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Saïd Safi, Miloud Frikel, Mathieu Pouliquen, B. Bouikhalne, A. Boumezough

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Said Safi
Sultan Moulay Slimane University
LIRST Laboratory, Po.Box. 592
Beni Mellal, Morroco
Email: safi.said@gmail.com

Miloud Frikel
and Mathieu Pouliquen
ENSICAEN School, GREYC Laboratory,
6, B. Marchal Juin, 14050 Caen France
Email: miloud.frikel@ensicaen.fr

Belaid Bouikhalne
and Ahmed Boumezzough
Sultan Moulay Slimane University
LIRST Laboratory, Po.Box. 592
Beni Mellal, Morroco

***Abstract*—A very high-speed wireless access of 100 Mb/s to 1 Gb/s is required for fourth-generation mobile communication systems. However, for such high-speed data transmissions, the channel is severely frequency-selective due to the presence of many interfering paths with different time delays. In this paper we discusses the identification problem of the normalized channel for 4^{th} generation mobile communication, representing the indoor scenario (European Telecommunications Standards Institute Broadband Radio Access Networks (ETSI BRAN A) channel model) and outdoor scenario (ETSI BRAN E channel model). The identification problem is performed using the Least Mean Squares (LMS) algorithm and the Takagi-Sugueno (TS) fuzzy system. The comparison between these techniques, for the channel identification, will be made for different Signal to Noise Ratios (SNR).**

keywords–LMS algorithm; Takagi-Sugeno Fuzzy System; Selective radio Communication Channels; System identification.

I. Introduction

Wireless or cellular mobile communication systems have been evolving according to advancements in wireless technologies and changes in user demands. In fixed and cellular networks, voice conversation was the dominant service for a long time. In line with the recent explosive expansion of Internet traffic in fixed networks, demands for broad ranges of services are becoming stronger even in mobile communication networks. A variety of services are now available over the second generation (2G) mobile communications systems, including email, Web access, and online services ranging from bank transactions to entertainment, in addition to voice conversation. People want to be connected anytime, anywhere with the networks, not only for voice conversation but also for data conversation (i.e., downloading/uploading information). 3G systems based on wide band direct sequence code-division multiple access (DSCDMA) [1], with much higher data rates of up to 384 kb/s (around 10 Mb/s in the later stage), were put into service in some countries, and their deployment speed has since accelerated. However, the capabilities of 3G systems will sooner or later be insufficient to cope with the increasing demands for broadband services that will soon be in full force in fixed networks. Demands for downloading of ever increasing volumes of information will become higher and higher. 4G systems that support extremely high-speed packet services are now expected to emerge [2, 3]. How cellular systems have evolved from 1G to 3G and will further evolve into 4G is shown in Fig. 1. 100 Mb/s 1 Gb/s class wireless packet access may be necessary for 4G systems.

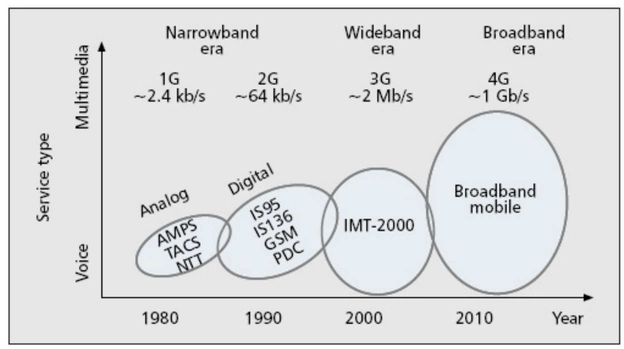

Fig. 1. mobile communications systems evolution

In this paper we focus on the channel identification representing the indoor propagation (ETSI BRAN A) and the outdoor propagation (BRAN E), these channel models are normalized for 4G systems. the propagation channel is introduced for better understanding of the frequency-selective channel. The identification problem is performed using the Least Mean Squares algorithm (LMS) [4] and the Takagi-Sugueno (TS) fuzzy system [5].

