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Abstract—**In this paper, we study a flexible routing strategy for demand protection and a corresponding optimization problem for networks that permanently experience fluctuations of the capacity available on their links. This is an important and novel topic as limited link availability is a fundamental feature of wireless networks; yet majority of work in survivable network design is restricted to total failures of single links. Hence, protection against partial failures of multiple links is considered as congestion avoidance. We assume a given finite set of network states. Each state is characterized by a vector of link availability coefficients specifying, for each link, the fraction of its nominal (maximum) capacity available in this state, and by a traffic coefficients vector specifying, for each demand, the proportion of its nominal traffic to be realized in the considered state. Our routing strategy allows for adjustment (thinning or thickening) of the reference path-flows. For a given nominal value x of a path-flow, its thickening is limited to τx where τ is a given constant greater than or equal to 1. Thus, in each state, the value of every path-flow can range from 0 to τ times its reference value. It turns out that the corresponding link cost minimization problem (where link capacities and state-dependant path-flows are decision variables) is NP-hard. We present a non-compact linear programming formulation of the problem together with a solution algorithm based on path generation. We illustrate the effectiveness of the introduced routing strategy by presenting numerical results for a set of representative network examples.**

Keywords-survivable networks; multiple partial link failures; multicommodity flow; linear and mixed integer programming; path generation.

I. INTRODUCTION

In this paper, we study a routing strategy for demand protection called Flow-Adjustment Routing (FAR) together with the corresponding network optimization problem. FAR aims at protecting traffic demands against Multiple Partial Link Failures (MPLF) by appropriate adjustment of the reference (nominal) path-flows. Strictly speaking, FAR is not a rerouting strategy because no paths can be created to restore the affected flows. Instead, the nominal path-flows are thinned or thickened according to the current state of the links. To account for MPLF, we consider a nominal state and a set of link availability states $\mathcal{S}$ (containing the nominal state) and a finite set A fractional numbers. For each link $e \in E$ and each availability state $s \in \mathcal{S}$, the corresponding link availability coefficient $\alpha_e^s \in A$ specifies the fraction of the nominal capacity of link e that is available in state s $(0 \le \alpha_e^s \le 1)$. FAR assumes that the nominal path-flows (i.e., path-flows defined for the nominal state with all links fully available) can be thinned or thickened to adapt to capacity fluctuations but not restored on paths not used in the nominal state. On top of this, we assume that the demand volumes to be realized in the availability states different from the nominal state are possibly reduced as compared to the nominal traffic. In the following, we use the terms *availability states* and *failure states* interchangeably.

For the so-specified setting, we study the corresponding optimization problem referred to as Flow-Adjustment Problem (FAP). FAP aims at minimizing the cost of links under the following constraints: (i) the nominal path-flows and the state-dependent path-flows satisfy the nominal and the state-dependent traffic demand requirement, respectively, (ii) path-flows obey the maximum thickening assumption of the protection strategy, and (iii) for each link and each availability state, the link load does not exceed the currently available link capacity. As FAP is NP-hard (because a special case of FAP shown to be NP-Hard is studied by Tomaszewski et al. [13]), its Linear Programming (LP) formulations (see Section III) are unavoidably non-compact and require path generation to be solved to optimality. For generating paths we need to apply a mixed-integer pricing problem—here the difficulty of FAP is manifested.

The introduced model is original. In fact, not much work has been done in survivable network optimization under the MPLF assumption. To the best of our knowledge, only the so called global rerouting (restoring flows from scratch in surviving capacity) has been studied in this context [1] – a strategy quite impractical in the considered wireless framework. It happens that multiple failures have so far been considered in survivable network design almost always (i.e., besides global rerouting) assuming total link failures $(\alpha_e^s \in \{0,1\})$. The work done here is substantial (again, see [1][5] and the references therein) but this case is, unfortunately, not relevant for our framework.

The rest of this paper is organized as follows. In the next section, we give motivation behind the considered flow-adjustment strategy. In Section III, we introduce the notation and formulate the basic FAP model. Section IV describes a path generation algorithm necessary to find optimal solutions for FAP. In Section V, we present results of a numerical study that illustrates a potential value of the considered protection strategy in terms of the cost of the resulting network in comparison with global rerouting. In Section VI, we present the performance of the restoration process using simulations. Finally, in Section VII, we present concluding remarks.

II. FLOW ADJUSTMENT- A SIMPLE PROTECTION STRATEGY FOR MULTIPLE PARTIAL FAILURES

The flow-adjustment routing strategy for MPLF proposed in this paper extends the Flow Thinning Routing strategy (FTR) presented in [2][3]. Both FTR and FAR are inspired by the idea of Elastic Rerouting (ER) presented in [4] and Path Diversity Protection (PDP) [1], [5]. We recall that both ER and PDP assume total link failures, i.e., failure scenarios which admit only binary vectors α_e^s characterizing the availability states (referred to as the failure states in this case) in $\mathcal{S}$ (i.e., $A = \{0,1\}$), that is, when links fail, then their entire capacity is lost. PDP is a protection strategy where in a failure state $s \in \mathcal{S}$ path-flows through the failing links are simply disconnected, and the surviving path-flows must be sufficient to realize the demand volumes, possibly decreased with respect to nominal demand volumes. ER allows for decreasing the flows of unaffected demands, as well as for increasing (to a certain threshold) path-flows of affected demands.

