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# On the solutions to complex parameter-dependent LMIs involved in the stability analysis of 2D discrete models

Olivier Bachelier<sup>1</sup>, Didier Henrion<sup>2,3,4</sup>, Nima Yeganefar<sup>1</sup>, Driss Mehdi<sup>1</sup>

## Abstract

The aim of this short communiqué is to adapt a result established by Bliman, related to the possible approximation of the solutions to real-parameter-dependent linear matrix inequalities (LMIs), to the special context of stability analysis of 2D discrete Roesser models. While Bliman considered the case of LMIs involving several *real* parameters, which is especially crucial for the analysis of linear systems against parametric deflections, the stability of Roesser models leads to consider LMIs with only one single *complex* parameter. Extending the results from real parameters to complex ones is not straightforward in our opinion. This is why the present note discusses precautions to be taken concerning this case before applying the results in a 2D context. Actually, it is shown that a well-known condition for *structural* stability of a 2D discrete Roesser can be relaxed into an LMI system whose solution polynomially depends on a single complex parameter over the unit circle.

## 1 Motivation and introduction

Multidimensional models raised a growing interest within the community of automatic control during the last three decades, due to the wide field of their applications (signal and image processing, seismic phenomena, ... see [13, 10]). All systems for which information propagates in more than one direction are eligible to be described by such models. Among them, the 2D models are probably the most studied ones, since they can be used to describe the behaviours of, *e.g.* Iterative Learning Control (ILC)-schemes or so-called repetitive processes [10, 18]. In the present contribution, we focus on 2D discrete Roesser models (although similar developments could be led for continuous or mixed continuous-discrete models). These models comply with

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$$\begin{bmatrix} x^1(j_1 + 1, j_2) \\ x^2(j_1, j_2 + 1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^1(j_1, j_2) \\ x^2(j_1, j_2) \end{bmatrix}. \quad (1)$$

The state vector is divided into two subvectors corresponding to the propagation of the information in two directions. The matrices  $A_{ij}$  are here assumed to be real. A frequently used stability criterion is as follows (see [15], the references and the discussion therein).

**Lemma 1** *System (1) is stable if and only if  $A_{22}$  is Schur-stable (i.e. has no eigenvalue outside the unit disc) and  $M(\delta) = A_{11} + A_{12}(I - \delta A_{22})^{-1}\delta A_{21}$  is Schur-stable for any  $\delta \in \mathbb{C}$  such that  $|\delta| = 1$ .*

From this let us derive the next lemma.

**Lemma 2** *System (1) is stable if and only if there exist a positive definite matrix  $Y$  and a parameter-dependent Hermitian matrix  $P(\delta)$  such that*

$$A'_{22}Y + YA_{22} < 0 \quad (2)$$

and

$$-P(\delta) + M'(\delta)P(\delta)M(\delta) < 0, \quad \forall \delta \in \mathbb{C} : |\delta| = 1 \quad (3)$$

**Proof.** Inequalities (2) and (3) are just Stein's inequalities [22] which are equivalent to the Schur-stability of  $A_{22}$  and  $M(\delta)$  respectively, thus matching the conditions of Lemma 1.  $\square$

**Remark 1** *There are similar results for 2D continuous or mixed continuous-discrete 2D systems, which are not detailed here.*

Lemma 2 is interesting but involves condition (3) which is not computationally tractable due to its dependence on the complex parameter  $\delta$ . However, if one could prove that  $P(\delta)$  might comply to a specified non conservative structure, it could allow the research for tractable relaxations. The establishment of such relaxations is the underlying motivation behind this work. But the present note focuses on the preliminary step *i.e.* the relevance of the structure that can be assumed for  $P(\delta)$ . In the sequel we discuss this issue in order to prove the main result which is now stated.

**Theorem 1** *System (1) is stable if there exist positive definite matrices  $Y$  and*

$$P(\delta) = \left( \sum_{i=1}^{\nu} P_i \delta^i \right)^H, \quad P_i \in \mathbb{C}^{n \times n}, \quad (4)$$

*(where  $X^H$  denotes  $X + X'$ ) such that (2) and (3) are satisfied. Moreover, the condition is also necessary for a sufficiently large value of the degree  $\nu$ .*

