Fig. 1. Permanent magnet machine models used for FE validation.

$$\frac{d\boldsymbol{\lambda}}{dt} = \mathbf{v} - \mathbf{R}\mathbf{L}^a(\boldsymbol{\theta}, \mathbf{i})^{-1} [\boldsymbol{\lambda} - \boldsymbol{\lambda}_m(\boldsymbol{\theta}, \mathbf{i})] \quad (5)$$

where $L_{jk}^a = \frac{\lambda_j}{i_k}$. From the above expressions, the limitation of inductance-based abc-modeling is clear. First, the necessary distinction between apparent inductance \mathbf{L}^a and differential inductance \mathbf{L}^d is error-prone during both parameters calculation and implementation. Secondly, \mathbf{L}^d contains partial derivatives of the flux linkages with respect to the currents. The numerical derivation increases both computation time and numerical errors. Thirdly, it is not necessarily clear how to calculate the "(generalized) back-emf". Finally, a 3-phase PMSM requires the specification of at least one 3×3 inductance matrix and one 3×1 matrix, which coefficients are each a function of the rotation angle θ and the 3 currents. Handling such a volume of data further increases the complexity of the model.

2.2. $i(\lambda)$ -based abc-model

As proposed in [7], we use the flux linkage as state variable, and the current-flux linkage $\mathbf{i}(\boldsymbol{\lambda})$ instead of the inductance. The nonlinear abc-model is then simply,

$$\frac{d\boldsymbol{\lambda}}{dt} = \mathbf{v} - \mathbf{R}\mathbf{i}(\boldsymbol{\theta}, \boldsymbol{\lambda}) \quad (6)$$

The existence of $\mathbf{i}(\boldsymbol{\theta}, \boldsymbol{\lambda})$ is guaranteed by the fact that a change of variable can be done between \mathbf{i} and $\boldsymbol{\lambda}$ under the hypothesis that (1) the losses of the coupling field (eddy current, hysteresis, or dielectric losses) are negligible [9], and (2) that the windings are not perfectly coupled [7]. The advantages of this model are that the variables meaning is clear and that, a 3-phase PMSM requires only the specification of one 3×1 matrix, which coefficients are each a function of the rotation angle θ and the 3 currents. One general way to define $\mathbf{i}(\boldsymbol{\theta}, \boldsymbol{\lambda})$ without making any further approximation is to use three 4-D lookup tables (LUT). The integration of Eq.6 is straightforward once the LUTs $\mathbf{i}(\boldsymbol{\theta}, \boldsymbol{\lambda})$ have been specified. As $\mathbf{i}(\boldsymbol{\theta}, \boldsymbol{\lambda})$ is not directly available, it must be obtained from $\boldsymbol{\lambda}(\boldsymbol{\theta}, \mathbf{i})$. We showed in [7] that this can be done using inversions and 3-D interpolations. The implementation of Eq.6 in Matlab/Simulink has been discussed in [7].

2.3. Electromagnetic torque

In previous works [2, 5, 6], the electromagnetic torque is generally obtained by separation of the synchronous/reluctance torque from the cogging torque. Once more, this introduces difficulties for both parameters calculation and implementation. Therefore, we use directly the electromagnetic torque [7]. It can be obtained by FE calculation using the Maxwell stress tensor at the same time as the $\boldsymbol{\lambda}(\boldsymbol{\theta}, \mathbf{i})$ characteristic, and stored in one 4-D LUT.

3. ABC-MODEL VALIDATION

In this section, we validate the $i(\lambda)$ -based nonlinear abc-model using N-D LUTs for permanent magnet synchronous machines (Eq.6) by comparison with the circuit-coupled finite element model. Three different types of machines are considered : interior permanent magnet machine (IPM), surface mounted permanent magnet machine (SPM) and flux switching permanent magnet machine (FSPM). The machines are 3-phase PMSM, and therefore three 4-D LUTs are used for the electric equation. One 4-D LUT is used for the electromagnetic torque equation. The LUTs are calculated by static 2-D finite element analysis.

Because the abc-model deals with the electrical part of the machine, we consider generator operation at fixed speed. The armature windings are connected to a three phase resistive load. We do not use an inductive/capacitive load to ensure that the current/voltage waveforms harmonics are due only to the space harmonics.

A complete validation should (1) cover a wide range of stator currents and (2) include various transient operations. The former is obtained for each machine by varying the rotor speed at constant load. The latter is done, to keep the article concise, by simulating one different transient for each machine. Even so, this allows us to evaluate both balanced and unbalanced operation, with star point floating or grounded.

