Adaptive high gain observers for a class of nonlinear systems with nonlinear parametrization

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***Abstract*— In this paper, an adaptive observer is proposed for a class of uniformly observable nonlinear systems with nonlinear parametrization. The main novelty of the proposed observer is the introduction of characteristic indices, which allow a natural construction of the gain matrix. The main properties of the proposed observer are highlighted in simulation through an example dealing with the identification of an engine transient fuel characterized by a delay on the input.**

I. Introduction

Over the last decades, adaptive observers design has become a wide and active research field. Generally, the aim of adaptive observers is to simultaneously provide an estimation of the non measured state variables and the unknown (constant) parameters. These observes are particularly used in challenging applications, namely the adaptive control and fault detection and isolation (see for instance [1] and [25]). The seminal contributions related to adaptive observers design have been devoted to linear time invariant systems (see for instance [14] and [12]). The case of linear time varying systems has been investigated in recent works within a deterministic and stochastic contexts [24], [18]. Several approaches have been adopted to tackle the adaptive observer design for nonlinear systems. The usual approach is based on appropriate coordinate transformation which allows to obtain linear error dynamics up to output injection [2], [15], [16], [19]. An optimization based approach has been presented in [6], the existence of the underlying adaptive observer depends on the feasibility of a set of linear matrix inequalities. Some results have been proposed in [3], [5] and [21] without requiring the considered nonlinear systems to be linearizable. These results have been established assuming the existence of some Lyapunov functions satisfying particular conditions. A switching variable structure approach has been pursued in [17]. Adaptive versions of high gain observers have been proposed in [4] and [7]. Though most results on adaptive (nonlinear) observer design deal with linear parametrization, some results dealing with nonlinear parameterizations are available in the literature [13], [11], [20], [10], [7], [23], [22].

In this paper, we propose an adaptive observer for nonlinearly parameterized systems. The class of systems considered is the same than that given in [7]. The main novelty of the proposed observer design with respect to that proposed in [7] is the introduction of the characteristic indices associated to the unknown parameters which allows to derive a new persistent excitation condition more admissible than that considered in [7].

The paper is organized as follows. In the next section, we introduce the class of considered systems and the notations used throughout the paper. Section III is dedicated to the observers design. We first define the characteristic indices associated to the unknown parameters and then the observer design is detailed with a full convergence analysis. In section IV, the performances of the proposed observer and its main properties are illustrated in simulation through an example dealing with an identification problem involving a linear system with delayed input. Finally, concluding remarks are given in section V.

II. Preliminaries

We present first the class of systems which will be in consideration in the paper and the notations that will be used throughout the paper.

A. Presentation of the class of systems

Consider the class of single input single ouput systems which are diffeomorphic to the following form:

$$\begin{cases} \dot{x} = Ax + \varphi(u, x, \rho) \\ y(t) = Cx(t) = x_1(t) \end{cases} \tag{1}$$

with

$$A = \begin{pmatrix} 0 & I_{n-1} \\ 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix} \tag{2}$$

where $x(t) = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$ and $\rho = \begin{pmatrix} \rho_1 \\ \vdots \\ \rho_m \end{pmatrix} \in \mathbb{R}^m$ denote the state and the unknown parameters of the system respectively, $u(t) \in D$ a compact subset of $\mathbb{R}$ is the input of the system and $y(t) \in \mathbb{R}$ denotes the systems output. The function φ has the following structure:

$$\varphi(u, x, \rho) = \begin{pmatrix} \varphi^1(u, x_1, \rho) \\ \varphi_2(u, x_1, x_2, \rho) \\ \vdots \\ \varphi_{n-1}(u, x_1, \dots, x_{n-1}, \rho) \\ \varphi_n(u, x, \rho) \end{pmatrix} \tag{3}$$

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The adaptive observer design needs the adoption of some hypothesis that shall be stated at due courses. At this step, we assume the following:

(A1) The state $x(t)$, the control $u(t)$ and the unknown parameters ρ are bounded, i.e. $x(t) \in X$, $u(t) \in U$ and $\rho \in \Omega$ where $X \subset \mathbb{R}^n$, $U \subset \mathbb{R}$ and $\Omega \in \mathbb{R}^m$ are compacts sets.

(A2) The function φ is continuous on $U \times X \times \Omega$.