Theorem 1 was used (but not proved) in [1] to derive a simple and rather tractable hierarchy of sufficient LMI conditions for (2-4), which tends to necessity, by using a classical S-procedure as formulated in [21]. To fully motivate Theorem 1, we have to explain the interest in applying the S-procedure whereas there exist more sophisticated relaxation schemes such as moment-SOS (sum of squares), or Lasserre’s hierarchy [14]. Indeed, it could be possible to write  $\delta = a + \mathbf{i}b$  with  $(a; b) \in \mathbb{R}^2$  and  $\mathbf{i}$  being the imaginary unit, and then to prove the existence of a solution to (3) that would be polynomial with respect to  $a$  and  $b$  (see for instance [20]). Such a polynomial could be computed by using SOS techniques. But the interest in using the S-procedure associated with the Linear Fractional Representation (LFR) framework to eliminate  $\delta \in \mathbb{C}$  from the condition is that this relaxation not only possibly leads to non conservative LMIs in the problem that we aim at tackling, but, moreover, straightforwardly yields clear expressions of these LMIs [1], which can be a basis for an extension to stabilizing control. Hence a motivation to obtain an expression of  $P$  w.r.t  $\delta$ . Unfortunately, in [1], Theorem 1 was taken at a face value with no actual proof. This paper aims at filling this gap.

The LMIs are a tool whose utility and the popularity are no longer to be demonstrated [5]. They constitute an effective approach to solve numerous problems in automatic control. Some of those problems unfortunately involve parameter-dependent LMIs. Typically, analyzing the stability of linear systems against parameter uncertainties amounts to assessing the existence of a solution (*i.e.* a Lyapunov function) which itself depends on these parameters [9, 11, 7], which is a difficult problem. In order to efficiently relax such a problem, the question of the way this solution could depend on the parameters is of course important. Bliman made a very important step in the understanding of such a question. Indeed, in [4], he proved that when the LMI depends on real parameters, its solution can be considered polynomial with respect to the parameters without conservatism. The question of its minimal degree is still open.

In the remainder of the paper, inspired by the work of Bliman [4] and motivated by our will to fully justify the work in [1], we discuss the possibility for the solutions to complex parameter-dependent LMIs to be assumed polynomial with respect to the complex parameters. The purpose is not to provide a complete complex counterpart with a complete proof, which would be very redundant with Bliman’s work, but to insist on the steps where some adaptations are required and to highlight the currently existing limits which prevent the complete generalization of Bliman’s result. But it must be kept in mind that the case with only one complex parameter is of special interest for us to prove Theorem 1.

## 2 Bliman’s result

In this section, we recall Bliman’s theorem and give a very brief outline of his proof. Let us define the following expression:

$$G(x, \delta) = G_0(\delta) + \sum_{i=1}^p x_i G_i(\delta), \delta \in K, x = \begin{bmatrix} x_1 \\ \vdots \\ x_p \end{bmatrix} \in \mathbb{R}^p, \quad (5)$$

where  $K$  is a compact subset of  $\mathbb{R}^m$  and  $G_0, G_1, \dots, G_p$  are mappings defined in  $K$  and taking values in the set of symmetric matrices of  $\mathbb{R}^{n \times n}$ .

**Theorem 2** [4] *Assume that mappings  $G_i$ ,  $i = 1, \dots, p$ , are continuous. If for any  $\delta \in K$ , there exists  $x(\delta)$  such that  $G(x(\delta), \delta) > 0$ , then there exists a polynomial function  $x^* : K \rightarrow \mathbb{R}^p$  such that for any  $\delta \in K$ ,  $G(x^*(\delta), \delta) > 0$ .*

The result is very strong for two reasons. First, it proves that there exists a function  $x^*(\delta)$  which can be used as a solution for every  $\delta$ . Moreover, it states that this function can be polynomial. It is not worth detailing the proof which can be found in [4] but it is useful, for the sequel, to recall an outline.

**Outline of the proof of Theorem 2.** The proof consists in four steps:

- i) It starts with noting that, if for any  $\delta \in K$ , there exists  $x(\delta)$  such that  $G(x(\delta), \delta) > 0$  holds, then there exists  $\alpha > 0$  such that for any  $\delta \in K$ , the inequality  $G(x, \delta) \geq 2\alpha I$  has a nonempty set of solutions. It leads to define the set-valued map  $F$  which, for any  $\delta \in K$ , associates the nonempty closed convex set

$$F(\delta) = \{x \in \mathbb{R}^p : G(x, \delta) \geq \alpha I\}. \quad (6)$$

(Note that any  $x \in F(\delta)$  satisfies  $G(x, \delta) > 0$ .)