3.1. Interior PMSM

We consider the interior PMSM shown in Fig.1(a), adapted from [10]. The abc-model parameters are calculated for all the combinations of $\{\theta, i_a, i_b, i_c\}$ such as,

$$\begin{cases} \theta \in \{0 : 1.875 : 90\} \text{ meca deg} \\ i_{abc} \in \{0.5, 1, 1.3\}^3 \times \pm 10 \text{ A} \end{cases} \quad (7)$$

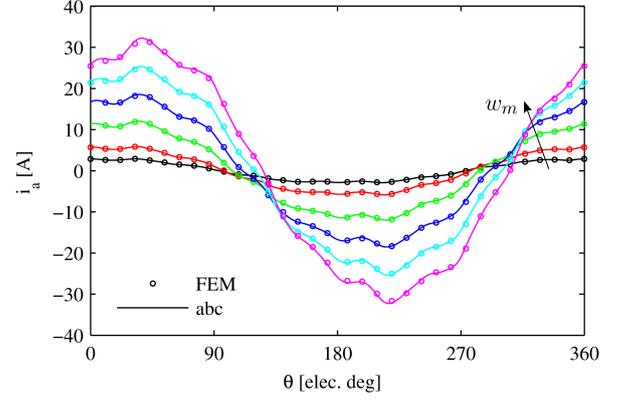
Fig.2(a) shows the steady-state armature current as a function of the rotor speed. To illustrate the ability of the abc-model in modeling unbalanced transients with star point floating, we simulate a single-phase-to-ground fault. Initially the machine is in steady-state operation ($w_m = 1040$ rpm, $R_{load} = 12.5 \Omega$). At t_0 , phase a is suddenly connected to the ground during 1 ms. Fig.2(b) shows the armature currents, voltage and the electromagnetic torque during the transient.

3.2. Surface mounted PMSM

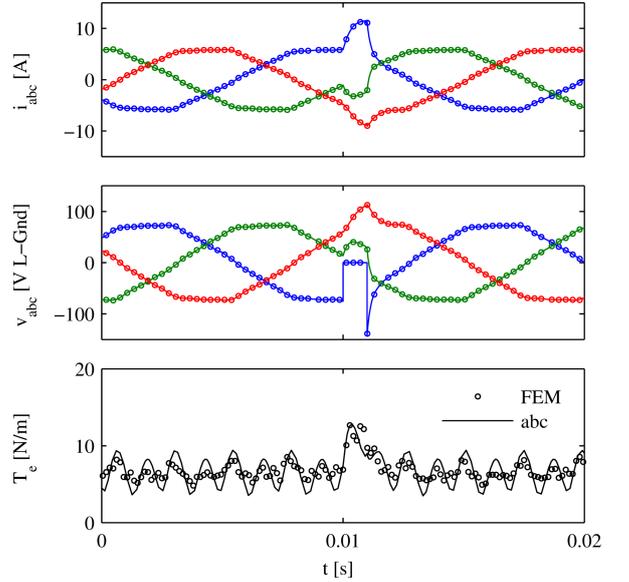
We consider the surface mounted PMSM shown in Fig.1(b), adapted from [11]. The abc-model parameters are calculated for all the combinations of $\{\theta, i_a, i_b, i_c\}$ such as,

$$\begin{cases} \theta \in \{0 : 3.75 : 180\} \text{ meca deg} \\ i_{abc} \in \{0.5, 1, 1.3\}^3 \times \pm 5 \text{ A} \end{cases} \quad (8)$$

Fig.3(a) shows the steady-state armature current as a function of the rotor speed. To illustrate the ability of the abc-model in modeling unbalanced transients with star point grounded, we simulate a phase-to-phase fault. Initially the machine is in steady-state operation ($w_m = 3000$ rpm, $R_{load} = 33.75 \Omega$). At t_0 , phases a and b are suddenly connected during 1 ms. Fig.3(b) shows the armature currents, voltage and the electromagnetic torque during the transient.



(a) Phase a current for various rotor speed. Steady-state, star point grounded, $w_m = 520, 1040, 2080, 3120, 4160, 5200$ rpm, $R_{load} = 12.5 \Omega$.



(b) Armature currents, voltage and the electromagnetic torque. Single-phase-to-ground fault, star point floating, $w_m = 1040$ rpm, $R_{load} = 12.5 \Omega$.