In this paper, we follow the concept of ER, but no distinction will be made between unaffected and affected demands. Hence, our strategy is allowed to thin or thick the nominal path-flow on any routing path. Thus, FAR assumes that each demand is in general routed over several paths, not necessarily disjoint, with over-dimensioned nominal path-flows to ensure an assumed level of demand survivability. Contrary to conventional end-to-end restoration strategies, no flow is rerouted to recover from a failure. In fact, it seems that allowing only for thinning makes the resulting protection strategy less complicated for network implementations. Our strategy is an extension of FTR in which the path-flows are allowed only to be decreased. Instead, FAR admits both thinning and thickening allowing for less costly networks as compared with FTR. Yet, FAR and FTR are similar from the mathematical modeling point of view, especially the mixed-integer pricing problem.

To clarify the idea of how FAR works, we wish to emphasize a few points. First, note that a common pool of link capacities is used for the nominal path-flows and in the remaining availability states. Hence, the selected routing paths carrying the flows are dimensioned so that the total traffic realized by the demand's path-flows could in general be greater than the nominal traffic. The most important feature of FAR is handling partial failures without any flow rerouting at all. In other words, no paths besides the nominal paths are used for handling other availability states. Therefore, the proposed approach results in using a sort of limited dynamic routing, adapted to the network states. To summarize, for each demand there is a fixed set of nominal routing paths carrying nominal flows. In an availability state in general only a part of the total nominal demand flow will be realized on these paths, depending on the available capacity and the required demand restoration ratio. Consequently, the restoration will be practically done by thickening some of the path-flows, and no new (re)routing paths are allowed.

III. FAP: FLOW-ADJUSTMENT PROBLEM

The basic problem considered in this paper is referred to as FAP and is as follows. We minimize the cost of link capacity assuming that in the nominal state of network operation all demand volumes are realized by means of (nominal) path-flows. When the network is subject to a failure from a given set of failure states (we assume that a failure state consists of partial failures of multiple links) then the demand volumes, possibly reduced, are realized for the duration of the failure state by appropriate thinning or thickening of the nominal flows. The detailed formulation of FAP will be given in Subsection III-B.

A. Notation

The considered network is modeled using a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, undirected or directed, composed of a set of nodes $\mathcal{V}$ and a set of links $\mathcal{E}$. In the sequel, we will always consider directed graphs unless stated explicitly otherwise. Thus, each link $e \in \mathcal{E}$ represents a directed pair (v,w) of some nodes $v,w \in \mathcal{V}$, and is assigned a non-negative unit capacity cost ξ_e which is a parameter, and a capacity reservation y_e which is an optimization variable. The cost of the network is given by the quantity $C = \sum_{e\in\mathcal{E}} \xi_e\, y_e$. The demands are represented by the set $\mathcal{D}$. Each demand $d \in \mathcal{D}$ is associated with a directed pair of nodes (end-nodes of d) $(o(d)\,;\,t(d))$ for some $o(d), t(d) \in \mathcal{V}$; a volume h_d^0 (a parameter) has to be sent from $o(d)$ to $t(d)$ (demand volumes and link capacities are expressed in the same units). Also, each demand d is assigned a set of admissible paths $\mathcal{P}_d$ composed of selected elementary paths from $o(d)$ to $t(d)$ in graph $\mathcal{G}$. (Recall that an elementary path does not traverse any node more than once.) Paths from $\mathcal{P}_d$ are used to realize the demand volume h_d^0 by means of path-flows $x_{dp}^0, p \in \mathcal{P}_d$, which are optimization variables.

The given sets of admissible paths define the link-path incidence coefficients $\delta_{edp}, e \in \mathcal{E}, d \in \mathcal{D}, p \in \mathcal{P}_d$, where $\delta_{edp} = 1$ if path $p \in \mathcal{P}_d$ traverses link $e \in \mathcal{E}$ (i.e., if $e \in p$, treating the paths as subsets of links: $p \subseteq \mathcal{E}$), and $\delta_{edp} = 0$ if path $p \in \mathcal{P}_d$ does not traverse link e. It is important to note that the sets of admissible paths $\mathcal{P}_d, d \in \mathcal{D}$, are parameters in the FAP problem formulation considered in the sequel, although in general it assumes that all possible elementary paths can potentially be used if this is required to reach the optimum.

Network links are subject to (partial) failures. The failure-less state (with all links fully available) is called the nominal state and is denoted by 0, and the flows $x_{dp}^0, p \in \mathcal{P}_d$

are referred to as nominal flows. The set of failure states is denoted by $\mathcal{S}$ (we call set $\mathcal{S}$ the failure scenario). Each failure state $s \in \mathcal{S}$ is specified by a vector of link availability ratios $\alpha^s = (\alpha_e^s, e \in \mathcal{E})$, where $0 \le \alpha_e^s \le 1$ for $e \in \mathcal{E}$. The link capacity available in state s is assumed to be equal to $\alpha_e^s y_e$. Hence, α_e^s is the fraction of capacity y_e that survives in state s. Certainly, in state 0 all the ratios α_e^0 are equal to 1. In general, in a failure state more than one link can have its availability ratio less that 1 so in fact we are considering multiple partial failures of the network links.

