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Excitation of prestressed multi-wire helical waveguides

Fabien Treyssède, Laurent Laguerre

Abstract Elastic guided waves are of interest for the non-destructive evaluation (NDE) of cables. Guided waves are multimodal and dispersive. Cables are complex structures, helical, multi-wired and highly prestressed. This further complicates the interpretation of measurement. Numerical models are required for understanding guided wave propagation and optimizing inspection systems. This paper reports on the modeling of wave propagation inside seven-wire strands, typically encountered in civil-engineering cables, including complicating effects such as geometry curvature and prestress. Special attention is given to the interwire energy transfer occurring in a seven-wire strand excited by a source localized in a single wire. Numerical results show how the energy transfer decreases with frequency, which leads to the discovery of a compressional mode of local type. This mode may have interesting features for the NDE of cables.

1 Introduction

Understanding wave propagation inside cables is a complicated task due to the structural complexity in addition to the multimodal and dispersive nature of guided waves. Cables are generally made of individual helical steel wires that are in contact and subjected to high tensioning forces. This work focuses on seven-wire strands, constituted by one central cylindrical wire and six peripheral helical wires, typically encountered in civil-engineering cables. Numerical tools are required for modeling wave propagation inside such complex structures. In this work, a semi-analytical finite element (SAFE) formulation is proposed. The formulation is specifically written in a non trivial twisting coordinate system and accounts for the effects of axial load.

IFSTTAR; GERS, AI, Route de Bouaye, 44344 Bouguenais, France, e-mail: fabien.treyssede@ifsttar.fr, laurent.laguerre@ifsttar.fr

2 Computation of guided modes

First, we briefly review the twisting SAFE method for the computation of guided modes in prestressed helical structures [1]. A SAFE approach consists in applying a time Fourier transform as well as a spatial Fourier transform along the waveguide axis z before discretizing the cross-section (x, y) by a finite element method. Inside one finite element e , the displacement field is thus expressed as follows:

$$\mathbf{u}(x, y, z, t) = \mathbf{N}^e(x, y) \mathbf{U}^e e^{i(kz - \omega t)} \quad (1)$$

where \mathbf{U}^e is the nodal displacement vector and \mathbf{N}^e is the matrix of nodal interpolating functions of the element e . The variational formulation of three-dimensional elastodynamics yields an eigenvalue problem of the following form:

$$\{\mathbf{K}_1 - \omega^2 \mathbf{M} + ik(\mathbf{K}_2 - \mathbf{K}_2^T) + k^2 \mathbf{K}_3\} \mathbf{U} = \mathbf{0} \quad (2)$$

In this work, one emphasizes that the equilibrium equations must be rewritten in a twisting coordinate system, defined by a non zero torsion τ . For clarity, the strain-displacement relation written in a twisting system is given by [2]:

$$\boldsymbol{\varepsilon} = (\mathbf{L}_{xy} + \mathbf{L}_z \partial / \partial z) \mathbf{u}, \quad \text{with: } \mathbf{L}_{xy} = \begin{bmatrix} \partial / \partial x & 0 & 0 \\ 0 & \partial / \partial y & 0 \\ 0 & 0 & \Lambda \\ \partial / \partial y & \partial / \partial x & 0 \\ \Lambda & -\tau & \partial / \partial x \\ \tau & \Lambda & \partial / \partial y \end{bmatrix}, \quad \mathbf{L}_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (3)$$

where $\Lambda = \tau y \partial / \partial x - \tau x \partial / \partial y$. One points out that the expressions for \mathbf{L}_{xy} and \mathbf{L}_z do not depend on s , which proves that guided waves truly exist in helical structures [3].

Furthermore, the variational formulation must account for prestress effects and is augmented by the following additional term, sometimes referred to as geometric stiffness:

$$\int_{V_0} \text{tr}(\nabla_0 \delta \mathbf{u} \cdot \boldsymbol{\sigma}_0 \cdot \nabla_0 \mathbf{u}^T) dV_0 \quad (4)$$

where subscripts 0 are used for the prestressed configuration. $\boldsymbol{\sigma}_0$ denotes Cauchy prestress. In Eq. (2), each matrix must hence be augmented by a term related to the above geometric stiffness operator.