II. Characterization of Selective (Broadband) Channel

Between a base station and a mobile station (MS) there are many obstacles, and also many local scatterers (e.g., neighboring buildings) in the vicinity of the MS. Reflection of the signal by large obstacles creates propagation paths with different time delays; each path is a cluster of irresolvable multipaths created by reflection or diffraction, by local scatterers, of the transmitted signal reaching the surroundings of an MS. They interfere with each other, producing multipath fading, and the received signal power changes rapidly in a random manner with a period of about half-carrier wavelength when the MS moves. Such a multipath channel can be viewed as a time varying linear filter of impulse response $h(\tau)$ observed at time t, which can be expressed as [6]

$$h(\tau) = \sum_{i=0}^{L-1} \xi_i \delta(\tau - \tau_i) \tag{1}$$

where $\delta(n)$ is Dirac function, ξ_i is the magnitude of the target i, $L = 18$ the number of target and τ_i is the time delay (from the origin) of target i.

A. ETSI BRAN A Mobile Channel Model

In this paragraph we consider the ETSI BRAN A model representing the propagation in an indoor case. In the Table I we have summarized the measured values corresponding the ETSI BRAN A radio channel impulse response Eq. 1.

TABLE I
DELAY AND MAGNITUDES OF 18 TARGETS OF BRAN A RADIO CHANNEL

delay $\tau_i(ns)$	mag. $C_i(dB)$	delay $\tau_i(ns)$	mag. $C_i(dB)$
0	0	90	-7.8
10	-0.9	110	-4.7
20	-1.7	140	-7.3
30	-2.6	170	-9.9
40	-3.5	200	-12.5
50	-4.3	240	-13.7
60	-5.2	290	-18
70	-6.1	340	-22.4
80	-6.9	390	-26.7

B. ETSI BRAN E Mobile Channel Model

In this paragraph we consider the ETSI BRAN E model representing the fading radio channels, where the data corresponding to this model are measured in outdoor environment. In the table II we represent the values corresponding to the ETSI BRAN E radio channel impulse response.

TABLE II
DELAY AND MAGNITUDES OF 18 TARGETS OF BRAN E CHANNEL

delay $\tau_i(ns)$	mag. $C_i(dB)$	delay $\tau_i(ns)$	mag. $C_i(dB)$
0	-4.9	320	0
10	-5.1	430	-1.9
20	-5.2	560	-2.8
40	-0.8	710	-5.4
70	-1.3	880	-7.3
100	-1.9	1070	-10.6
140	-0.3	1280	-13.4
190	-1.2	1510	-17.4
240	-2.1	1760	-20.9

III. CHANNEL IDENTIFICATION USING THE LMS ALGORITHM AND TS FUZZY SYSTEM

A. Description of the LMS Algorithm

From the method of steepest descent, the weight vector equation is given by [4],

$$w(n+1) = w(n) + \frac{1}{2}\mu[-\nabla(E\{e^2(n)\})] \tag{2}$$

Where μ is the step-size parameter and controls the convergence characteristics of the LMS algorithm; $e^2(n)$ is the mean square error between the beamformer output $y(n)$ and the reference signal which is given by,

$$e^2(n) = [d^*(n) - w^h x(n)]^2 \tag{3}$$

The gradient vector in the above weight update equation can be computed as

$$\nabla_w(E\{e^2(n)\}) = -2r + 2Rw(n) \tag{4}$$

In the method of steepest descent the biggest problem is the computation involved in finding the values r and R matrices in real time. The LMS algorithm on the other hand simplifies this by using the instantaneous values of covariance matrices r and R instead of their actual values i.e.

$$R(n) = x(n)x^h(n) \tag{5}$$

$$r(n) = d^*(n)x(n) \tag{6}$$

Therefore the weight update can be given by

$$\begin{aligned} w(n+1) &= w(n) + \mu x(n)[d^*(n) - x^h(n)w(n)] \quad (7)\\ &= w(n) + \mu x(n)e^*(n) \end{aligned}$$

The LMS algorithm is initiated with an arbitrary value $w(0)$ for the weight vector at $n = 0$. The successive corrections of the weight vector eventually leads to the minimum value of the mean squared error. Therefore the LMS algorithm can be summarized in the following equations

$$Output, \quad y(n) = w^h(n)x(n) \tag{8}$$

$$Error, \quad e(n) = d^*(n) - y(n) \tag{9}$$

$$Weight, \quad w(n+1) = w(n) - \mu x(x)e^*(n) \tag{10}$$

The LMS algorithm initiated with some arbitrary value for the weight vector is seen to converge and stay stable for

$$0 < \mu < 1/\lambda_{max} \tag{11}$$

Where λ_{max} is the largest eigenvalue of the correlation matrix R. The convergence of the algorithm is inversely proportional to the eigenvalue spread of the correlation matrix R. When the eigenvalues of R are widespread, convergence may be slow. The eigenvalue spread of the correlation matrix is estimated by computing the ratio of the largest eigenvalue to the smallest eigenvalue of the matrix. If μ is chosen to be very small then the algorithm converges very slowly. A large value of μ may lead to a faster convergence but may be less stable around the minimum value. The literatures [7,8] also provides an upper bound for μ based on several approximations as $\mu \leq 1/(3trace(R))$.

