Multiresolution analysis of incompressible flows interaction with forced deformable bodies

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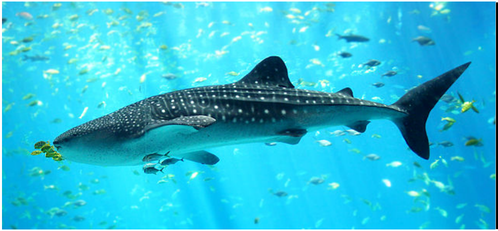
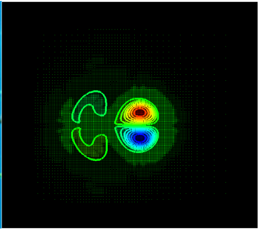
Outline

Motivation

Numerical simulation of moving/deformable bodies interacting with incompressible flows via multi-resolution analyses

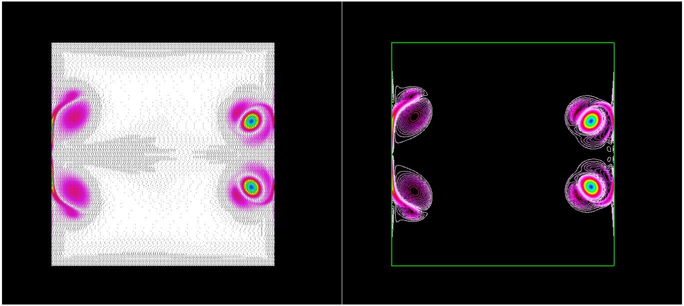




- Flow with different scales
- Better accuracy

Motivation

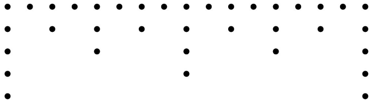
Grid adaptation methods



- Error indicator based
- Error control based

Multiresolution analysis

Biorthogonal wavelet transform

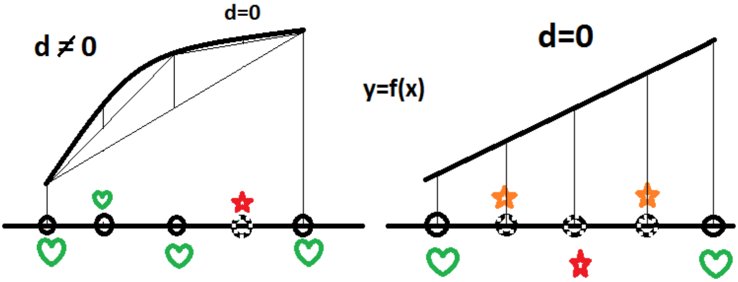


$$f(x) = \sum_{i=0}^{2^J} f_{0,i}\Phi_{0,i}(x) + \sum_{j=0}^{J}\sum_{i=0}^{2^J} d_{j,i}\Psi_{j,i}(x) \tag{1}$$

- Applicable to non-periodic functions

Multiresolution analysis

Filtering of wavelet coefficients

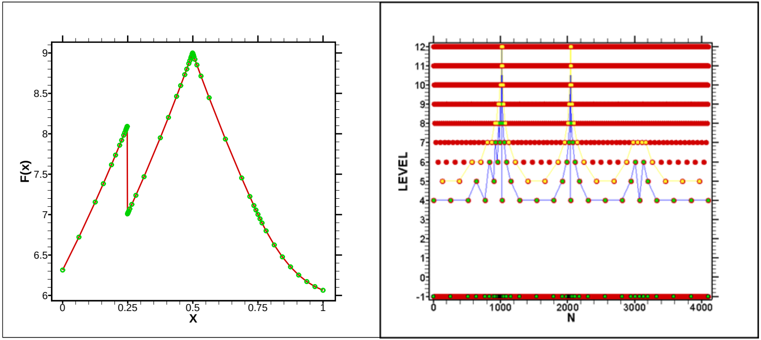


$$d = \begin{cases} 0 & \text{if } |d| \leq \epsilon \\ f - \hat{f} & \text{else} \end{cases} \quad (2)$$

$$||f(x) - \hat{f}(x)|| \leq \epsilon \quad (3)$$

Multiresolution analysis

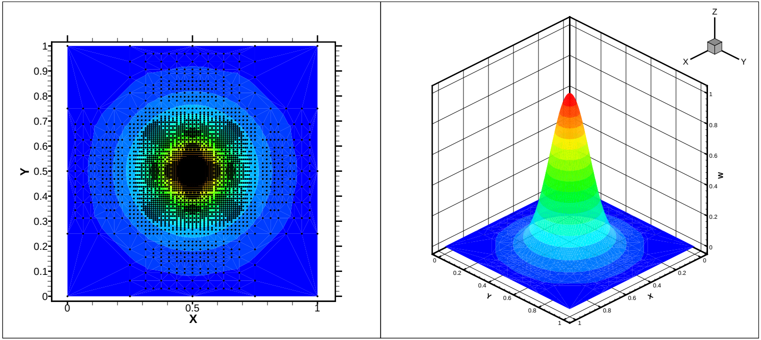
Regularity analysis for data compression



$$\epsilon = 10^{-3}, \quad L_\infty - error \leq 5 \times 10^{-5}, \quad Compression = 94\%$$