(A3) The function φ is Lipschitz with respect to x and ρ, uniformly in u, where $u \in U$, $x \in X$ and $\rho \in \Omega$ and its Lipschitz constant will be denoted L_φ.

B. Notations

Throughout the paper, we denote:

- ρ_M the upper bound of ρ, i.e.
$$\rho_M = \max_{1 \leq i \leq m} |\rho_i|. \tag{4}$$
- $\lambda_m(A)$ and $\lambda_M(A)$ are respectively the lowest and the greatest eigenvalues of A, where A is a square matrix.
- Δ_θ the diagonal matrix defined by:
$$\Delta_\theta = diag\left[1, \frac{1}{\theta}, \ldots, \frac{1}{\theta^{n-1}}\right] \tag{5}$$
where $\theta > 0$ is a real number. Classical computations allow to check the following identities:
$$\begin{aligned} \Delta_\theta A \Delta_\theta &= \theta A \\ C\Delta_\theta^{-1} &= C \end{aligned} \tag{6}$$
where the matrices A and C are defined in (2).
- S the unique solution of the algebraic Lyapunov equation:
$$S + A^T S + SA - C^T C = 0 \tag{7}$$
It has been shown in [8] that S is Symmetric Positive Definite (SPD) and that the matrix $(A - S^{-1}C^T C)$ is Hurwitz.

III. Observers design

One of the novelty of this paper lies in the introduction of m characteristic indices ν_j associated to the m unknown parameters ρ_j for $j = 1, \ldots, m$.

Definition 1: The characteristic index ν_j associated to the parameter ρ_j for system (1) is the smallest integer i such that
$$\frac{\partial \varphi_i}{\partial \rho_j}(u, x, \rho) \neq 0,$$
that is:
$$\begin{aligned} \frac{\partial \varphi_k}{\partial \rho_j}(u, x, \rho) &= 0 \quad \text{for } k = 1, \ldots, \nu_j - 1, \\ \frac{\partial \varphi_{\nu_j}}{\partial \rho_j}(u, x, \rho) &\neq 0. \end{aligned}$$

This allows to introduce the following diagonal $m \times m$ matrix
$$\Omega_\theta = diag\left[\frac{1}{\theta^{\nu_1}}, \frac{1}{\theta^{\nu_2}}, \ldots, \frac{1}{\theta^{\nu_m}}\right] \tag{8}$$
According to the structure of Δ_θ and Ω_θ respectively given by (5) and (8), one has
$$\Delta_\theta \frac{\partial \varphi(u, x, \rho)}{\partial \rho} \Omega_\theta^{-1} = \theta \Phi(u, x, \rho) + R\left(u, x, \rho, \frac{1}{\theta}\right) \tag{9}$$
where $\Phi(u, x, \rho)$ and $R\left(u, x, \rho, \frac{1}{\theta}\right)$ are $n \times m$ rectangular matrices whose respective entries $\Phi_j^i(u, x)$ and $R_j^i\left(u, x, \frac{1}{\theta}\right)$, for $(i, j) \in [1, n] \times [1, m]$, are defined as follows:
$$\Phi_j^i(u, x, \rho) = 0 \quad \text{if } i \neq \nu_j \tag{10}$$
$$\Phi_j^{\nu_j}(u, x, \rho) = \frac{\partial \varphi^{\nu_j}}{\partial \rho_j}(u, x, \rho) \tag{11}$$
$$R_j^i\left(u, x, \rho, \frac{1}{\theta}\right) = 0 \quad \text{if } i \leq \nu_j \tag{12}$$
$$R_j^i\left(u, x, \rho, \frac{1}{\theta}\right) = \frac{1}{\theta^{i-1-\nu_j}} \frac{\partial \varphi_i}{\partial \rho_j}(u, x, \rho) \text{ otherwise} \tag{13}$$
The matrix $\Phi(u, x, \rho)$ does not depend on θ at all while this parameter appears with non positive powers in the entries of the matrix $R\left(u, x, \rho, \frac{1}{\theta}\right)$. Moreover, these matrices verifiy the following property:
$$\begin{aligned} \Delta_\theta \Phi(u, x, \rho) \Omega_\theta^{-1} &= \theta \Phi(u, x, \rho) \\ R_j^1\left(u, x, \rho, \frac{1}{\theta}\right) &= 0 \ for \ j = 1, \ldots, m \end{aligned} \tag{14}$$
The candidate adaptive observer for sytem (1) is given by the following dynamical system:
$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + \varphi(u(t), \hat{x}(t), \hat{\rho}(t)) \\ \qquad -\theta \Delta_\theta^{-1}\left(S^{-1} + \Upsilon(t) P(t) \Upsilon^T(t)\right) C^T (C\hat{x}(t) - y(t)) \\ \dot{\hat{\rho}}(t) = -\theta \Omega_\theta^{-1} P(t) \Upsilon^T(t) C^T (C\hat{x}(t) - y(t)) \\ \dot{\Upsilon}(t) = \theta(A - S^{-1}C^T C)\Upsilon(t) + \theta \Phi(u(t), \hat{x}(t), \hat{\rho}(t)) \\ \Upsilon(0) = 0 \\ \dot{P}(t) = -\theta P(t) \Upsilon^T(t) C^T C \Upsilon(t) P(t) + \theta P(t) \\ P(0) = P^T(0) > 0 \end{cases} \tag{15}$$
where $\hat{x} \in \mathbb{R}^n$ and $\hat{\rho} \in \mathbb{R}^m$ respectively denote the state and parameter estimates, $\Phi(u, \hat{x}, \hat{\rho})$, S, C and Δ_θ are respectively given by (11), (7), (2) and (5) and θ is a positif scalar which is the sole observer design parameter.