B. Description of the TS Fuzzy model

The fuzzy model proposed by Takagi and Sugeno [9] is described by fuzzy IF-THEN rules which represents local input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models [12, 13].

The fuzzy Takagi-Sugeno model uses the linear functions in the consequent part. So it can be seen as a combination of language model [9, 14, 15, 16] and the mathematical regression model in the sense that the antecedents describe fuzzy regions in the space of input functions in which consequences are valid. Such fuzzy models, rule-based, provide a lot of possibilities for approximating nonlinear systems. In this work, we use particular fuzzy models of Takagi-Sugeno approach that allow non-linear systems by a combination of several local linear models [16, 17]. These models are written as follows

$$\begin{aligned} R_i: \ if\ x_t\ is\ A_i\ \ Then\ \hat{y}_{t,i} = \alpha_{0i} + x_t^T \alpha_i \\ where\ \ i = 1, 2, ...c\ \ and\ \ t = 1, 2, ...N \end{aligned} \tag{12}$$

$R_i (i = 1, 2, ..., c)$ indicates the i^{th} fuzzy rule, x_t is the input variable ($x_t \in \mathbf{R^n}$), $\hat{y}_{t,i}$ is the output of the rule i relative to input x_i, A_i is the fuzzy set and $\alpha_i = (\alpha_1, \alpha_2, ..., \alpha_n)^T$.
The output $\hat{y}_t$ relative to input x_t after aggregating of c TS fuzzy rules, can be written as a weighted sum of the individual conclusions

$$\hat{y}_t = \sum_{i=1}^{c} \pi_i(x_t) \hat{y}_{t,i} \tag{13}$$

with

$$\pi_i = \frac{\mu_{A_i}(x_t)}{\sum_{j=1}^{c} \mu_{A_j}(x_t)} \tag{14}$$

where μ_{A_i} is the membership function related to the fuzzy set A_i. The identification of the TS fuzzy systems requires two types of tuning:
- Structural tuning: concerns the determination of the number of rules c and the fuzzy sets to be used in the fuzzy system, for that we used the Gustafson-Kessel (GK) fuzzy clustering algorithm [18] with the following fuzzy validity criterion[19]

$$S(c) = \sum_{t=1}^{N} \sum_{i=1}^{c} (\mu_{kt})^m (\|z_t - \nu_i\|^2 \|\nu_i - \overline{z}\|^2) \tag{15}$$

Where z_t is the t^{th} data point, ν_i is the centre of the i^{th} cluster and $\overline{z}$ is the average of data and m is the fuzzification exponent. For the functional Eq.15, the two terms inside the bracket represent the variance of data inside each clusters and the variance of the clusters themselves, respectively. So the optimum number of clusters is determined as a minimum of the fuzzy validity criterion $S(c)$ as c increases.
- Parametric tuning: consists to identify the parameters of the TS fuzzy model: Generally two methods are used for the estimation of the linear parameters (α_k) [14]. The first one is the Weighted Least Squares (WLS) algorithm, called also the local method. The second one is the Global Least Square (GLS) algorithm called also the global method. The determination of the best strategy to apply is not clearly established.
However if we want to construct a most legible system, the local approach is preferable, since we can separate (interpret) the contributions of every rule (expert). On the contrary, if we want to adjust the linear parameters so that the global model resulting approximate to best the data base, the global approach is preferable, in this approach, all rules (experts) act simultaneously and we can not discriminate their action. The nonlinear parameters are estimated using the Marquardt-Levenberg (LM) method [19]. In this work, we use the global least squares approach (GLS).

IV. Simulation Results

In this section we show the performance results obtained by computer simulation for different SNR and assuming that the input channel is driven by non Gaussian signal $x(n)$. The output channel $y(n)$ is corrupted by a gaussian noise $N(n)$.