Multiresolution analysis

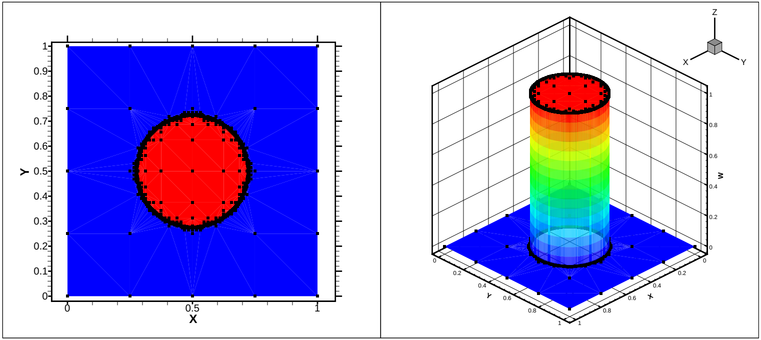
Two dimensional Gaussian function



$$J_{max} = 9, \quad \epsilon = 10^{-3}, \quad L_1 - error \leq 10^{-4}, \quad Compression = 98\%$$

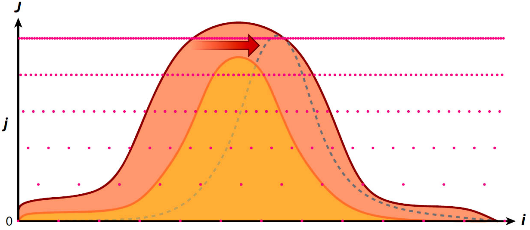
Multiresolution analysis

A function with a Jump in value



$$J_{max} = 9, \quad \epsilon = 10^{-3}, \quad L_1 - error \leq 10^{-15}, \quad Compression = 99.4\%$$

Multiresolution analysis

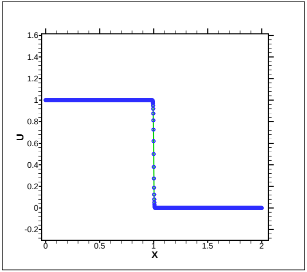


$$\omega^{n+1} = E(\Delta t)\Big[M^{-1} \cdot S \cdot T(\epsilon) \cdot M\Big]\omega^{n} \tag{4}$$

$$E(\Delta t)\omega^{n} = \omega^{n} + \Delta t \, RHS(\omega^{n}) \tag{5}$$

Multi-resolution analyses of Burgers equation

$$u_t + uu_x = \nu u_{xx}$$

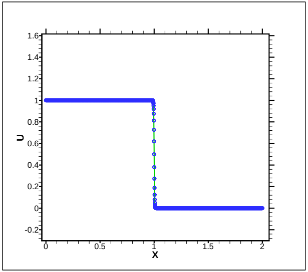


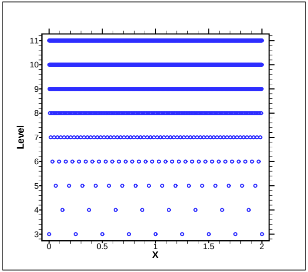
$$u_{\mathrm{exact}}(x,t) = \frac{1}{2}\left[1 - \tanh\left(\frac{x-1-t/2}{4\nu}\right)\right], \quad x \in \mathbb{R} \qquad (6)$$