In order to prove the convergence of the observer, we need the following assumption:

(A4) The input u is such that for any trajectory of system (15) starting from $(\hat{x}(0), \hat{\rho}(0)) \in X \times \Omega$, the matrix $C\Upsilon(t)$ is persistently exciting i.e.
$$\exists \delta_1, \delta_2 > 0; \exists T > 0; \forall t \geq 0 : \\ \delta_1 I_m \leq \int_t^{t+T} \Upsilon^T(\tau) C^T C \Upsilon(\tau) d\tau \leq \delta_2 I_m$$
Now, due to the nonlinear parametrization, the following additional assumption is also needed:

(A5) The function $\Phi(u,x,\rho)$ satisfies the following condition:

$$\begin{aligned}&\forall\ (u,\hat{x})\in U\times X,\ \forall\ (\hat{\rho},\rho)\in\Omega^2:\\&|\Phi(u,\hat{x},\hat{\rho})-\Phi(u,\hat{x},\rho)|\leq\nu\sqrt{\tfrac{\lambda_m(P)}{\lambda_M(S)}}\end{aligned}\tag{16}$$

where S and P are respectively given by (7) and (15) and $\nu 0$ belongs to $]0,1[$.

One now states the following.

Theorem 1: Consider the system (1) subject to assumptions **(A1)** to **(A5)**. Then, for every bounded input, there exists a constant θ_o such that for every $\theta>\theta_o$, system (15) is an adaptive observer for system (1) with an exponential error convergence to the origin.

Remark 1:

- The persistent excitation condition considered in Assumption **(A4)** involves the observers states, unlike classical assumptions where the state of the system is generally considered. Such a formulation renders the underlying condition checkable on line.
- Assumption **(A5)** reveals the local nature of the adaptive observer (15) since inequality (16) cannot generally be satisfied if the initial observation error related to the unknown parameter has a high magnitude. However, in practice, the observer (15) still work with relatively high values of these initial observation error, miming the behaviour of global observers, as it shall be illustrated through simulation results given in the next section.
  Note that in the case of a linearly parameterized system, assumption **(A5)** is always satisfied.

The proof of this theorem is given after some comments and facts that will be used throughout the proof.

- The matrix $\Upsilon(t)$ is bounded with upper and lower bounds that do not depend on θ. To prove this, let us change the time scale by setting $\tau=t/\theta$ and let $\bar{\Upsilon}(t)=\Upsilon\left(\frac{t}{\theta}\right)$. From equations (15), one has:

$$\dot{\bar{\Upsilon}}(t)=(A-S^{-1}C^TC)\bar{\Upsilon}(t)+\Phi\left(u\left(\frac{t}{\theta}\right),\hat{x}\left(\frac{t}{\theta}\right),\hat{\rho}\left(\frac{t}{\theta}\right)\right)\tag{17}$$

  Since the matrix $A-S^{-1}C^TC$ is Hurwitz and from the fact that $\Phi\left(u\left(\frac{t}{\theta}\right),\hat{x}\left(\frac{t}{\theta}\right),\hat{\rho}\left(\frac{t}{\theta}\right)\right)$ is bounded with bounds independent of θ, one naturally concludes that $\Upsilon(t)$ is bounded with upper and lower bounds independent of θ. In fact, the matrix Υ is a filtered version of Φ. The initial condition of Υ is taken equal to zero in order to make shorter the transient behavior.