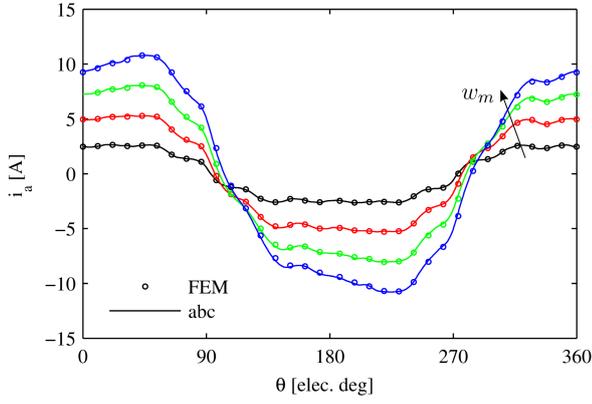
Fig. 2. Interior permanent magnet machine model validation

3.3. Flux switching PMSM

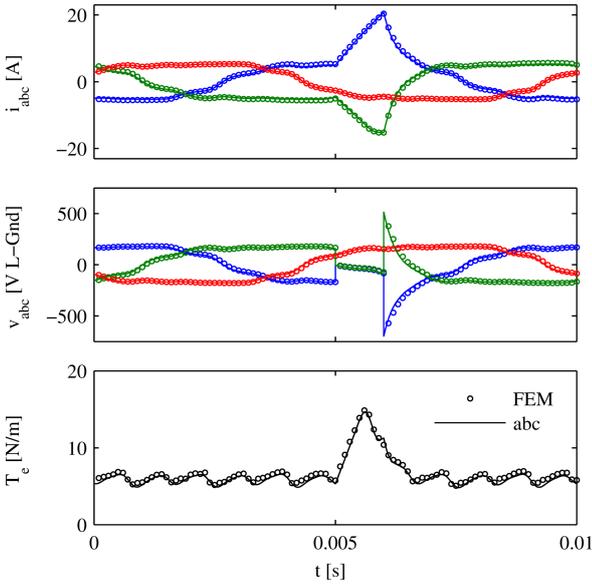
We consider the flux switching PMSM shown in Fig.1(c), adapted from [12]. The abc-model parameters are calculated for all the combinations of $\{\theta, i_a, i_b, i_c\}$ such as,

$$\begin{cases} \theta \in \{0 : 1 : 36\} \text{ meca deg} \\ i_{abc} \in \{0.5, 1, 1.3\}^3 \times \pm 20 \text{ A} \end{cases} \quad (9)$$

Fig.4(a) shows the steady-state armature current as a function of the rotor speed. To show the ability of the abc-model in modeling reluctance torque ripples and balanced transients, we simulate a sudden R load switching. Initially the machine is in steady-state operation at no-load ($w_m = 1000$ rpm, $R_{load} = \infty$). At t_0 , the 3-phase resistive load ($R_{load} = 7.5 \Omega$) is suddenly connected. Fig.4(b) shows the armature currents, voltage and the electromagnetic torque during the transient.



(a) Phase a current for various rotor speed. Steady-state, star point grounded, $w_m = 1500, 3000, 4500, 6000$ rpm, $R_{load} = 33.75 \Omega$.

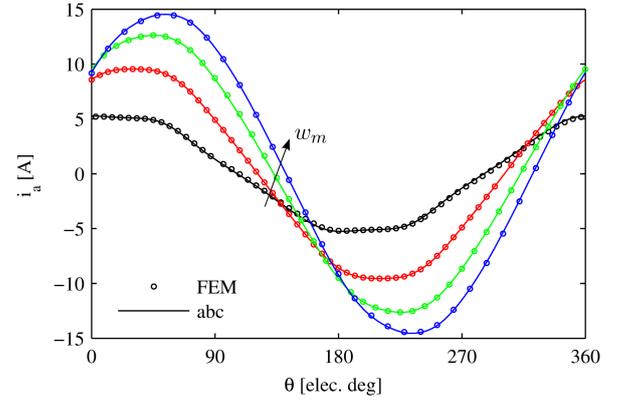


(b) Armature currents, voltage and the electromagnetic torque. Phase-to-phase fault, star point grounded, $w_m = 3000$ rpm, $R_{load} = 33.75 \Omega$.