Multi-resolution analyses of Burgers equation

$$u_t + uu_x = \nu u_{xx}$$

$$L_{max} = 11$$



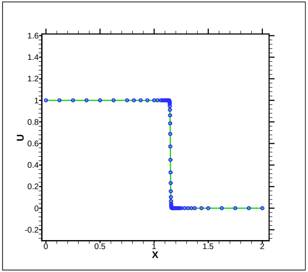


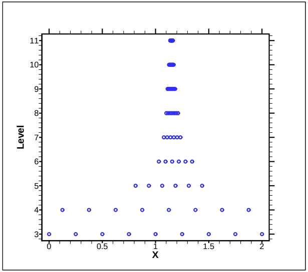
$$t = 0$$

Multi-resolution analyses of Burgers equation

$$u_t + uu_x = \nu u_{xx}$$

$$L_{max} = 11$$



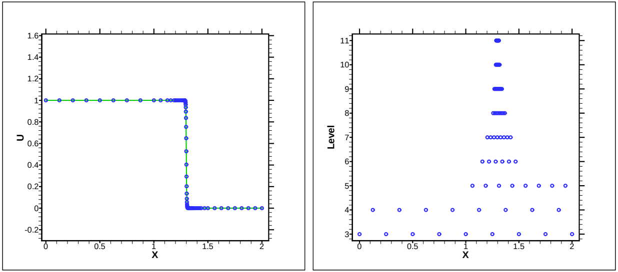


$$t = 0.3$$

Multi-resolution analyses of Burgers equation

$u_t + uu_x = \nu u_{xx}$

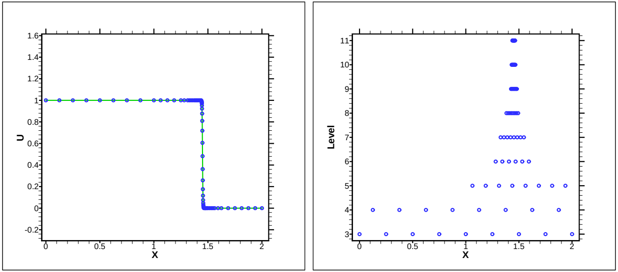
$L_{max} = 11$



$t = 0.6$

Multi-resolution analyses of Burgers equation

$u_t + uu_x = \nu u_{xx}$ $\qquad$ $L_{max} = 11$

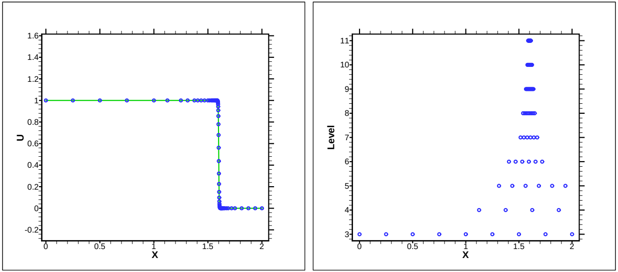


$t = 0.9$

Multi-resolution analyses of Burgers equation

$$u_t + uu_x = \nu u_{xx}$$

$$L_{max} = 11$$

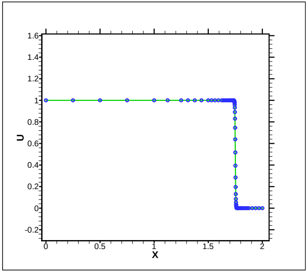


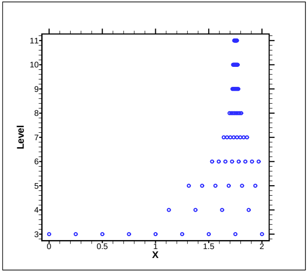
$$t = 1.2$$

Multi-resolution analyses of Burgers equation

$$u_t + uu_x = \nu u_{xx}$$

$$L_{max} = 11$$

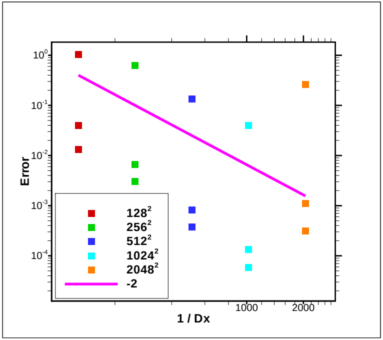




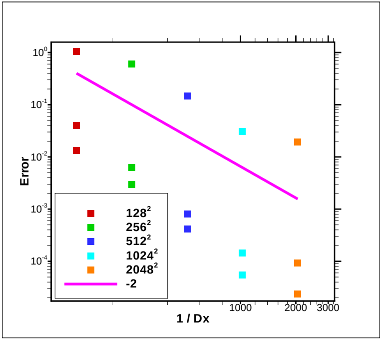
$$t = 1.5$$

Multi-resolution analyses of Burgers equation

$$u_t + uu_x = \nu u_{xx}$$



(a) $\epsilon = 5 \times 10^{-3}$

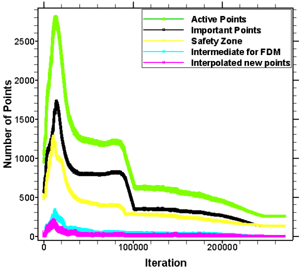


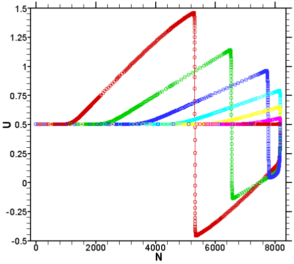
(b) $\epsilon = 1 \times 10^{-3}$

Error convergence

Multi-resolution analyses of Burgers equation

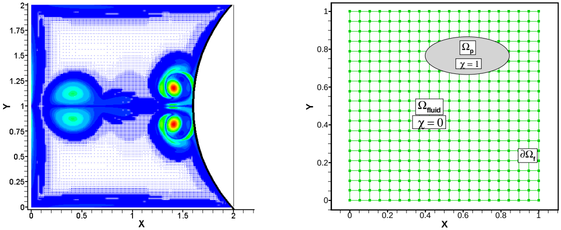
$$u_t + uu_x = \nu u_{xx}$$
$$u(x,0) = \sin(x)$$





Evolution of the points number

Vorticity transport equation including penalization



$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = \nu \nabla^2 \omega + \nabla \times \left(\frac{\chi(\mathbf{u}_P - \mathbf{u})}{\eta} \right) \ , \quad \mathbf{x} \in \Omega \in \mathbb{R}^2 \tag{7}$$

$$\chi(\mathbf{x}, t) = \begin{cases} 1 & \mathbf{x} \in \Omega_b \\ 0 & \mathbf{x} \in \Omega_f \end{cases} \tag{8}$$

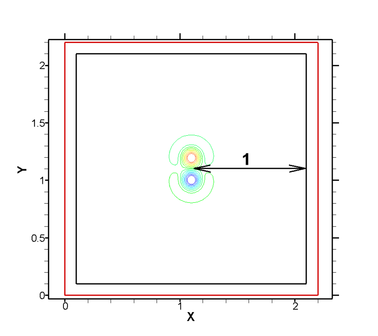
E. Arquis et J.P. Caltagirone. Comptes Rendus de l'Academie des Sciences, Paris, 2(299), 1-4, 1984.