- ii) Then the lower semicontinuity of  $F$  is proved.
- iii) Michael's selection theorem [17] is invoked to prove that, from  $F$ , there exists a continuous selection  $f$  mapping elements of  $K$  into elements of  $\mathbb{R}^p$  and such that, for any  $\delta \in K$ ,  $G(f(\delta), \delta) \geq \alpha I$ .
- iv) The last step applies Weierstrass approximation theorem [8] to prove that each real-valued entry of  $f$  is the limit of a sequence of polynomials, in the sense of uniform convergence.

Our purpose leads us to discuss the extension of Theorem 2 where  $K$  is a compact subset of  $\mathbb{C}^m$  and where each  $G_i(\delta)$  belongs to  $\mathbb{C}^{n \times n}$ . The three first steps of Bliman's proof can be preserved. All the discussion concerns the fourth step. We will show that the extension to the complex case is not so obvious and deserves a little attention.

### 3 The complex case

In this section, we discuss to what extend complex parameters can be considered in Theorem 2. But before doing so, we remind the reader of the needed mathematical theorems.

**Theorem 3** (Weierstrass approximation theorem [19]) *Suppose that  $f$  is a continuous real-valued function defined on the real range  $[a, b]$ . Then there exists a sequence of real-valued polynomial functions which uniformly converges to  $f$ .*

In other words, it is possible to approximate a real-valued function  $f$  by a real-valued polynomial function with any desired accuracy, provided that the degree of the underlying polynomial is large enough. Bliman actually applies an extension which is the Stone-Weierstrass Theorem.

**Theorem 4** (Stone-Weierstrass theorem [23, 8, 19]) *Let  $K$  be a compact Hausdorff space and let  $\mathcal{P}$  be a subalgebra of the set  $\mathcal{C}(K, \mathbb{R})$  of continuous functions from  $K$  to  $\mathbb{R}$ , then  $\mathcal{P}$  is dense in  $\mathcal{C}(K, \mathbb{R})$  if and only if it includes a non zero constant function and separates the points.*

Indeed, a subset  $K \in \mathbb{R}^m$  is a compact Hausdorff space and the set of polynomial functions with arguments  $\delta$  taken in  $K$  is a subalgebra of  $\mathcal{C}(K, \mathbb{R})$ . It would also be possible to consider  $K$  as a subset of  $\mathbb{C}^m$ . However, we also want to consider matrices  $G(\delta)$  in  $\mathbb{C}^{n \times n}$  and therefore  $\mathcal{C}(K, \mathbb{C})$ , the set of continuous functions from  $K$  to  $\mathbb{C}$ , should be involved in Theorem 4 instead of  $\mathcal{C}(K, \mathbb{R})$ . But such a substitution in the writing of the theorem is not possible. The extension is not so simple but it exists and it is due to Glimm [12]. We here propose a simplified version which is especially taylorred for our needs.

**Theorem 5** (Complex version of Stone-Weierstrass theorem [12]) *Let  $K$  be a compact Hausdorff space and let  $\mathcal{C}(K, \mathbb{C})$  be the set of continuous functions from  $K$  to  $\mathbb{C}$ . Also let  $\mathcal{P}$  be a subalgebra of  $\mathcal{C}(K, \mathbb{C})$  including a non zero constant function and which separates the points. Then  $\mathcal{P}$  is dense in  $\mathcal{C}(K, \mathbb{C})$  if and only if it is a  $*$ -algebra with  $'*$ ' being the relation which associates two complex numbers by conjugation.*

This theorem allows the consideration of complex functions but also adds an additional property to be checked by  $\mathcal{P}$ . This is a key issue in our motivation to write this paper since the set of polynomial functions from  $K$  to  $\mathbb{C}$  is not stable under conjugation. Therefore, we have to carefully choose the set  $\mathcal{P}$  to make this theorem valid.

Before stating our result, we also recall the next theorem, which is another avatar of Weierstrass approximation theorem.

**Theorem 6** (Mergelyan's theorem [16, 19]) *Let  $K$  be a compact subset of  $\mathbb{C}$  such that  $\mathbb{C} \setminus K$  is connected. Any continuous function  $f : K \rightarrow \mathbb{C}$  which is holomorphic on its interior  $\text{int}(K)$  can be uniformly approximated on  $K$  by a polynomial function.*

Bliman's theorem (Theorem 2) relies on the two first theorems recalled in this section. With the two other ones, we are now able to state the next theorem which is a key result before introducing Theorem 1.