1) ETSI BRAN A Identification using the LMS algorithm: In Fig. 2 we represent the estimation of the ETSI BRAN A parameters using the LMS algorithm, for an SNR varying between $0dB$ and $40dB$ the data length is 2048 and for 100 iterations. From the Fig. 2 we observe a very low influence

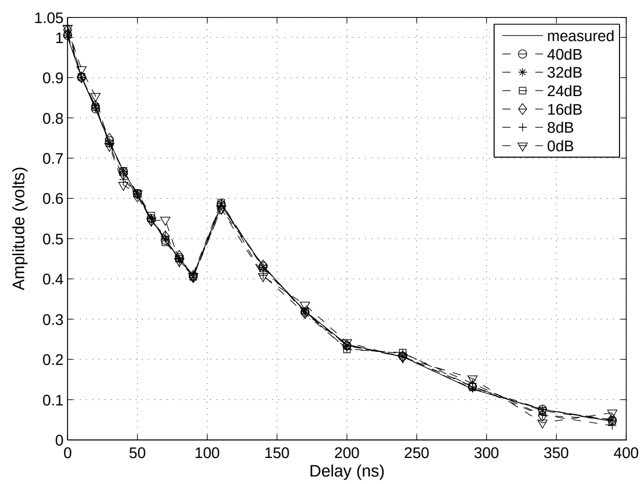

Fig. 2. ETSI BRAN A channel identification for different SNR, using the LMS algorithm

of the noise on the parameters estimation even for a $SNR = 0dB$, this is due to a slow variance of the impulse response of the ETSI BRAN A channel.
In order to know the differences between values estimated by the LMS algorithm and the measured values of the model ETSI BRAN A, we represent in the Fig. 3 the root-mean-square error (RMSE).

The Fig. 3 show that the $RMSE$ values decrease approximately linearly from $0dB$ to $24dB$, but if the $SNR > 24dB$ we remark that the $RMSE$ values are approximately constant and have the value $\approx 2.10^{-3}$.

2) ETSI BRAN E Identification: In this section we consider the ETSI BRAN E channel model. The Fig. 4 show the impulse response estimation for this channel using the LMS algorithm for different SNR.

From the Fig. 4, we remark a slight influence of the noise in the impulse response parameters estimation principally if the $SNR < 24dB$, but if the $SNR > 24dB$ the estimated

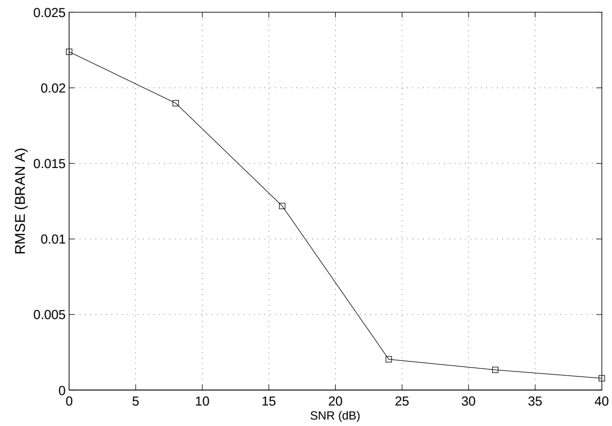

Fig. 3. RMSE (RMSE values (using the LMS algorithm) as a function of SNR

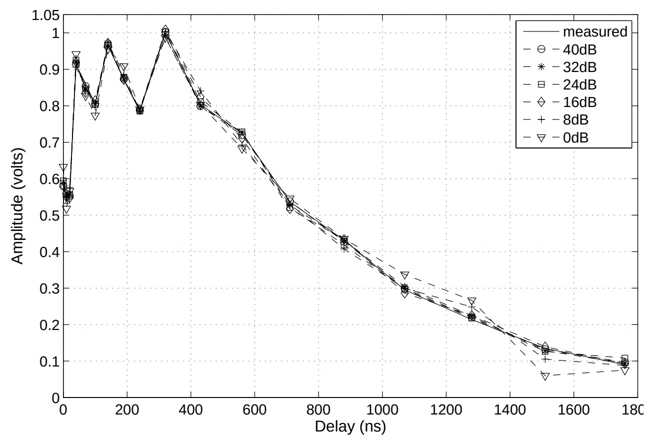

Fig. 4. ETSI BRAN E channel identification for different SNR, using the LMS algorithm

parameters are very closed to the measured one.
In Fig. 5 we represent the RMSE values for different SNR; we can conclude the same remark like the RMSE calculated