Numerical method

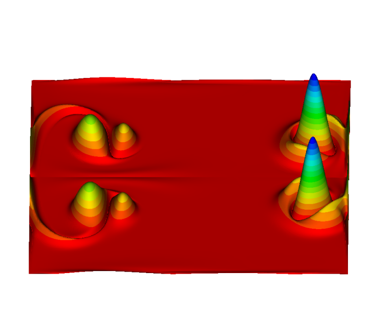
$$-\nabla^2 \psi = \omega \tag{9}$$

- 2^{nd}-order FDM for spatial derivatives
- RK4 for parabolic vorticity Eq. (7)
- PSOR method for Eq. (9)

Multi-resolution analyses of Dipole-wall collision



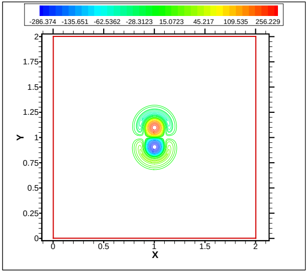
(c) Initial condition $t = 0$

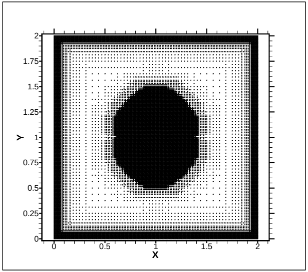


(d) Collision time $t = 0.33$

Multi-resolution analyses of Dipole-wall collision

$L_{max} = 11$

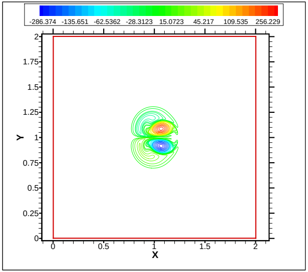


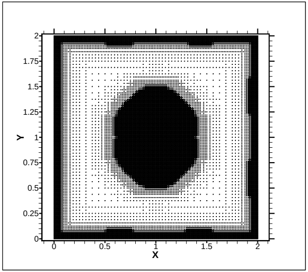


$t = 0$

Multi-resolution analyses of Dipole-wall collision

$L_{max} = 11$

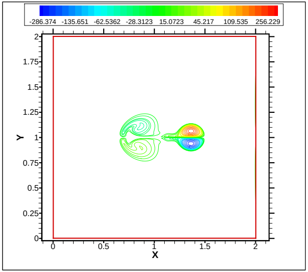


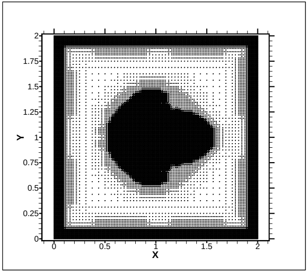


$t = 0.05$

Multi-resolution analyses of Dipole-wall collision

$L_{max} = 11$

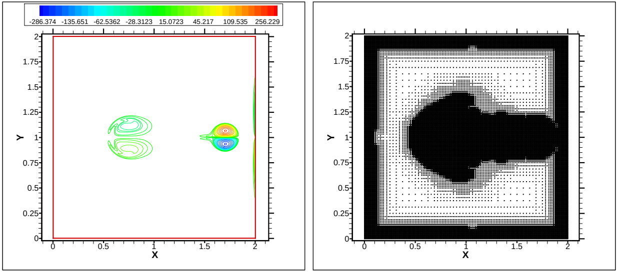




$t = 0.15$

Multi-resolution analyses of Dipole-wall collision

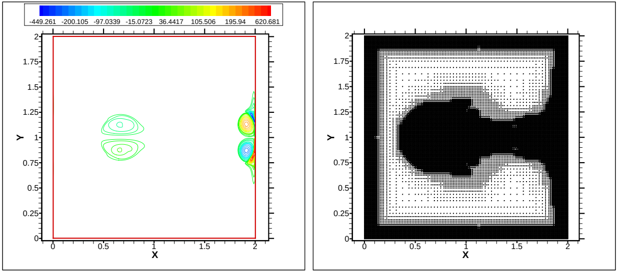
$L_{max} = 11$



$t = 0.25$

Multi-resolution analyses of Dipole-wall collision

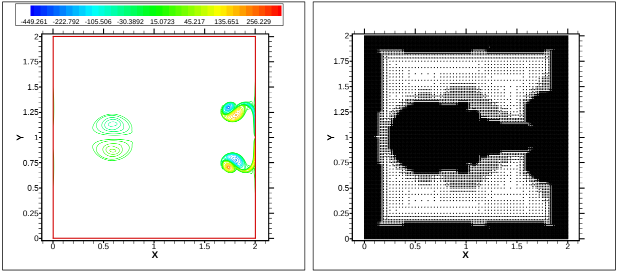
$L_{max} = 11$



$t = 0.35$

Multi-resolution analyses of Dipole-wall collision

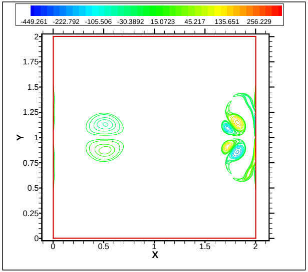
$L_{max} = 11$

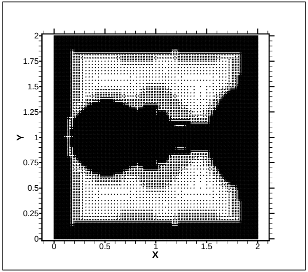


$t = 0.45$

