Finite element computation of leaky modes in straight and helical elastic waveguides

Khac-Long Nguyen, Fabien Treyssede, Christophe Hazard, Anne-Sophie Bonnet-Ben Dhia

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Abstract Elastic guided waves are of interest for inspecting structures due to their ability to propagate over long distances. However, waveguides are often embedded in a large solid domain, considered as unbounded. The waves can attenuate strongly along the guide axis due to the energy leakage into the surrounding medium, which reduces the propagating distance. Searching modes with low attenuation becomes necessary. The goal of this work is to propose a numerical approach to compute modes in embedded elastic waveguides (straight and helical structures), combining the so-called semi-analytical finite element method (SAFE) and a perfectly matched layer (PML) method. The application of this work is the non destructive evaluation of multi-wire strands, which constitute cables in civil structures.

1 Introduction

For embedded structures, the energy leakage into the surrounding medium yields wave attenuation along the guide axis, which limits the application of guided wave technique for non destructive evaluation (NDE). An accurate determination of low leakage modes becomes necessary to maximize the inspecting range. Analytical tools have been developed to model simple geometries (plates and cylinders) [1, 2].

For complex geometries, modeling often relies on numerical approaches. A classical method is the so-called semi-analytical finite element (SAFE) method restricting the computational domain on the discretized waveguide cross-section. Yet, two difficulties are encountered for the numerical simulation of embedded waveguides : the unbounded cross-section and the presence of leaky modes whose amplitudes grow in transverse directions. To circumvent these difficulties, the SAFE method

K. L. Nguyen, F. Treyssède
IFSTTAR; Route de Bouaye, 44344 Bouguenais, France, e-mail: fabien.treyssede@ifsttar.fr

C. Hazard, A.-S. Bonnet-BenDhia
ENSTA Paris Tech; 828, Boulevard des Maréchaux, 91762 Palaiseau, France

has been combined with other techniques such as absorbing layer [3] and boundary element methods [4]. An alternative approach proposed in this paper is the perfectly matched layer (PML) method. A SAFE-PML approach has already been applied to model embedded solid plate waveguides in two dimensions [5].

The goal of this work is to extend this technique to model three-dimensional elastic waveguides buried in a solid matrix. First of all, the modeling of embedded straight waveguides is considered. The next step consists in applying the PML method in transverse twisting coordinates [6] to compute modes in embedded helical structures.

2 Straight waveguides

A PML applied along the Cartesian transverse directions (X,Y) consists in extending the elastodynamic formulation into complex coordinates $(\tilde{X}(X),\tilde{Y}(Y))$ to absorb waves in the surrounding medium: $\tilde{X}=\int_0^X \gamma_X(\xi)d\xi$, $\tilde{Y}=\int_0^Y \gamma_Y(\xi)d\xi$. A change of variable is then performed to transform the formulation back to real coordinates. In addition to the PML technique, the SAFE method is applied, which assumes an e^{ikZ} dependence, where k is the axial wavenumber and Z is the axial direction. The FE dicretization yields the quadratic eigenvalue problem :

$$\{\mathbf{K}_1-\omega^2\mathbf{M}+ik(\mathbf{K}_2-\mathbf{K}_2^T)+k^2\mathbf{K}_3\}\mathbf{U}=0 \tag{1}$$

Numerical results are computed for a steel bar in concrete. The eigenspectrum at a given frequency shows that the PML method not only computes physical modes (leaky) but also non-intrinsic modes (radiation modes) which depend on PML characteristics and resonate mainly in the PML region. In order to eliminate radiation modes, a filtering criterion is applied, defined by the ratio of kinetic energy in the PML over the kinetic energy in the whole domain. As shown in Fig. 1, dispersion curves of leaky modes are well identified after filtering. Numerical results obtained by the SAFE-PML method are in agreement with Refs. [3, 4].

3 Helical waveguides

Helical waveguides must be considered in twisting coordinates (x,y,z) defined as :

$$x=X\cos(\tau Z)+Y\sin(\tau Z),\quad y=-X\sin(\tau Z)+Y\cos(\tau Z),\quad z=Z \tag{2}$$

where τ denotes the torsion of the (x,y) plane around the Z-axis.