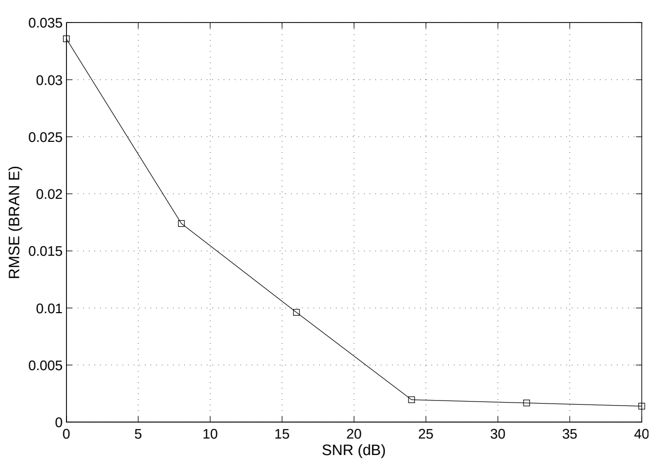

Fig. 5. RMSE (BRAN E channel using the LMS algorithm)for different SNR

for the ETSI BRAN A, i.e the RMSE values for the ETSI BRAN A channel model are decrease linearly if the SNR between $0dB$ and $24dB$ and slowly varying and take the values $\approx 2.10^{-3}$.

A. Channel Identification Using the TS Fuzzy System

In this section we use Takagi-Sugueno fuzzy system to identify the impulse response of ETSI BRAN (A and E) channel model, for different SNR.

1) ETSI BRAN A Identification: The Fig. 6 represent the impulse response of ETSI BRAN A channel model, for various SNR.
This figure shows clearly the influence of noise on parameter

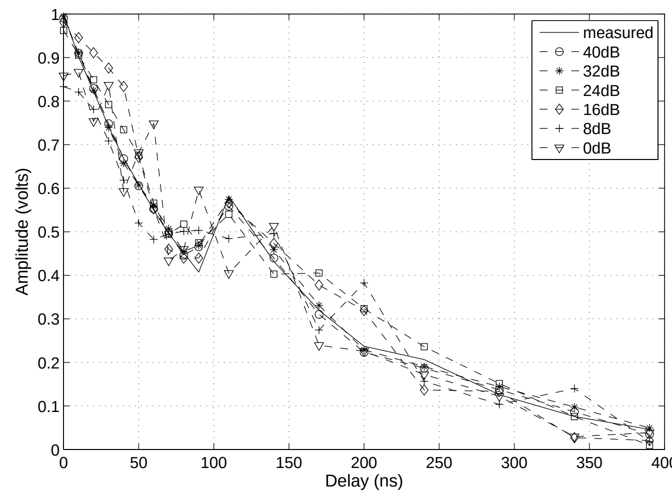

Fig. 6. ETSI BRAN A channel identification for different SNR, using TS fuzzy system

estimation, principally when the noise is important, i.e. $SNR < 24dB$, This is due to any noise filtering is realized in the TS fuzzy system. Which means if the noise is important, we observe that the estimated parameters are not closed to the measured one.
In Fig. 7 we represent the RMSE for different SNR, the results show that the standard deviation is important this imply RMSE values are most significant, for example: $RMSE = 2.10^{-2}$.

2) ETSI BRAN E Identification: The problem of the noise filtering is most clear in the impulse response identification of the ETSI BRAN E channel model using the TS fuzzy system, i.e. Fig. 8, This figure demonstrates the identification problem of the ETSI BRAN E impulse response, when we have a rapid variance of the impulse response and in presence of noise, using the TS fuzzy system. This problem is clear principally for the first six values of the ETSI BRAN E impulse response, where the estimated parameters do not follow those measured. But, if the impulse response decrease ”slowly”, i.e after the sixth values we observe that the estimated values are closed to those measured.