Multi-resolution analyses of Dipole-wall collision

$L_{max} = 11$

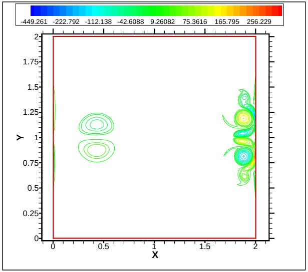


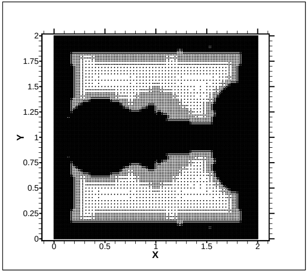


$t = 0.55$

Multi-resolution analyses of Dipole-wall collision

$L_{max} = 11$

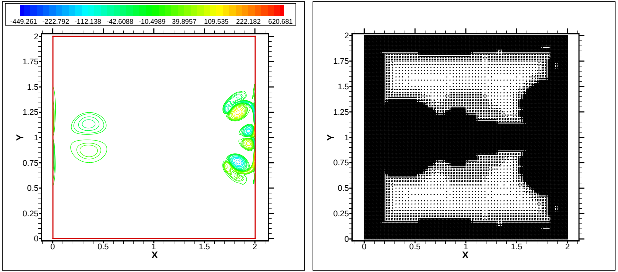




$t = 0.65$

Multi-resolution analyses of Dipole-wall collision

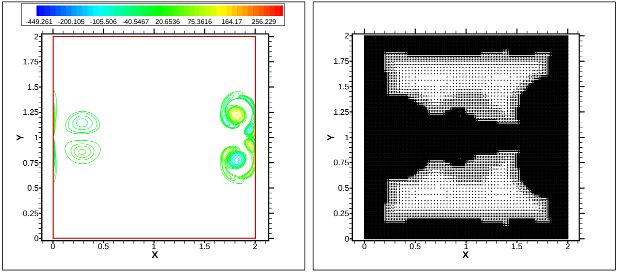
$L_{max} = 11$



$t = 0.75$

Multi-resolution analyses of Dipole-wall collision

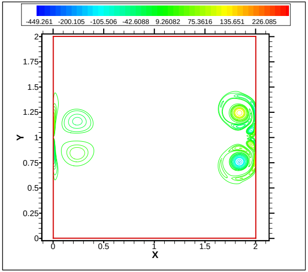
$L_{max} = 11$

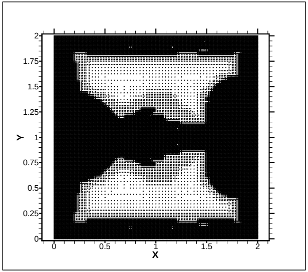


$t = 0.85$

Multi-resolution analyses of Dipole-wall collision

$L_{max} = 11$

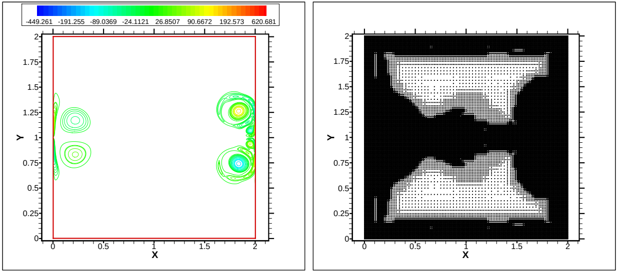




$t = 0.95$

Multi-resolution analyses of Dipole-wall collision

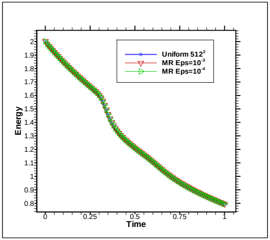
$L_{max} = 11$



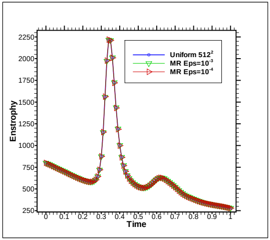
$t = 1$

Multi-resolution analyses of Dipole-wall collision

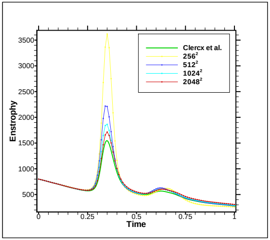
Convergence study



(e) Total energy $E(t)$

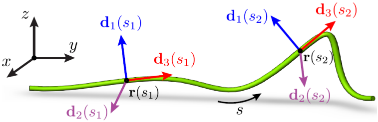


(f) Total enstrophy $E(t)$

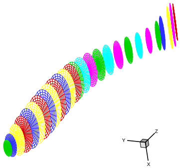


(g) Total enstrophy convergence

Fish's backbone as a 1D Cosserat medium



(h) picture from Lazarus et al. 2012



(i) A 3D example