- Under assumption **(A4)**, the matrix $P(t)$ governed by the ordinary differential equation described in (15) is SPD and bounded. Moreover, its corresponding upper and lower bounds are independent of θ. To prove this, let us again change the time scale by setting $\tau=t/\theta$ and let $\bar{P}(t)=P\left(\frac{t}{\theta}\right)$. Then, one has:

$$\begin{aligned}\dot{\bar{P}}(t)&=-\bar{P}(t)\Upsilon^T\left(\frac{t}{\theta}\right)C^TC\Upsilon\left(\frac{t}{\theta}\right)\bar{P}(t)+\bar{P}(t)\\\bar{P}(0)&=\bar{P}^T(0)>0\end{aligned}\tag{18}$$

  Under assumption **(A4)**, it has been shown in [25] that $\bar{P}$ is SPD and bounded and its corresponding bounds (obviously) do not depend on θ. The same result is trivially valid for $P(t)$.

Proof: [Proof of theorem 1] Set $\tilde{x}(t)=\hat{x}-x$ and $\tilde{\rho}(t)=\hat{\rho}(t)-\rho$. Using (1) and (15), one has

$$\begin{aligned}\dot{\tilde{x}}&=A\tilde{x}-\theta\Delta_\theta^{-1}S^{-1}C^TC\tilde{x}+\Delta_\theta^{-1}\Upsilon(t)\Omega_\theta\dot{\tilde{\rho}}(t)\\&\quad+(\varphi(u,\hat{x},\hat{\rho})-\varphi(u,x,\rho))\end{aligned}\tag{19}$$

$$\dot{\tilde{\rho}}=-\theta\Omega_\theta^{-1}P(t)\Upsilon^T(t)C^TC\tilde{x}\tag{20}$$

Set $\bar{x}=\Delta_\theta\tilde{x}$ and $\bar{\rho}=\Omega_\theta\tilde{\rho}$. Using the identities (6), one obtains:

$$\begin{aligned}\dot{\bar{x}}&=\Delta_\theta A\Delta_\theta^{-1}\bar{x}-\theta S^{-1}C^TC\Delta_\theta^{-1}\bar{x}+\Upsilon(t)\dot{\tilde{\rho}}(t)\\&\quad+\Delta_\theta(\varphi(u,\hat{x},\hat{\rho})-\varphi(u,x,\rho))\\&=\theta(A-S^{-1}C^TC)\bar{x}+\Upsilon(t)\dot{\tilde{\rho}}(t)\\&\quad+\theta\Phi(u,\hat{x},\hat{\rho})\bar{\rho}\\&\quad+\theta(\Phi(u,\hat{x},\rho_\xi)-\Phi(u,\hat{x},\hat{\rho}))\bar{\rho}\\&\quad+R\left(u,\hat{x},\rho_\xi,\frac{1}{\theta}\right)\bar{\rho}\\&\quad+\Delta_\theta(\varphi(u,\hat{x},\rho)-\varphi(u,x,\rho))\\\dot{\tilde{\rho}}&=-\theta P(t)\Upsilon^T(t)C^TC\Delta_\theta^{-1}\bar{x}\\&=-\theta P(t)\Upsilon^T(t)C^TC\bar{x}\end{aligned}\tag{21}$$

The equality (21) results from the following decomposition of φ, which uses the mean value theorem and property (9):

$$\begin{aligned}&\Delta_\theta(\varphi(u,\hat{x},\hat{\rho})-\varphi(u,x,\rho))\\&=\Delta_\theta(\varphi(u,\hat{x},\hat{\rho})-\varphi(u,\hat{x},\rho))+\Delta_\theta(\varphi(u,\hat{x},\rho)-\varphi(u,x,\rho))\\&=\theta(\Phi(u,\hat{x},\hat{\rho}))\bar{\rho}+\theta(\Phi(u,\hat{x},\rho_\xi)-\Phi(u,\hat{x},\hat{\rho}))\bar{\rho}\\&\quad+R\left(u,\hat{x},\rho_\xi,\tfrac{1}{\theta}\right)\bar{\rho}+\Delta_\theta(\varphi(u,\hat{x},\rho)-\varphi(u,x,\rho))\end{aligned}$$