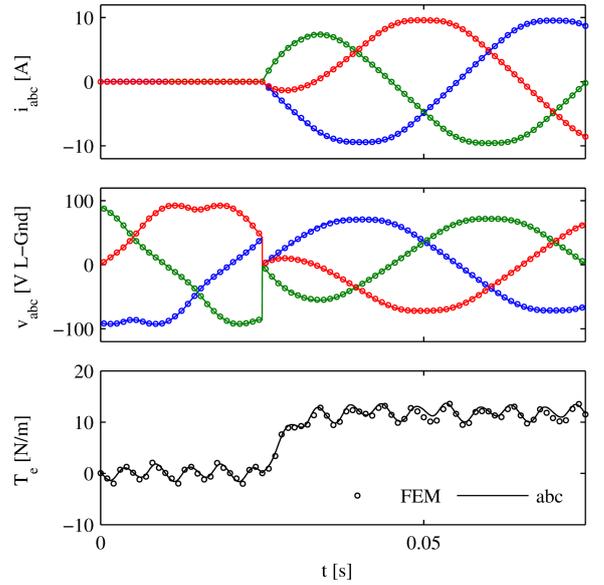
Fig. 3. Surface mounted permanent magnet machine model validation

3.4. Discussion

The nonlinear abc-model using $i(\lambda)$ N-D LUTs and the circuit-coupled finite element model are in very good agreement for all machines and all operating conditions. The slight torque difference between the two models is attributed to the difficulty of torque calculation with the FE model; both the static FE model used to fill up the LUTs and the transient FE model. Note that the abc-model is able to correctly predict operation for LUT out of the range (Figs.2(a), 3(a) and 3(b)), using LUT cubic spline extrapolation for the currents and LUT linear extrapolation for the electromagnetic torque. This validates the ability of the abc-model in modeling geometry- and saturation-induced space harmonics, during steady and transient state, for balanced and unbalanced operation, with star point floating or grounded. Comparison of the models in terms of computation time has been discussed in [7].



(a) Phase a current for various rotor speed. Steady-state, star point grounded, $w_m = 500, 1000, 1500, 2000$ rpm, $R_{load} = 7.5 \Omega$.



(b) Armature currents, voltage and the electromagnetic torque. R load switching, star point grounded, $w_m = 1000$ rpm, $R_{load} = 7.5 \Omega$.

Fig. 4. Flux switching permanent magnet machine model validation

4. APPLICATIONS

4.1. Simulation of integrated power system

The dynamic simulation of integrated power systems involving switching devices and closed-loop control requires small time steps. This makes the use of the circuit-coupled finite element model impractical [13]. On the contrary, the use of the lumped-parameter abc-model keeps the computation time low while maintaining high accuracy. The abc-model can then be used to obtain a realistic estimation of the operating characteristics of a machine-drive system before building a prototype [14]. Another important application is the design of advanced controllers: torque ripple minimization [1], nonlinear sensorless control [15], etc. Finally thanks to its generality, it is able to model machines with complex topologies and to include 3-D effects, without modification.

4.2. Real time simulator

If an electrical machine can be simulated in real time, it is possible to interface the machine model to the physical power

converter (not its model) [16]. This allows to test equipments before on-site installation, to tune controllers for best performance and to shorten development cycles. In this context, specialized hardware-in-the-loop real time simulation tools, such as RTDS® or eDRIVESim®, have been developed. They are able to simulate PM synchronous machines in real-time ($< 1\mu\text{s}$ time step) with high accuracy. The proposed abc-model is a good candidate for such implementations because it uses fewer lookup tables than inductance-based models [4].

4.3. Condition monitoring and fault diagnosis

Machine condition monitoring and fault diagnosis are important for the reliable operation of power systems. A fast and accurate machine model can be used to : obtain machine fault signatures contained in operating signals, evaluate on-line fault diagnosis strategies, or investigate the impact of machine faults on external circuits (excitation and loads). Besides, internal faults in synchronous machines can be simulated by minor modifications of the abc-model [3, 17, 18].

4.4. Noise analysis

Because it includes geometry- and saturation-induced space harmonics, the abc-model can be used for modeling, assessing, and minimizing acoustic noise in electric machines and drives. In reference [19], Boesing *et al.* calculate the electromagnetic forces acting in the air-gap of the machine using a similar model. In combination with structural vibration or radiation models, one could then estimate the acoustic noise of the machine under arbitrary operating conditions.

5. CONCLUSION

We validated the ability of the lumped-parameter nonlinear abc-model using $i(\lambda)$ N-D lookup tables in modeling permanent magnet machines, including IPM, SPM, and FSPM machines. The model estimates with the same accuracy as the circuit-coupled finite element model both geometry- and saturation-induced space harmonics, during steady and transient state, for balanced and unbalanced operation, with star point floating or grounded. Such a model is suited for applications requiring both high accuracy and high computing speed, such as simulation of integrated power system, hardware-in-the-loop real time simulation, machine condition monitoring and fault diagnosis, or noise analysis.