$$\frac{\partial Q}{\partial s} = \frac{1}{2} M^{\vee}(K)\ Q \tag{10}$$

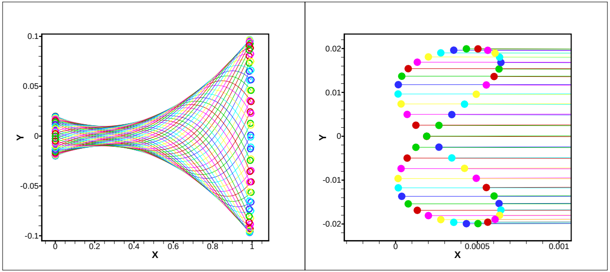
$$\frac{\partial \mathbf{X}}{\partial s} = Rot(Q)\ \Gamma \tag{11}$$

$$V, \Omega, a, \alpha, \ldots$$

F. Boyer, M. Porez and W. Khalil. *IEEE Transactions on Robotics*, Vol. 22, No. 4, 763-775, August 2006.

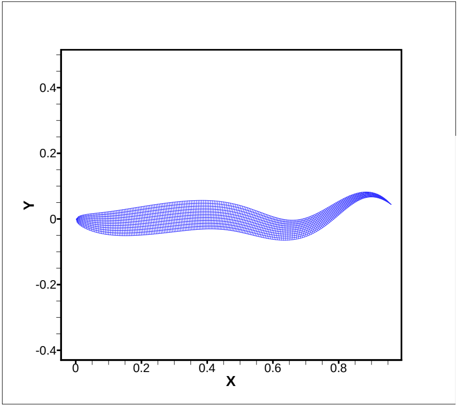
Anguilliform swimmers

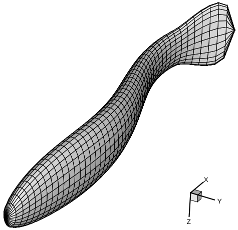
Motion of the head/tail over an butterfly-like curve



$$y(x(s),t) = f_{\text{decay}}(t)\ f_{\text{growth}}(t)\ a(x)\sin\Big(2\pi(x/\lambda \pm ft)\Big) \tag{12}$$

Lagrangian structured grid covering the body





Anguilliform swimmers undulate the majority of their body

Interpolation to Eulerian grid

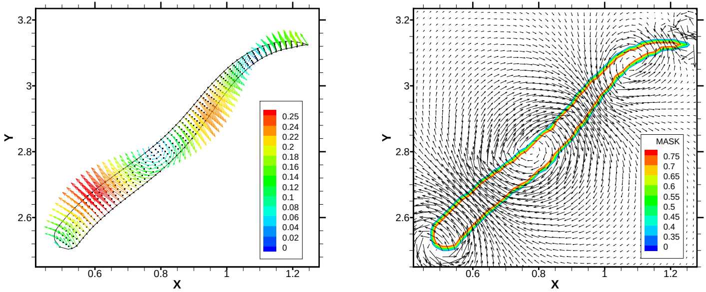
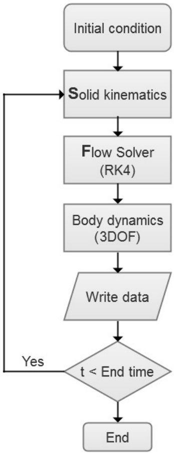


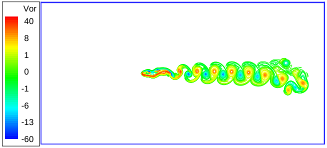
Figure: (left) Lagrangian structured grid. (right) Interpolated smoothed mask function $\bar{\chi}$ on the Eulerian grid.

Flowchart of the algorithm



Fish swimming in forward gait

(a) $t = 1$

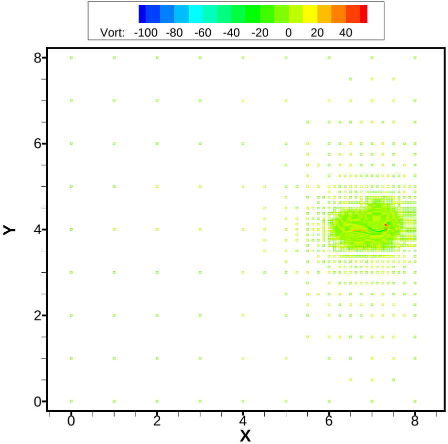


(b) $t = 9$

Figure: Snapshots of vorticity isolines, $(x, y) \in [0\,, 10 l_{\text{fish}}] \times [0\,, 5 l_{\text{fish}}]$, resolution 2048×1024, $Re \approx 3800$.