The FE mesh of the seven-wire strand cross-section is shown in Fig. 1a. We suppose that friction is high enough to prevent interwire slip. Figure 1b shows the energy velocity dispersion curves of a strand subjected to an axial tensile strain of 0.6%. Although dispersion curves exhibit a complex pattern, the figure particularly shows an apparent cut-off of the fastest mode (compressional-like $L(0, 1)$ mode) around $\omega a / c_s = 0.44$, i.e. 86kHz for steel strands of 15.7mm nominal diameter. This phenomenon, sometimes referred to as 'notch frequency', coincides with experimental results of the literature [4, 5]. It actually corresponds to a sudden veering between two distinct branches [2], denoted as $L(0, 1)a$ and $L(0, 1)b$. Without load,

the notch frequency is centered around 0.36, i.e. 68kHz (results not shown for conciseness). Numerical experiments have shown that the phenomenon responsible for this frequency shift is indeed the increase of interwire contact area with the axial load.

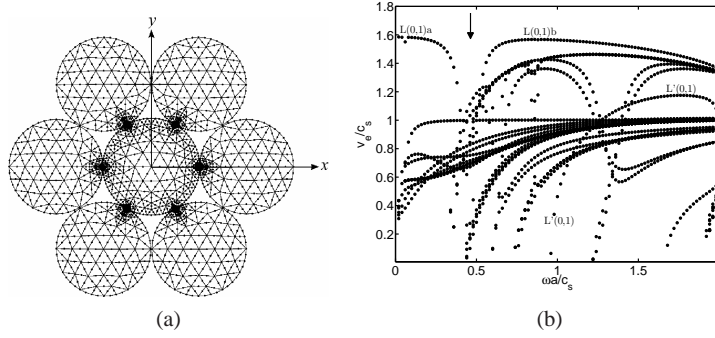


Fig. 1: (a) Cross-section FE mesh of a seven-wire strand, (b) normalized energy velocity versus frequency under 0.6% tensile strain (the arrow indicates the notch frequency).

3 Response under excitation

Now, we are interested in solving the forced response problem, similar to Eq. (2) but with a non-zero excitation $\mathbf{F}(k)$ at the right hand side. The waveguide response can be obtained by expanding the solution as a sum of guided modes. Taking advantage of biorthogonality relations and applying the Cauchy residue theorem, the authors have shown in Ref. [6] that the SAFE solution as a function of $z > 0$ can be written as:

$$\mathbf{U} = \sum_{m=1}^M \alpha_m \frac{\mathbf{U}_m}{\sqrt{P_m}} e^{ik_m z}, \quad \text{with: } \alpha_m = \frac{i\omega}{4\sqrt{P_m}} \mathbf{U}_m^* \mathbf{F}(k_m) \quad (5)$$

The summation in Eq. (5) is performed over positive-going modes. P_m denotes the power flow of the m th mode. The above solution neglects the contribution of non-propagating modes. Viscoelasticity is not considered here.

One considers an excitation \mathbf{F} , normal to the cross-section, distributed over the cross-section of the central wire (half-sine function vanishing at its boundary) and concentrated at $z = 0$. One focuses on the transfer of energy from the central wire to the peripheral ones. The acoustic field of interest to quantify the wave energy is the power flow. We define η , the ratio of the power flow inside the six peripheral wires to the power flow of the whole seven-wire strand. Therefore, a small η means a weak energy transfer to the peripheral wires and inversely.

Figure 2a shows η as a function of frequency for a loaded strand (0.6% tensile strain). In the low frequency region, the energy transfer to peripheral wires is high. This is an expected result since the $L(0,1)$ -like mode of a strand has a global behavior [2]. Then from $\omega a/c_s = 1$, η suddenly drops. This drop is indeed explained by the excitation of a new compressional-like mode, denoted as $L'(0,1)$ in Fig. 1b, whose motion is localized into the central wire. Figure 2b plots the modulus of the modal coefficients α_m of propagating modes. The power of the localized $L'(0,1)$ mode after its cut-on is clearly greater than the global $L(0,1)$ mode. This new mode could be of interest for NDE applications.

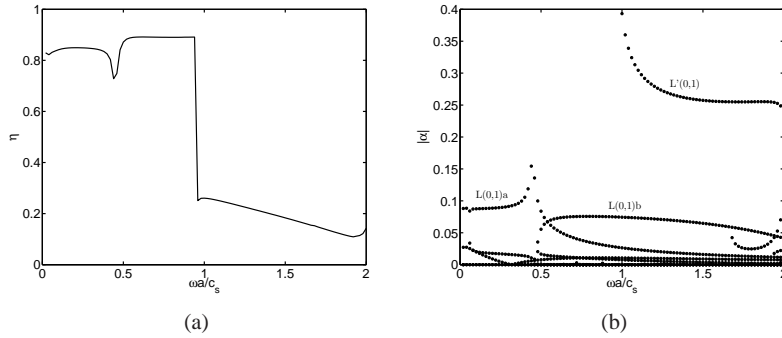


Fig. 2: (a) Energy transfer ratio, (b) modulus of modal coefficients for a strand subjected to a 0.6% tensile strain

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