When a failure state $s \in \mathcal{S}$ affects the network, in general not all the nominal flows $x_{dp}^0, d \in \mathcal{D}, p \in \mathcal{P}_d$ can be realized anymore as the available link capacity is decreased. It is assumed that the demand volumes to be realized in state $s \in \mathcal{S}$ can be decreased, to values $h_d^s, d \in \mathcal{D}$. The demand volumes assumed for a failure state must be realized by means of the nominal flows that are appropriately thinned (decreased) or thickened (increased) so to fit to the reduced link capacity. The thinned nominal flows for state $s \in \mathcal{S}$ are denoted by $x_{dp}^s, d \in \mathcal{D}, p \in \mathcal{P}_d$. These flows are allocated to the admissible paths for the duration of the failure state. The state-dependent flows $x_{dp}^s, s \in \mathcal{S}, d \in \mathcal{D}, p \in \mathcal{P}_d$ are optimization variables. The relation, for each demand $d \in \mathcal{D}$, is defined by a given demand-dependent flow control factor τ_d, where $\tau_d \ge 1$. Note that when $\tau_d = 1$, then, the reference path-flows of demand d can only be thinned. Still, when $\tau_d > 1$, they can be as well thickened, but only up to the factor τ_d so that, $x_{dp}^s \le \tau_d x_{dp}^0, s \in \mathcal{S}, d \in \mathcal{D}, p \in \mathcal{P}_d$.

B. *Formulation of FAP*

FAP assumes that in each failure state $s \in \mathcal{S}$, only a part h_d^s of the nominal volume h_d^0 has to be realized for each demand $d \in \mathcal{D}$. This is achieved by thinning or thickening the nominal flows x_{dp}^0 to values x_{dp}^s, ($p \in \mathcal{P}_d$) so that the links capacities $\alpha_e^s y_e, e \in \mathcal{E}$ available in state s are not exceeded. For given sets of admissible paths $\mathcal{P}_d, d \in \mathcal{D}$, problem FAP can be represented by the following path-flow linear programming (LP) formulation involving variables x_{dp}^0 $(d \in \mathcal{D}, p \in \mathcal{P}_d)$, x_{dp}^s $(s \in \mathcal{S}, d \in \mathcal{D}, p \in \mathcal{P}_d)$ and $y_e (e \in \mathcal{E})$:

$$\min \quad C = \sum_{e \in E} \xi_e y_e \qquad (1a)$$

$$[\lambda_d^0 \ge 0] \quad \sum_{p \in \mathcal{P}_d} x_{dp}^0 \ge h_d^0,\ d \in \mathcal{D} \qquad (1b)$$

$$[\pi_e^0 \ge 0] \quad \sum_{d \in \mathcal{D}} \sum_{p \in \mathcal{P}_d} \delta_{edp} x_{dp}^0 \le y_e, e \in \mathcal{E} \qquad (1c)$$

$$[\lambda_d^s \ge 0] \quad \sum_{p \in \mathcal{P}_d} x_{dp}^s \ge h_d^s,\ d \in \mathcal{D}, s \in \mathcal{S} \qquad (1d)$$

$$[\pi_d^s \ge 0] \quad \sum_{d \in \mathcal{D}} \sum_{p \in \mathcal{P}_d} \delta_{edp} x_{dp}^s \le \alpha_e^s y_e, e \in \mathcal{E}, s \in \mathcal{S} \qquad (1e)$$

$$[\sigma_{dp}^s \ge 0] \quad x_{dp}^s \le \tau_d x_{dp}^0, d \in \mathcal{D}, p \in \mathcal{P}_d, s \in \mathcal{S} \qquad (1f)$$

$$\text{all variables } x^0,\ x^s, y \text{ continuous},\ x^s \ge 0. \qquad (1g)$$

In the formulation, the quantities in brackets on the left-hand sides are dual variables associated with the constraints (see Section I.2.2 in [5]). All these variables are, by assumption, nonnegative. Objective (1a) minimizes the total cost of links. Constraint (1b) makes sure that for each demand its paths have jointly sufficient capacity to satisfy the demand volume assumed for the nominal state. Constraint (1c) does not allow the nominal link loads to exceed the nominal link capacities. Next, constraint (1d) assures that in each failure state $s \in \mathcal{S}$, the adjusted flows are sufficient to realize the (possibly reduced) volume of each demand $d \in \mathcal{D}$. Then, constraint (1e) makes sure that in each state $s \in \mathcal{S}$, the surviving capacity of each link $e \in \mathcal{E}_s$ is not exceeded. Finally, constraint (1f) relates the state-dependent path-flows to the reference flows.

IV. Path generation

Formulation (1) is a link-path LP formulation. It is non-compact because of exponentially many path-flow variables x as we potentially consider all possible paths in the admissible path-sets $\mathcal{P}_\text{d}, d \in \mathcal{D}$.

Thus, in order to consider all possible routing paths in graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ in FAP, we need to apply the technique of linear programming known as column generation [7], [8], called Path Generation (PG) in our context. With PG, starting from some initial path-sets $\mathcal{P}_\text{d}, d \in \mathcal{D}$, we generate new paths (corresponding to variables/columns $x_{dp}^0, d \in \mathcal{D}, p \in \mathcal{P}_d$ and $x_{dp}^s, s \in \mathcal{S}, d \in \mathcal{D}, p \in \mathcal{P}_d$), and iteratively add them to the path-sets. As discussed below, this is done by solving an appropriate pricing problem using, as parameters, optimal dual variables associated with constraints (1b)-(1f), i.e., an optimal solution $\lambda^*, \pi^*, \sigma^*$ of the problem dual to FAP formulated in the next subsection.