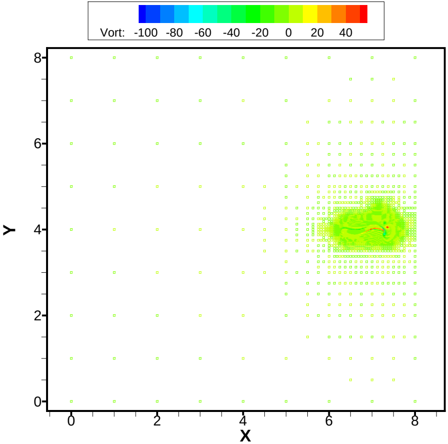
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 0.5$



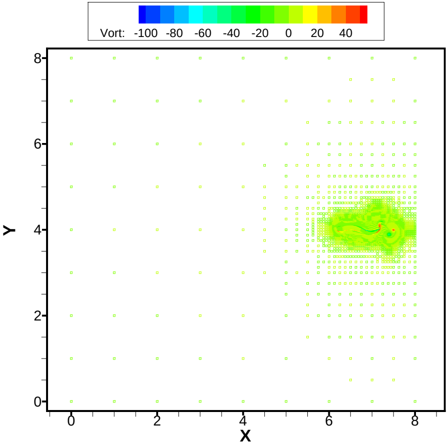
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $\quad$ $t = 1.0$



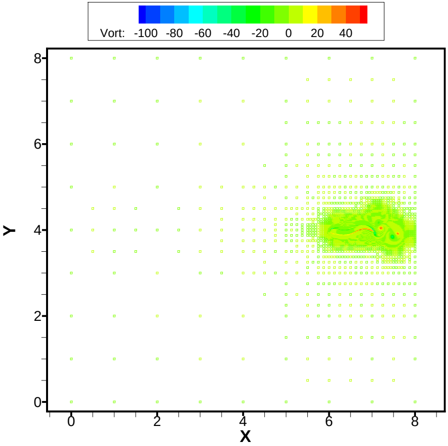
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 1.5$



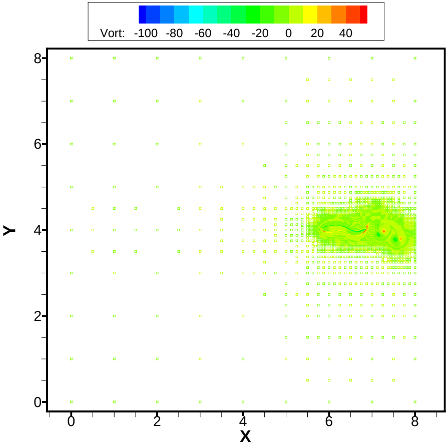
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 2.0$



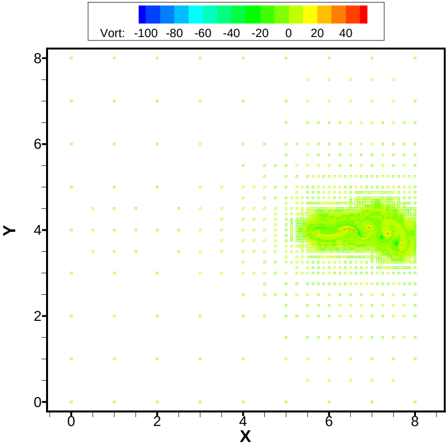
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 2.5$



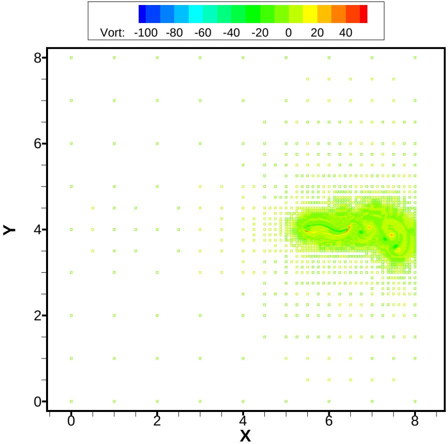
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 3.0$



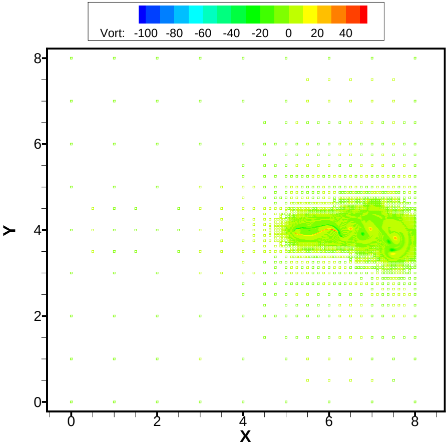
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 3.5$