Now, set:

$$\eta(t)=\bar{x}(t)-\Upsilon(t)\bar{\rho}(t)\tag{22}$$

For writing convenience and as long as there is no ambiguity, the time variable t shall be omitted in the sequel. One gets:

$$\begin{aligned}\dot{\eta}(t)&=\theta(A-S^{-1}C^TC)(\eta+\Upsilon\bar{\rho})+\Upsilon\dot{\tilde{\rho}}+\theta\Phi(u,\hat{x},\hat{\rho})\bar{\rho}\\&\quad+\theta(\Phi(u,\hat{x},\rho_\xi)-\Phi(u,\hat{x},\hat{\rho}))\bar{\rho}+R(u,\hat{x},\hat{\rho},\frac{1}{\theta})\\&\quad+\Delta_\theta(\varphi(u,\hat{x},\rho)-\varphi(u,x,\rho))-\Upsilon\dot{\tilde{\rho}}-\dot{\Upsilon}\bar{\rho}\\&=\theta(A-S^{-1}C^TC)\eta+R\left(u,\hat{x},\rho_\xi,\frac{1}{\theta}\right)\bar{\rho}\\&\quad+\theta(\Phi(u,\hat{x},\rho_\xi)-\Phi(u,\hat{x},\hat{\rho}))\bar{\rho}\\&\quad+\Delta_\theta\left(\varphi(u,\hat{x},\rho)-\varphi(u,x,\rho)\right)\end{aligned}$$

Note that the last equality comes from the fact that Υ is governed by the ordinary differential equation given in (15). Set $V_1(\eta(t)) = \eta^T(t)S\eta(t)$, $V_2(\bar{\rho}(t)) = \bar{\rho}^T(t)P^{-1}(t)\bar{\rho}(t)$ where S and $P(t)$ are respectively given by (7) and (15) and let $V(\eta(t), \bar{\rho}(t)) = V_1(\eta(t)) + V_2(\bar{\rho}(t))$ be the Lyapunov candidate function. Using (7), one gets:

$$\begin{aligned}\dot{V}_1(\eta) &= 2\theta\eta^T S(A - S^{-1}C^TC)\eta \\ &\quad +2\eta^T SR\left(u, \hat{x}, \rho_\xi \frac{1}{\theta}\right)\bar{\rho} \\ &\quad +2\theta\eta^T S(\Phi(u,\hat{x},\rho_\xi) - \Phi(u,\hat{x},\hat{\rho}))\bar{\rho} \\ &\quad +2\eta^T S\Delta_\theta\left(\varphi(u,\hat{x},\rho) - \varphi(u,x,\rho)\right) \\ &= -\theta V_1(\eta) - \theta\eta^T C^TC\eta \\ &\quad +2\eta^T SR\left(u, \hat{x}, \rho_\xi, \frac{1}{\theta}\right)\bar{\rho} \\ &\quad +2\theta\eta^T S(\Phi(u,\hat{x},\rho_\xi) - \Phi(u,\hat{x},\hat{\rho}))\bar{\rho} \\ &\quad +2\eta^T S\Delta_\theta\left(\varphi(u,\hat{x},\rho) - \varphi(u,x,\rho)\right)\end{aligned}$$

It is clear from (22) that

$$|\bar{x}| \leq |\eta| + |\Upsilon(t)||\bar{\rho}| \leq |\eta| + \gamma_M|\bar{\rho}| \tag{23}$$

where

$$\gamma_M = \sup_{t\geq 0}|\Upsilon(t)| \tag{24}$$

Proceeding as in [7], one can show that for $\theta \geq 1$:

$$\begin{aligned}2\eta^T S\Delta_\theta(\varphi(u,\hat{x},\rho) - \varphi(u,x,\rho)) \\ \leq 2\sqrt{\lambda_M(S)}\sqrt{V_1(\eta)}L_\varphi|\bar{x}| \\ \leq k_1V_1 + c_1\sqrt{V_1}\sqrt{V_2}\end{aligned}$$

$$2\theta\eta^T S\left(\Phi(u,\hat{x},\rho_\xi) - \Phi(u,x,\rho)\right)\bar{\rho} \leq 2\nu\theta\sqrt{V_1(\eta)}\sqrt{V_2(\bar{\rho})}$$