A. *Dual LP formulation of FAP*

The problem dual to LP (1) is as follows (see for example [1]):

$$\max W = \sum_{d \in \mathcal{D}} \left(h_d^0 \lambda_d^0 + \sum_{s \in \mathcal{S}} h_d^s \lambda_d^s \right) \qquad (2a)$$

$$\pi_e^0 + \sum_{s \in \mathcal{S}_e} \alpha_e^s \pi_e^s \le \xi_e, e \in \mathcal{E} \qquad (2b)$$

$$\lambda_d^0 + \sum_{s \in \mathcal{S}} \tau_d \sigma_{dp}^s \le \sum_{e \in \mathcal{E}} \delta_{edp} \pi_e^0, d \in \mathcal{D}, p \in \mathcal{P}_d \qquad (2c)$$

$$\lambda_d^s \le \sigma_{dp}^s + \sum_{e \in \mathcal{E}_s} \delta_{edp} \pi_e^s, s \in \mathcal{S}, d \in \mathcal{D}, p \in \mathcal{P}_d \qquad (2d)$$

$$\text{all variables } \lambda, \pi, \sigma \text{ continuous and nonnegative.} \qquad (2e)$$

Let $\mathcal{D}(\mathcal{P})$ denote the problem defined by (2) for a given set of admissible paths $\mathcal{P} = \bigcup_{d \in \mathcal{D}} \mathcal{P}_d$ and let $\Pi(\mathcal{P})$ denote the (dual) polyhedron defined by constraints (2b)-(2e). Essentially, path generation is related to the so called dual separation problem. The problem consists in trying to find, for each demand d $\in \mathcal{D}$, a path $q(d)$ between $o(d)$ and $t(d)$ such that adding the constraints (2c)-(2d) for $q(d)$ to formulation (2) makes the current optimal solution of $\mathcal{D}(\mathcal{P})$ infeasible for $\mathcal{D}(\mathcal{P} \cup \{q(d)\})$ (see for example [1]). If there is no demand for which a path exists, then the current set $\mathcal{P}$ is sufficient to find the optimum of FAP admitting all possible admissible paths. The problem of finding paths $q(d), \mathrm{d} \in \mathcal{D}$ is called the pricing problem.

B. Pricing problem

Suppose λ, π, σ form an optimal solution of the dual problem $\mathcal{D}(\mathcal{P})$ defined by (2). Then, to be sure that π and λ form an optimal solution also for the dual problem with all possible paths, we need to check whether for each demand $\mathrm{d} \in \mathcal{D}$, and for each path q in graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ between the nodes $o(d)$ and $t(d)$, there exist nonnegative numbers $\sigma^s, s \in \mathcal{S}$ such that:

$$\lambda^0 + \sum_{s \in \mathcal{S}} \tau \sigma^s \leq |q|^0 \quad (3a)$$

$$\lambda^s \leq \sigma^s + |q|^s, s \in \mathcal{S} \quad (3b)$$

$$\sigma^s \geq 0, s \in \mathcal{S} \quad (3c)$$

Above, $|q|^0 = \sum_{e \in q} \pi_e^0$ and $|q|^s = \sum_{e \in q} \pi_e^s, s \in \mathcal{S}$ denote the state-dependent dual length of path q (note that to simplify the notation we do not use the link-path incidence coefficients δ_{eq} here).

Certainly, for all paths $p \in \mathcal{P}_d$ the above inequalities fulfilled by $\sigma^s = \sigma_{dp}^s$ are a part of the considered optimal solution of (2). So the question now is how to find a path q outside the assumed set $\mathcal{P}_d$, if any, for which inequalities (3) are infeasible with respect to variables $\sigma^s, s \in \mathcal{S}$. In fact, in path generation it is advantageous (to speed up the PG algorithm, see Subsection III-D) to find, for each $\mathrm{d} \in \mathcal{D}$, not only a path (if any) in $\hat{\mathcal{P}}_d$ that just separates the current dual solution λ, π, but rather a path $q \in \hat{\mathcal{P}}_d$ for which the dual constraints (2c) and (2d) are maximally violated by the considered dual solution λ, π (see [7], [8], [9]). This is especially true when finding such a path is not substantially more complex than finding an arbitrary path that violates the dual constraints. In our setting, the (negative) measure of violation of the dual constraints corresponding to path q is equal to

$$\|q\| = \left(\tau^{-1}|q|^0 - \sum_{s \in \mathcal{S}_q} |q|^s\right) - \left(\tau^{-1}\lambda^0 + \sum_{s \in \mathcal{S}_q} \lambda^s\right) \quad (5)$$

provided it is negative. The quantity $\|q\|$, is referred to as the generalized dual length of path q. Thus, the pricing problem for demand $\mathrm{d} \in \mathcal{D}$ can be specified as:

$$\min_{q \in \hat{\mathcal{P}}_d} \|q\| \quad (6)$$

The so-formulated pricing problem (6) is difficult because of the particular form of the dual length $\|q\|$. Nevertheless, the problem can be stated as a binary program using formulation (7). In the formulation, binary variables $u_e, e \in \mathcal{E}$, specify the path q we are looking for: $q = \{e \in \mathcal{E}: u_e = 1\}$. Binary variables $z^s, s \in \mathcal{S}$, in turn, identify the set $\mathcal{S}_q$ corresponding to the so defined path $q: \mathcal{S}_q = \{s \in \mathcal{S}: z^s = 1\}$. Besides, $\delta^+(v)$ and $\delta^-(v)$ denote the sets of all links outgoing from, and all links incoming to, respectively, node $v \in \mathcal{V}$, and o is the originating node of the considered demand and t is its terminating node.