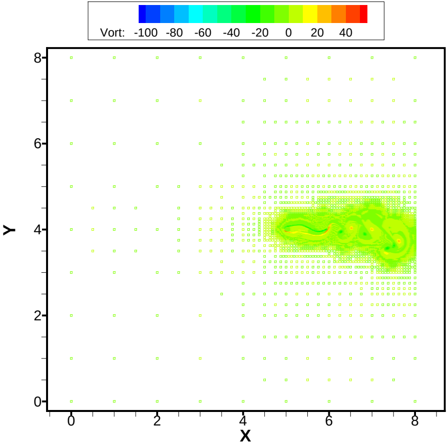
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 4.0$



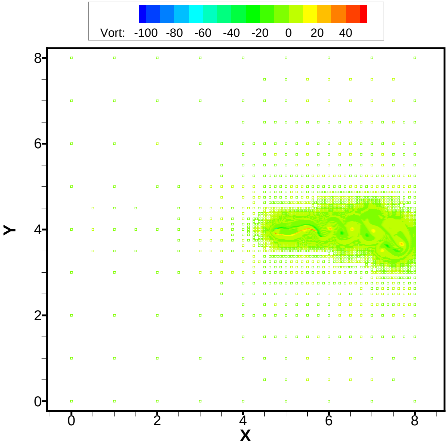
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 4.5$



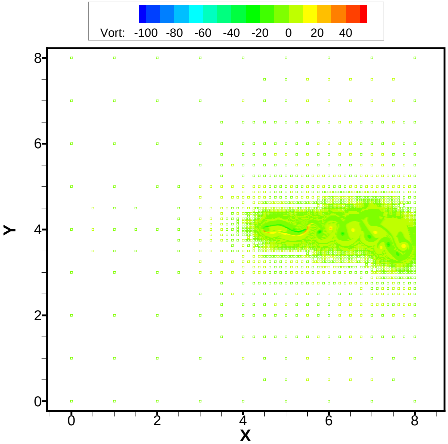
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 5.0$



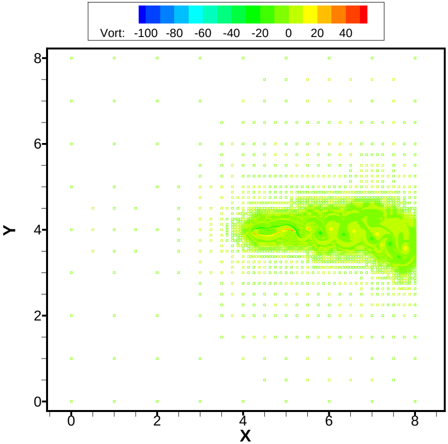
MR analyses of fish swimming in forward gait

$L_{max} = 10$ $t = 5.5$



MR analyses of fish swimming in forward gait

$L_{max} = 10$ $\quad\quad$ $t = 6.0$



Fish swimming in forward gait

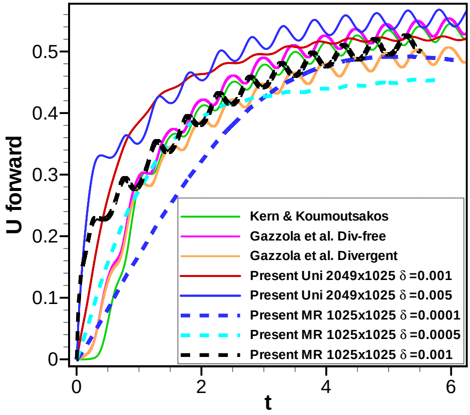
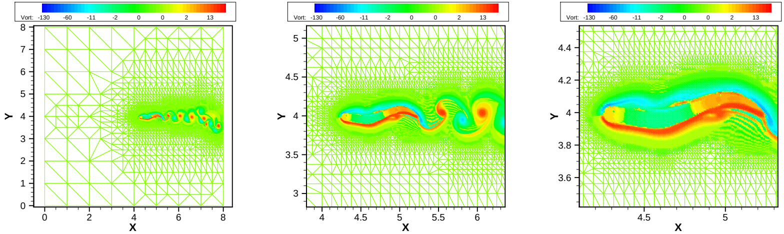


Figure: Forward velocity *U* comparisons of 2D anguilliform swimmer.

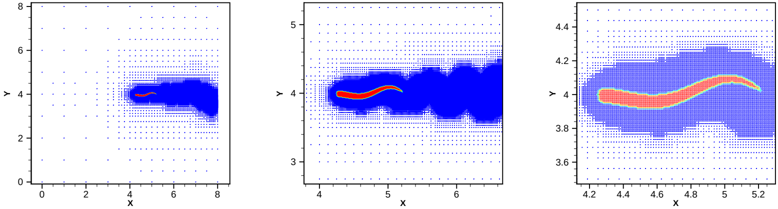
Multi-resolution analyses of fish swimming

The adaptive grid colored by vorticity at $t = 6$



Multi-resolution analyses of fish swimming

The corresponding mask function $\bar{\chi}$ at $t = 6$



Conclusion

- An efficient algorithm for simulation of deformable bodies interacting with 2D incompressible flows is proposed.
- The volume penalization method is applied to the N-S equations in vorticity stream-function formulation.
- Multi-resolution analyses allow to reduce the CPU-time of the simulations while the the accuracy-order of the numerical method is preserved
- The FORTRAN code is open source and is accessible upon request.

Thanks for your attention