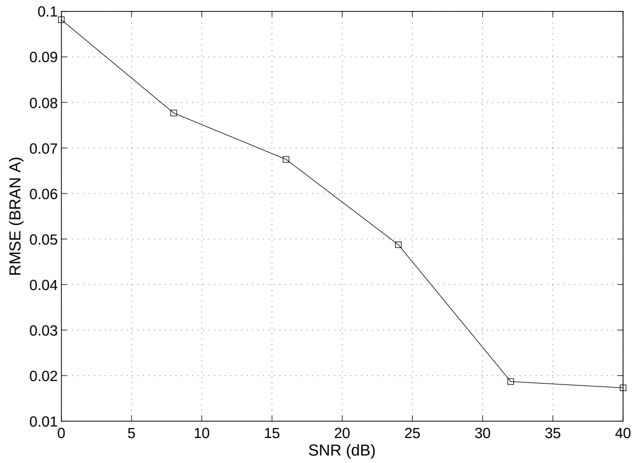

Fig. 7. RMSE (BRAN A channel using TS Fuzzy system) for different SNR

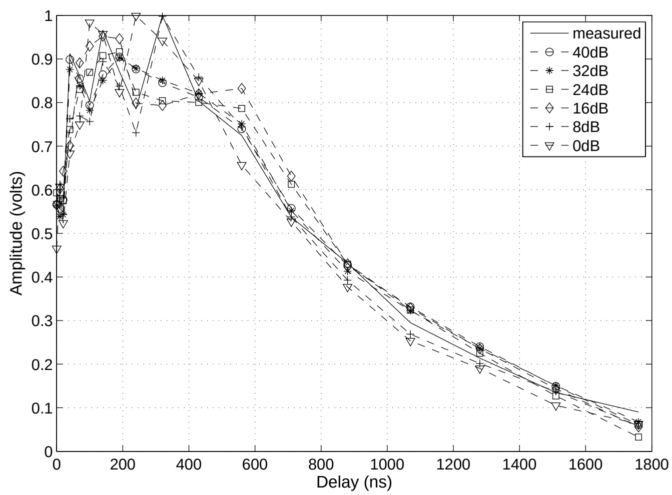

Fig. 8. ETSI BRAN E channel identification for different SNR, using TS fuzzy system

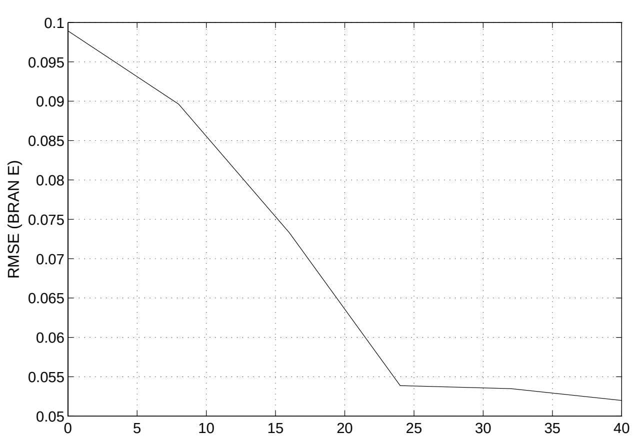

Fig. 9. RMSE (BRAN E channel using TS fuzzy system) for different SNR

Finely we represent in Fig. 9 the RMSE, for BRAN E impulse response using TS fuzzy systems, for different SNR.
From the Fig. 9 we remark a slow decrease of the RMSE values and take the value $\approx 5.10^{-2}$ for $SNR = 24dB$, which demonstrates that the estimated impulse response values are very closed to the measured one.

V. Conclusion and perspectives

In this paper we have presented two techniques: LMS algorithm and TS fuzzy system. Both of those techniques are used to identify the impulse response of the ETSI BRAN (A and E) selective channel. Through this paper, we have used the mentioned techniques for different SNR and in order to show the estimation quality we calculated the RMSE for each SNR value. From simulation results, we may conclude the following:
First, the LMS algorithm show their efficiency in the impulse response channel (ETSI BRAN (A and E)) identification with high precision (the estimated parameters are very closed to the measured one) for various SNR, eventually for a $SNR = 0dB$.
Second, the TS fuzzy system give good results for the impulse response ETSI BRAN A selective channel, for different $SNR > 16dB$. But for ETSI BRAN E selective channel we have some variance for the sixth values of the impulse response.
In conclusion the LMS algorithm is very adequate for the selected application (identification of the values of the impulse response ETSI BRAN (A and E)) than the TS fuzzy system, principally in noisy environment, because the LMS algorithm use a noise filtering in each iteration.
The future work of this paper is the MC-CDMA equalization using the presented techniques (LMS and TS fuzzy system).

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