$$\min \quad L = \sum_{e \in \mathcal{E}} \left(\tau^{-1}\pi_e^0 + \sum_{s \in \mathcal{S}_e} \pi_e^s z^s\right) u_e - \tau^{-1}\lambda^0 - \sum_{s \in \mathcal{S}} \lambda^s z^s \quad (7a)$$

$$\sum_{e \in \delta^+(o)} u_e - \sum_{e \in \delta^-(o)} u_e = 1 \quad (7b)$$

$$\sum_{e \in \delta^+(v)} u_e - \sum_{e \in \delta^-(v)} u_e = 0, \ v \in \mathcal{V} \backslash \{o, t\} \quad (7c)$$

$$M z^s \geq \lambda^s - \sum_{e \in \mathcal{E}_s} \pi_e^s u_e, s \in \mathcal{S} \quad (7d)$$

$$u_e \in \{0,1\}, e \in \mathcal{E}, \ z^s \in \{0,1\}, s \in \mathcal{S} \quad (7e)$$

where M is a “big M” constant. Constraints (7b) and (7c) assure that variables u_e that are equal to 1 form a path from o to t. Constraints (7d) force each variable $z^s, s \in \mathcal{S}$, to be equal to 1 when the length, with respect to π^s, of the path q defined by variables u is smaller than λ^s. If we for a while assume that also $z^s = 0$ when the length, with respect to π^s, of path q is greater than or equal to λ^s, then it is clear that the objective function computes the value of the quantity $\|q\| = \tau^{-1}(|q|^0 - \lambda^0) - \sum_{s \in \mathcal{S}_q}(|q|^s - \lambda^s)$ (see definition (5)). The optimal solution u, z, L of (7) defines the optimal path $q = \{e \in \mathcal{E}: u_e = 1\}$ we are looking for, with L equal to its reduced cost.

C. Path Generation algorithm

The path generation algorithm for FAP is as follows:

Algorithm 1. Flow-adjustment problem

Step 1: Initialization
Define initial feasible path-sets $\mathcal{P}_d, d \in \mathcal{D}$.

Step 2: Solving
Solve the dual problem $\mathcal{D}(\mathcal{P})$ given by (2) to obtain dual variables λ and π.

Step 3: Update
For each $d \in \mathcal{D}$ solve the pricing problem (7). If the optimal objective L is negative, then add the resulting path q to the path-sets $\mathcal{P}_d$. If for any demand no new path has been added then stop: the resulting path-sets are sufficient to solve FAP to optimality. Otherwise, go to Step 2.

End

Figure 1. Path generation algorithm for FAP

V. NUMERICAL RESULTS

In this section, we present the results of a computational study illustrating the performance of FAR. The undirected network instances used in our tests and listed in Table 1 are taken from SNDlib [10].

TABLE I. NETWORK DESCRIPTION

Network	***Nodes***	***Links***	***Demands***
Net_10 (dfn-bwin)	10	45	90
Net_11 (di-yuan)	11	42	22
Net_14 (nobel-us)	14	21	91
Net_17 (nobel-germany)	17	26	121

In the following subsections, we will compare the Global Rerouting strategy (GR), the FTR strategy, and the FAR strategy in terms of cost-effectiveness. Recall that in a non-nominal link availability state, GR is allowed to restore flows for all demands in the available capacity from scratch, and that FTR is equivalent to FAR with $\tau = 1$. For the experiments reported in this section, we assume a uniform availability ratio α for all the affected links in a given availability state, i.e., $\alpha_e^s = \alpha$ for all $s \in \mathcal{S}$ and $e \in \mathcal{E}_s$ (where $\mathcal{E}_s$ denotes the subset of links affected in state s), and $\alpha_e^s = 1, s \in \mathcal{S}, e \in \mathcal{E} \backslash \mathcal{E}_s$. Also, we assume the demand satisfaction ratio $\beta = 1$, i.e., $h_d^s = \beta h_d^0 = h_d^0$ for all $d \in \mathcal{D}$ and $s \in$. In the comparisons, C_{FA}^{α} (respectively C_{FT}^{α}) and C_{GR}^{α} will denote the optimal value of the network cost for FAR (respectively, the network cost for FTR and GR) with the availability ratio fixed to α. GapFA (respectively, GapGR) gives the relative gain of the cost indicated by C_{FA}^{α} (respectively, C_{GR}^{α}) with respect to C_{FT}^{α} for a given availability ratio α : $GapFA = \frac{C_{FT}^{\alpha} - C_{FA}^{\alpha}}{C_{FT}^{\alpha}}$ and $GapGR = \frac{C_{FT}^{\alpha} - C_{GR}^{\alpha}}{C_{FT}^{\alpha}}$.

A. Minimum network cost for single partial link failures

We first consider the single partial link failures, as a reference to compare FAR with other rerouting strategies. We assume that each link can fail but one at a time, and when it fails its availability ratio is equal to α. Hence, the availability states can be identified with the links. In Figures 2-5, we present GapFA and GapGR as a function of α for 19 selected cases, varying α from 95% to 5%.