The twisted PML method transforms twisted transverse coordinates (x,y) into complex ones $(\tilde{x}(x),\tilde{y}(y))$. Similarly to the previous section, a twisted SAFE-PML approach can then be applied.

3.1 Validation

Since a twisted cylinder remains a cylinder, this test case can be considered to validate the twisted SAFE-PML method.

The comparison between the axial wavenumbers computed in both coordinate systems (with twist and without) shows that the wavenumbers of compressional and torsional modes are unchanged while those of flexural modes are translated by $\pm\tau m$ in the twisting system, where m denotes the circumferential order (details in [6]).

Although the axial wavenumbers are different in twisting and straight coordinates, the physics of cylindrical modes are left unchanged. Figure 1 shows the agreement between the energy velocities in both coordinate systems.

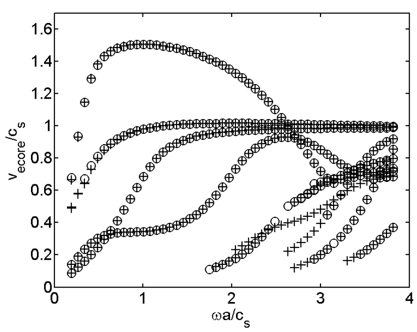

Fig. 1 Energy velocity for an embedded cylindrical bar obtained by SAFE-PML (circles) and twisted SAFE-PML (crosses) methods.

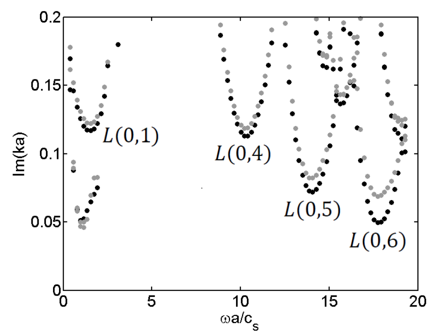

Fig. 2 Attenuation of modes in an embedded helical wire (black) and in an embedded cylindrical bar (gray).

3.2 Examples

Two examples of embedded helical waveguides are now considered : a steel helical wire and a steel seven-wire strand buried in concrete.

Figure 2 compares the axial attenuation of compressional $L(0,n)$ modes in an embedded helical wire with those of an embedded cylindrical bar. This comparison concentrates on compressional modes, which have low attenuations and high energy velocities. Compressional modes are more attenuated in a helical wire than in a straight one. This difference becomes more significant for high order modes. This result shows that the curvature of helical wires reduces the wave propagation distance and can make their NDE more difficult. This phenomenon has been observed for other kinds of curved waveguides [7].

The dispersion curves of energy velocity for a steel seven-wire strand in concrete is given by Fig. 3. The strand consists of one cylindrical bar surrounded by six

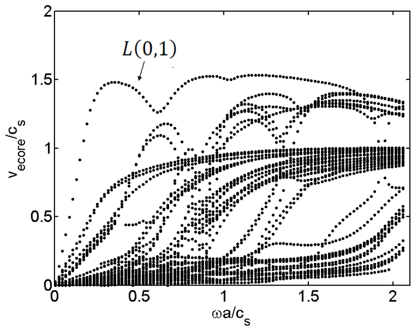

Fig. 3 Energy velocity of an embedded seven-wire strand.

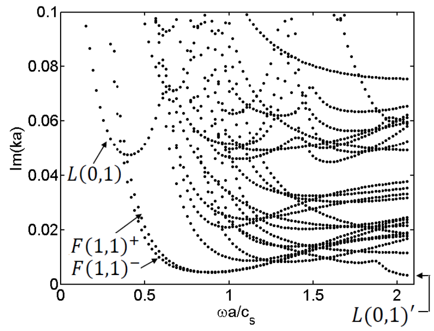

Fig. 4 Attenuation of modes in an embedded seven-wire strand.

helical wires. Comparing with the results of a free strand [6], the modal behaviour is strongly modified due to the introduction of the surrounding medium. The wave attenuation is shown in Fig. 4. $F(1,1)^{\pm}$ and $L(0,1)'$ modes have the lowest attenuations in the frequency range $[0, 1.5]$ and $[1.5, 2]$ respectively, which may be of interest for NDE. In addition to the attenuation due to the energy leakage, the surrounding medium has the other effect to make helical wires almost in contact, which further modifies the dispersion curves.

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