6. REFERENCES

- [1] O.A. Mohammed, S. Liu, Z. Liu, "Physical modeling of PM synchronous motors for integrated coupling with machine drives," *IEEE Trans. on Magnetics*, vol. 41, no. 5, pp. 1628-1631, 2005.
- [2] O.A. Mohammed, S. Liu, Z. Liu, "A phase variable model of brushless dc motors based on finite element analysis and its coupling with external circuits," *IEEE Trans. on Magnetics*, vol. 41, no. 5, pp. 1576-1579, 2005.
- [3] O.A. Mohammed, S. Liu, Z. Liu, "FE-based physical phase variable model of PM synchronous machines under stator winding short circuit faults," *Science, Measurement & Technology, IET*, vol. 1, no. 1, pp. 12-16, 2007.
- [4] O.A. Mohammed, S. Liu, N. Abed, "Simultaneous simulation of PM machine drive using the physics-based phase variable model and the hardware-in-the-loop simulation approach," *IEEE Vehicle Power and Propulsion Conf. (VPPC 2007)*, pp. 132-139, 2007.
- [5] F. Poltschak, W. Amrhein, "A dynamic nonlinear model for permanent magnet synchronous machines," *IEEE Int. Symp. on Industrial Electronics (ISIE 2008)*, 2008, pp. 724-729.
- [6] Y. Kano, K. Watanabe, T. Kosaka, N. Matsui, "A new modeling approach for circuit-field-coupled time-stepping electromagnetic analysis of satura-

- ted interior permanent magnet synchronous motors," *Electrical Engineering in Japan*, vol. 174, no. 1, pp. 49-58, 2011.
- [7] L. Quéval, H. Ohsaki, "Nonlinear abc-model for electrical machines using N-D lookup tables," *under review*, 2014.
- [8] E. Deng, N.A. Demerdash, "A coupled finite-element state-space approach for synchronous generators. I. model development," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 32, no. 2, pp. 775-784, 1996.
- [9] P.C. Krause, O. Wasynczuk, S.D. Sudhoff, *Analysis of electric machinery and drive systems*, IEEE Press, 2002, pp. 11-35.
- [10] R.H. Staunton, S.C. Nelson, P.J. Otaduy, J.M. McKeever, J.M. Bailey, S. Das, R.L. Smith, "PM motor parametric design analyses for a hybrid electric vehicle traction drive application," Oak Ridge National Laboratory report, 2004.
- [11] J. Güdelhöfer, "Evaluierung von Comsol Multiphysics für die Berechnung elektrischer Maschinen," Bachelor thesis, Fachhochschule Düsseldorf, Düsseldorf, Germany, 2001.
- [12] G.J. Li, J. Ojeda, E. Hoang, M. Gabsi, "Thermal-electromagnetic analysis of a fault-tolerant dual-star flux-switching permanent magnet motor for critical applications," *IET Electric Power Applications*, vol. 5, no. 6, pp. 503-513, 2011.
- [13] E. Lange, F. Henrotte, K. Hameyer, "An efficient field-circuit coupling based on a temporary linearization of FE electrical machine models," *IEEE Trans. on Magnetics*, vol. 45, no. 3, 2009.
- [14] L. Quéval, H. Ohsaki, "AC losses of a grid-connected superconducting wind turbine generator," *IEEE Trans. on Applied Superconductivity*, vol. 23, no. 3, pp. 5201905, 2013.
- [15] P. Guglielmi, M. Pastorelli, A. Vagati, "Cross-saturation effects in IPM motors and related impact on sensorless control," *IEEE Trans. on Industry Application*, vol. 42, no. 6, pp. 1516-1522, 2006.
- [16] P. Forsyth, M. Trevor, R. Kuffel, "Real time digital simulation for control and protection system testing," *IEEE 35th Annual Power Electronics Specialists Conf. (PESC 04)*, vol. 1, pp. 329-335, 2004.
- [17] A.I. Megahed, O.P. Malik, "Synchronous generator internal fault computation and experimental verification," *IEE Proc. Generation, Transmission and Distribution*, vol. 145, no. 5, pp. 604-10, 1998.
- [18] P. Subramaniam, O.P. Malik, "Digital simulation of a synchronous generator in direct-phase quantities," *Proc. of the IEE*, vol. 118, no. 1, pp. 153-160, 1971.
- [19] M. Boesing, M. Niessen, T. Lange, R. De Doncker, "Modeling spatial harmonics and switching frequencies in PM synchronous machines and their electromagnetic forces," *Int. Conf. on Electrical Machines (ICEM 2012)*, pp. 3001-3007, 2012.