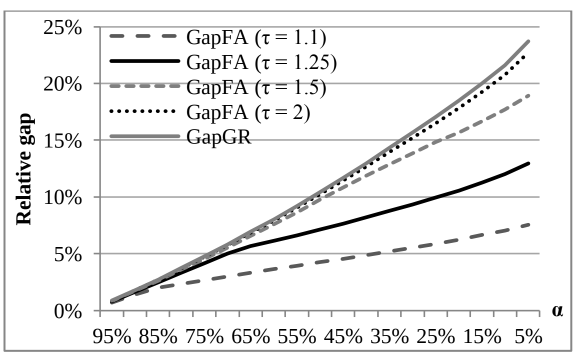

Figure 2. Relative gap between FAR, FTR and GR strategies for Net_10.

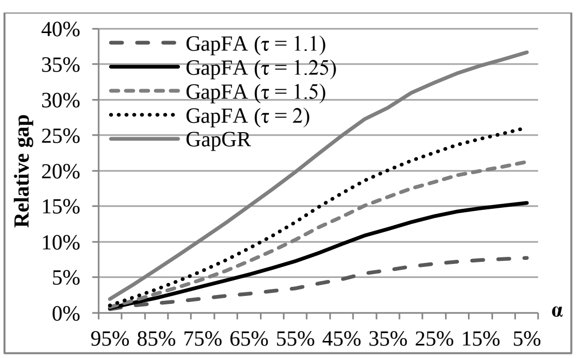

Figure 3. Relative gap between FAR, FTR and GR strategies for Net_11.

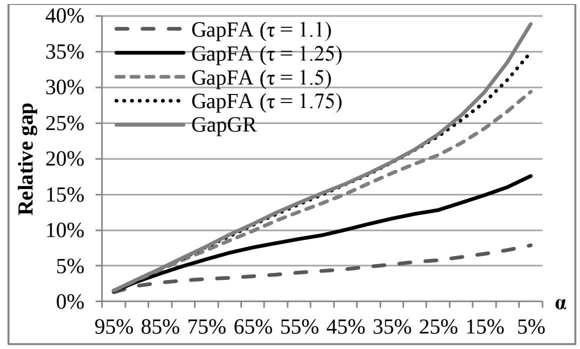

Figure 4. Relative gap between FAR, FTR and GR strategies for Net_14.

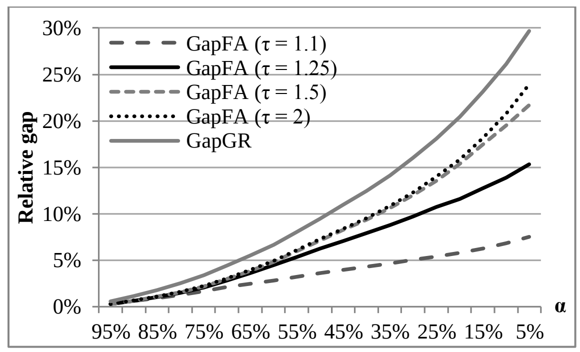

Figure 5. Relative gap between FAR, FTR and GR strategies for Net_17.

As illustrated by Figures 2 to 5, the overall cost of the network obtained when allowing thickening is substantially lower than that obtained using flow-thinning only, and this remains true for all considered availability scenarios and all networks.

It turns out that network topology and demand density characteristics have a significant impact on the efficiency of the flow-adjustment routing strategy. Net_10 and Net_14 have a high number of demands compared to their number of nodes (Net_10 has a full traffic matrix and Net_14 has a full undirected traffic matrix). For these networks the link cost optimized for FAR is close to the cost optimized for GR already for relatively low values of parameter τ (1.75 to 2). In contrast, Net_11 and Net_17 have a limited number of demands compared to their number of nodes and the optimal network cost for FAR requires a high value of parameter τ to meet the results of GR. We finally notice that for a sparse network with a high number of demands (as Net_17), allowing path thickening has a limited impact on the network cost for $\tau > 2$.

B. Minimum network cost for single partial node failures

We now consider single partial node failures (each node can fail but one at a time). This kind of failures represents a local perturbation in a network affecting several links at the same time. In this case, the availability states correspond to the nodes, and the link capacities in a failure state s are given by αy_e if e is incident to the failing node, and to y_e otherwise. In Figures 6-9, we present GapFA and GapGR as a function of α for 19 selected cases, varying α from 95% to 5%.

We notice that the graphs have similar shapes for all the considered networks. Indeed, the gain in the network cost using the flow-adjustment strategy increases until a certain threshold (which seems not to be dependent on the value of τ), and next decreases to 0% for full node failures (α=5%.) This shows that the flow-thinning strategy remains efficient for node failures for the extreme cases as sufficiently low or high perturbation levels. However, the gap between these two strategies and the global rerouting strategy is significant and reaches 80% to 90%, depending on the network, for a low availability ratio α=5%. (Recall that for $\tau \to +\infty$, the network cost for FAR converges to that of GR.)

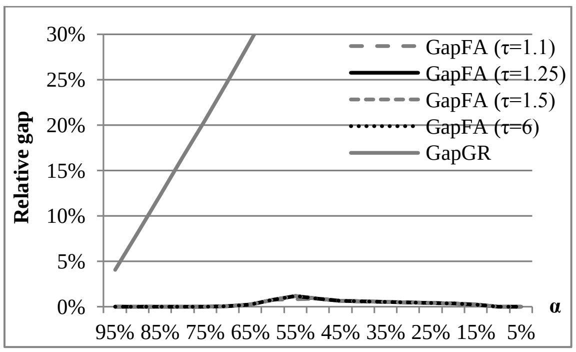

Figure 6. Relative gap between FAR, FTR and GR strategies for Net_10.