**Theorem 7** *Let the following expression be defined:*

$$G(x, \delta) = G_0(\delta) + \sum_{i=1}^p x_i G_i(\delta), \delta \in K, x = \begin{bmatrix} x_1 \\ \vdots \\ x_p \end{bmatrix} \in \mathbb{C}^p, \quad (7)$$

where  $K$  is a compact subset of  $\mathbb{C}^m$  and  $G_0, G_1, \dots, G_p$  are continuous mappings defined in  $K$  and taking values in the set of Hermitian matrices of  $\mathbb{C}^{n \times n}$ . Also define  $K' \subseteq \mathbb{C}^m$  as the image of  $K$  under conjugation. If for any  $\delta \in K$ , there exists  $x(\delta)$  such that  $G(x(\delta), \delta) > 0$ , then there exists  $x^* : K \times K' \rightarrow \mathbb{R}^p$ , a polynomial function with respect to  $\delta$  and  $\delta'$  such that, for any  $\delta \in K$ ,  $G(x^*(\delta, \delta'), \delta) > 0$ . Moreover, if  $m = 1$ , if  $\mathbb{C} \setminus K$  is connected, and if  $\text{int}(K) = \emptyset$ , there exists  $x^\bullet : K \rightarrow \mathbb{R}^p$ , a polynomial function with respect to  $\delta$ , such that, for any  $\delta$  in  $K$ ,  $G(x^\bullet(\delta), \delta) > 0$ .

**Proof.** Once again, we remind the reader that the three first steps of the proof of Theorem 2 [4] can be reproduced *mutatis mutandis*. Only the fourth step deserves attention. Once we know that there exists  $p$  continuous functions from  $K$  to  $\mathbb{C}$  which are the entries of a solution  $x(\delta)$  such that  $G(x(\delta), \delta) \geq \alpha I$  for some  $\alpha > 0$ , it remains to prove that, for any given complex entry of  $f$ , there exists an approximation. Define the set  $\mathcal{P}$  of functions of  $\delta$  defined through the expressions

$$f(\delta) = \sum_{i=0}^{\mu} \sum_{j=0}^{\mu} c_{ij} \delta^i \delta'^j, \quad c_{ij} \in \mathbb{C}. \quad (8)$$

These are polynomial functions w.r.t. both  $\delta$  and  $\delta'$ . It is quite easy to check that  $\mathcal{P}$  is a subalgebra of  $\mathcal{C}(K, \mathbb{C})$  which separates the points and contains non zero constant function. But in addition,  $\mathcal{P}$  is closed under conjugation so it is a  $*$ -algebra. Therefore, Theorem 5 can be invoked to claim that each entry  $x_i(\delta)$ ,  $i = 1, \dots, p$ , of  $x(\delta)$  is the limit of a sequence of polynomial functions complying with (8). Let  $x^*(\delta, \delta')$  denote (for short  $x^*(\delta)$ ) the  $\mathbb{C}^p$ -valued function resulting from the concatenation of these limits. Then it comes

$$\forall \delta \in K, G(x^*(\delta), \delta) \geq \alpha I \Rightarrow \forall \delta \in K, G(x^*(\delta), \delta) > 0. \quad (9)$$

This proves the first part of the theorem.

To prove the remainder, it can be noticed that if  $\text{int}(K) = \emptyset$ , the elements of  $\mathcal{P}$  do not need to be holomorphic on  $K$  but just continuous to invoke Theorem 6. Therefore, each entry  $x_i^*(\delta)$ ,  $i = 1, \dots, p$  of  $x^*(\delta)$  can be uniformly approximated on  $K$  by a polynomial function

$$x_i^\bullet(\delta) = \sum_{i=0}^{\nu} b_i \delta^i, \quad b_i \in \mathbb{C}. \quad (10)$$

Let denote  $x^\bullet(\delta)$  the  $\mathbb{C}^p$ -valued function resulting from the concatenation of the functions  $x_i^\bullet(\delta)$ . It follows

$$\forall \delta \in K, G(x^\bullet(\delta), \delta) > 0. \quad (11)$$

This concludes the proof.  $\square$

The previous theorem deserves several comments. The first statement in Theorem 7 is what will really be exploited in the sequel. It shows that the solution to complex