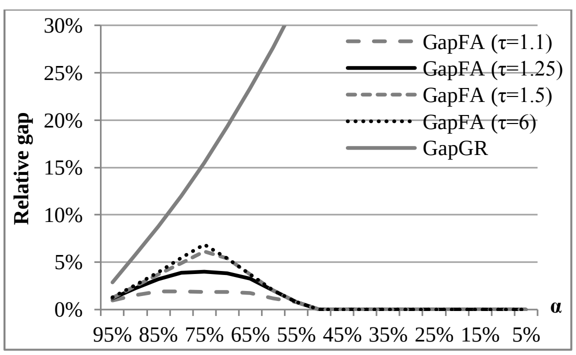

Figure 7. Relative gap between FAR, FTR and GR strategies for Net_11.

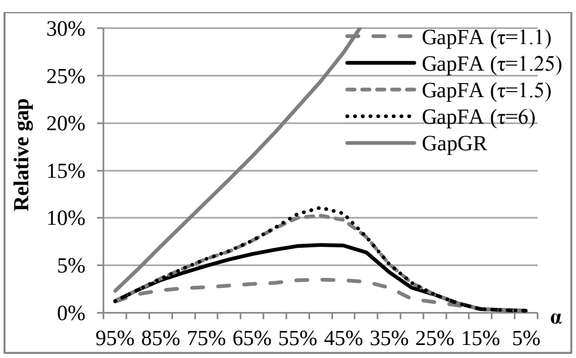

Figure 8. Relative gap between FAR, FTR and GR strategies for Net_14.

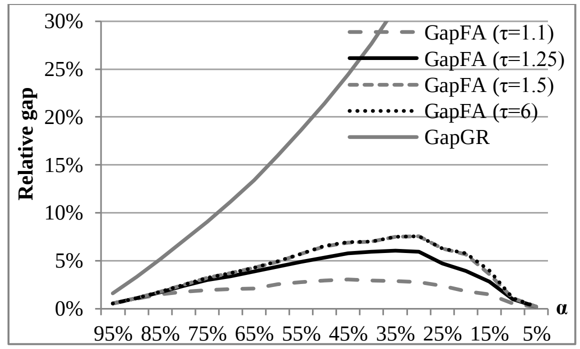

Figure 9. Relative gap between FAR, FTR and GR strategies for Net_17.

Observe that whereas sparse networks (as Net_14 and Net_17) display a decrease of the network cost from 8% to 10% with $\tau < 2$ (compared to flow-thinning), dense networks (as Net_11) have a limited decrease of the cost (maximum 5%). In fully meshed networks (as Net_10), it is virtually impossible to improve the network cost—the gain is less than 1%. However, further investigations are required to analyze this effect.

C. *Range of the perturbation*

Finally, we study the range of the perturbation, i.e., the number of paths which have their flow adjusted when a failure occurs.

We have two routing paths update processes: thinning and thickening. Figures 10-11 show the average number of paths that will have their flow adjusted, for all failure states with a fixed capacity availability ratio α set to 50% for single link failures (Figure 10) and for single node failures (Figure 11). In these figures, the percentage of paths which has to be thinned (resp. thickened) is represented in solid (resp. dashed) bars.

The figures show that FAR requires modifying flows on almost every path when a failure occurs. The percentage of "thickening operations" depends on the value of τ and exhibits a similar behavior for all networks.

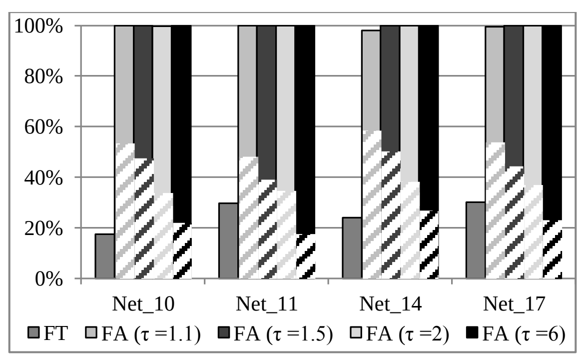

Figure 10. Fraction of paths to be thinned (solid bar) and thickened (dashed bar) with respect to the total number of paths per link failures.

Net_10 Net_11 Net_14 Net_17

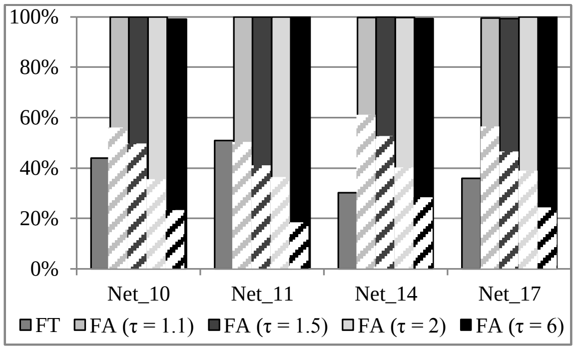

Figure 11. Fraction of paths to be thinned (solid bar) and thickened (dashed bar) with respect to the total number of paths per node failures.

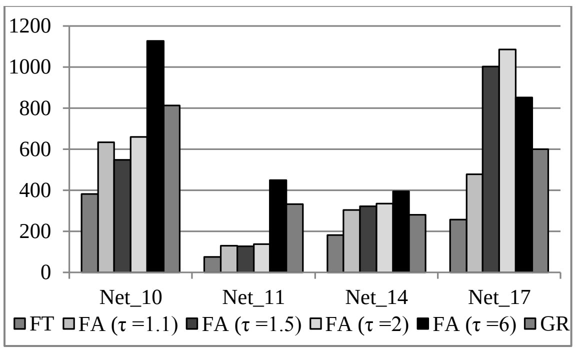

Figure 12. Average number of distincts paths used by FAR, FTR and GR per single link failures.

Figure 12 displays the number of distinct paths used by the other strategies: FTR and GR. We first notice that FTR requires less paths than FAR, and the gap increases with τ. Now we compare the number of paths used by FAR and GR. When τ is low, FAR requires less paths than GR. The difference decreases for higher values of τ, and for $\tau = 6$, for all networks, GR uses less paths than FAR to achieve an optimal solution. This is mainly due to the fact that FAR uses paths with very low value in the nominal state to be able to increase them in a failure state. In regard to scalability and management cost, the above observations suggest that FAR should be used with small values of τ. Another way to limit the number of paths per demand used by this scheme is to modify the formulation (1) by adding specific constraints restricting the number of routing paths per demand, (which would surely have an extra computational cost).

VI. FAILURE RECOVERY PROCESS

The main interest in the flow-thinning strategy stems from the simplicity of the path-flow handling process. Indeed, the reaction to a particular availability state basically consists in decreasing the flow on (some of) the perturbed paths. In the following, we assume that a signaling protocol sends a message from the end nodes of a link to the source nodes of the paths passing through this link when its capacity decreases and reaches a certain threshold. Hence, the time required to recover from a failure state is equal to the time required for the signaling messages to traverse the longest path from the end nodes of the perturbed links to the source node of the disturbed demands. In the following, we study the FAR restoration process.

A. Simulation of the FAR flow adjustment process

The flow adjustment process in FAR is not as simple as in FTR. This process is composed of two simultaneous stages managed by the source and the destination of the traffic demands [12]. The source nodes of the demands will first decrease the flow of some concerned paths, in order to make room for enlarging the flows on some other paths. Next, the destination nodes can increase the flow of the latter paths. Finally, we do not ensure full synchronization as this can be time consuming; hence, some routing paths may need a certain time to become fully operational. This will depend on the number of paths to restore, and on the quantity of flow required to be added to these paths.

B. Simulation results

We study the evolution of the ratio of demands that is perturbed when a failure occurs, and the time required for complete restoration by doing a basic simulation. This simulation is called “basic” for 2 reasons: we first assume that the buffer size is not modified by the perturbation (which is not correct for TCP where a failure is managed using congestion-avoidance algorithm) and we also consider that the time to pass through any link is the same, equal to 1 unit.

Figure 13 reports the ratio of perturbed demands for a fixed link failure as a function of the time, for network Net_14 with $\alpha = 0{,}5$. With FAR there are about 20% of demands perturbed for this link failure situation. Then, the bandwidth of some paths is decreased to allow the bandwidth of some other paths to be increased. This leads to further demands to be temporarily perturbed with about 55% of perturbed demands after 3 units of time. Nevertheless, the process converges very fast comparing to GR which reroutes all demands, perturbed or not.

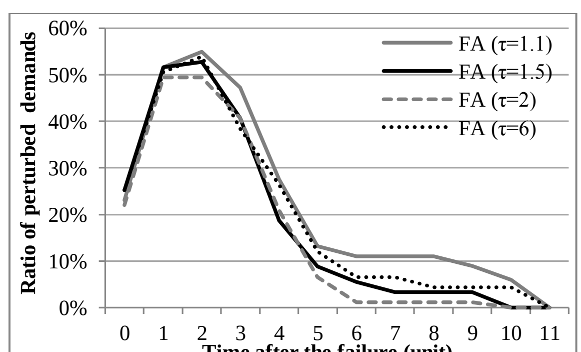

Figure 13. Evolution of the ratio of perturbed demands for Net_14 (α=0,5).

Finally, we notice that the ratio of perturbed demands remains too high compared to that with the flow-thinning strategy.

VII. CONCLUSION AND FUTURE WORK

In this paper, we have studied flow adjustment routing—a routing strategy for demand protection which allows for limited path-flow thickening on top of a commonly understood flow thinning mechanism. FAR is capable of handling multiple partial link failures that are observed in wireless networks or in upper layers of wired communication networks. FAR requires less capacity in the network as compared with a pure Flow-Thinning Strategy, and is cost-wise very close to Global Rerouting for some networks. Our computational results show that FAR achieves encouraging results in terms of the investment cost, at least for the sets of availability states consisting of partial single link failures or single node failures.

We have noticed that the FAR restoration process affects a large number of path flows in the network and may lead to additional perturbation at the very beginning of the process if thinning and thickening operations are not coordinated. Hence, some non-perturbed demands may be affected for a short time when some bandwidth needs to be released from its paths. Making a distinction between paths of perturbed and non-perturbed demands could help to avoid this. On one hand, paths of non-perturbed demands should not be allowed to decrease their total flow more than their traffic requirement. On the other hand, paths of perturbed demands should be able to thin or thick their bandwidth. This leads to a restricted flow-adjustment routing strategy which is nothing else than an extension of the elastic robust rerouting strategy [11], with some constraints relaxed, applied to multiple partial link failures. Moreover, in its current form, FAR and FTR to a less degree, could require significant implementation effort as a large volume of paths are concerned and an important quantity of information is required to maintain all the connections at the routing nodes; this may impact the scalability of the strategy. These issues need further investigation and will be the subject of future work.

ACKNOWLEDGMENT

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