parameter-dependent LMIs can be assumed polynomial with respect to both the parameter vector  $\delta$  and its conjugate  $\delta'$ , with no loss of generality. This is of course a very interesting consequence of Glimm's theorem but the research for computationally tractable relaxations of the original LMI conditions often suggests that the solutions should rather be polynomial or rational with the entries of  $\delta$  (not those of  $\delta'$ , see [1]). This additional step on the way to useful approximations is not so easy to make. Mathematical literature dedicated to approximations usually evokes Mergelyan's theorem (Theorem 6) as the most significant and advanced step but it can be seen that it is restricted to  $\delta \in \mathbb{C}$ . Furthermore,  $\delta$  should belong to a subset which should satisfy some special properties and these properties are not encountered in many practical problems, including the present one. For this reason, we claim that the research for efficient approximations of solutions to complex LMIs with one or several complex parameters, through simple functions, is a widely open problem. More precisely, consider  $G(x, \delta)$  expressed as in (5) or (7) and the underlying function  $g(x, \delta, y) = y'G(x, \delta)y$  which should be non negative: If there are various tools to numerically approximate solutions  $x(\delta, y)$  when  $\delta \in \mathbb{R}^m$  and  $y \in \mathbb{R}^n$ , the existence of such approximations when  $\delta \in \mathbb{C}^m$  and  $y \in \mathbb{C}^n$  is not as obvious as in the real case. Nevertheless, the very special case addressed in the present contribution offers a possibility to circumvent the obstacle.

## 4 Stability of the 2D discrete Roesser models

Now, we come back to our main motivation and prove Theorem 1.

**Proof of Theorem 1.** Sufficiency holds by virtue of Lemma 2. The interest lies in the second statement and in the assumed structure of  $P(\delta)$ . First note that if  $P(\delta)$  is solution, so is  $P'(\delta) = P(\delta)$  and thus  $P^H(\delta)$ . Therefore, there exists  $Q(\delta)$  (not necessarily Hermitian) such that

$$\begin{cases} -Q^H(\delta) + M'(\delta)Q^H(\delta)M(\delta) < 0 \\ Q^H(\delta) > 0 \end{cases} \quad \forall \delta : |\delta| = 1. \quad (12)$$

Such a system is an LMI system parametered by  $\delta$ . Following the notations of the previous sections, it can be rewritten as  $G(x(\delta), \delta) > 0$  where  $G(\delta)$  is deduced from  $M(\delta)$  and  $x(\delta)$  contains the decision variables *i.e.* the entries of  $Q(\delta)$  [5]. The parameter  $\delta$  describes the unit circle which is a compact set  $K$ . Matrix  $M(\delta)$  is rational w.r.t.  $\delta$  and, since  $A_{22}$  is Schur stable, it is continuous in  $K$  (it has no pole inside  $K$ ). So matrices  $G_i(\delta)$  are also continuous. Therefore, by virtue of Theorem 7,  $Q(\delta)$  can be chosen polynomial w.r.t.  $\delta$  and  $\delta'$  without loss of generality provided that the degree of the polynomial is large enough.  $Q(\delta)$  can then be written

$$Q(\delta) = \sum_{k=1}^{\eta} \sum_{l=1}^{\gamma} Q_{kl} \delta^k \delta'^l, \quad Q_{kl} \in \mathbb{C}^{n \times n}. \quad (13)$$



Besides, since  $|\delta| = 1$ , it comes  $\delta\delta' = 1$ , implying that each monomial of the form  $\delta^k\delta'^l$  equals either  $\delta^i$  or  $\delta'^i$  with  $i = \max(k, l) - \min(k, l)$ . Therefore  $Q(\delta)$  complies with

$$Q(\delta) = \sum_{i=1}^{\nu} X_i\delta^i + Y_i\delta'^i, \quad (X_i, Y_j) \in \{\mathbb{C}^{n \times n}\}^2. \quad (14)$$

Since  $P(\delta) = Q^H(\delta)$ , it can be written as in (4) with  $P_i = X_i + Y_i'$ .  $\square$

**Remark 2** *There are continuous and mixed continuous-discrete counterparts of Theorem 1 which can be addressed about the same way but their exposition requires slight changes which are not detailed here. The extensions also lead to LMI conditions, whose expressions are rather simple, to analyze the stability of 2D continuous or mixed Roesser models. [2].*

## 5 Conclusion

In this note, we proposed a necessary and sufficient condition for the stability of 2D discrete Roesser models which consists of an LMI system where the solutions are polynomial w.r.t. a complex parameter describing the unit circle. It has to be noticed that this result is in accordance with what was obtained through a completely different and more specific approach in [3] and that a recent paper proposed a quite similar result in the mixed continuous-discrete-case (but with a real parameter) which leads to an exploitable relaxation based upon an SOS approach [6]. We hope that the present approach involving a complex parameter can offer a general method to derive exact relaxations of 2D-stability conditions for all the cases, as it is more than suggested in [2]. Moreover, the research for different structures of  $P(\delta)$  other than polynomial ones, which could lead to non conservative LMI relaxations, is also a track for new investigations.

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