$$\begin{aligned}2\eta^T SR\left(u,\hat{x},\rho_\xi,\frac{1}{\theta}\right)\bar{\rho} &\leq 2\sqrt{\lambda_M(S)}\sqrt{V_1(\eta)}R_M|\bar{\rho}| \\ &\leq c_2\sqrt{V_1}\sqrt{V_2}\end{aligned}$$

where k_1, k_2, c_1, c_2 and c_3 are defined as:

$$\begin{gathered}k_1 = 2\sqrt{\tfrac{\lambda_M(S)}{\lambda_m(S)}}L_\varphi,\ c_1 = 2\sqrt{\tfrac{\lambda_M(S)}{\underline{\lambda}_m(P)}}L_\varphi\gamma_M, \\ c_2 = 2\sqrt{\tfrac{\lambda_M(S)}{\underline{\lambda}_m(P)}}R_M \\ \text{with } \underline{\lambda}_m(P) = \min_{t\geq 0}\lambda_m(P(t)) \\ \text{and } R_M = \sup_{t\geq 0}|R(\big(u(t),\hat{x}(t),\rho_\xi(t),\tfrac{1}{\theta}\big))|\end{gathered}$$

where L_φ is the Lipschitz constants of φ as stated in assumption **(A3)** and Ψ_M, ρ_M and γ_M are the upper bounds of Ψ, ρ and Υ given by (4) and (24), respectively. According to the above developments, one gets:

$$\dot{V}_1(\eta) \leq -(\theta - k)V_1 - \theta\eta^T C^TC\eta + (c + 2\nu\theta)\sqrt{V}_1\sqrt{V}_2 \tag{25}$$

where $k = k_1$ and $c = c_1 + c_2$.

Let us now derive the time derivative of V_2. From (15), one gets:

$$\begin{aligned}\dot{V}_2(\bar{\rho}) &= 2\bar{\rho}^T P^{-1}(t)\dot{\bar{\rho}} - \bar{\rho}^T P^{-1}(t)\dot{P}(t)P^{-1}(t)\bar{\rho} \\ &= -2\theta\bar{\rho}^T\ \Upsilon^T C^TC\bar{x} - \theta\bar{\rho}^T P^{-1}(t)\bar{\rho} \\ &\quad +\theta\bar{\rho}^T\Upsilon^T C^TC\Upsilon\bar{\rho} \\ &= -\theta V_2 - 2\theta\bar{\rho}^T\ \Upsilon^T C^TC\bar{x} + \theta\bar{\rho}^T\Upsilon^T C^TC\Upsilon\bar{\rho} \\ &= -\theta V_2 - 2\theta\bar{\rho}^T\ \Upsilon^T C^TC\left(\eta + \Upsilon\bar{\rho}\right) \\ &\quad +\theta\bar{\rho}^T\Upsilon^T C^TC\Upsilon\bar{\rho} \\ &= -\theta V_2 - \theta\bar{\rho}^T\Upsilon^T C^TC\Upsilon\bar{\rho} - 2\theta\bar{\rho}^T\ \Upsilon^T C^TC\eta\end{aligned} \tag{26}$$

Hence, using (25) and (26), one obtains

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(\eta) + \dot{V}_2(\bar{\rho}) \\ &\leq -(\theta - k)V_1 - \theta V_2 + (c + 2\theta\nu)\sqrt{V}_1\sqrt{V}_2 \\ &\quad -\theta\eta^T C^TC\eta - \theta\bar{\rho}^T\Upsilon^T C^TC\Upsilon\bar{\rho} \\ &\quad -2\theta\bar{\rho}^T\ \Upsilon^T C^TC\eta \\ &= -(\theta - k)V_1 - \theta V_2 + (c + 2\theta\nu)\sqrt{V}_1\sqrt{V}_2 \\ &\quad -\theta(\eta + \Upsilon\bar{\rho})^T C^TC(\eta + \Upsilon\bar{\rho}) \\ &\leq -(\theta - k)V_1 + (c + 2\theta\nu)\sqrt{V}_1\sqrt{V}_2 - \theta V_2 \\ &\leq -(\theta - k)(V_1 + V_2) + \frac{(c + 2\theta\nu)}{2}(V_1 + V_2) \\ &= -((1 - \nu)\theta - k - \frac{c}{2})V\end{aligned} \tag{27}$$

Now, it suffices to choose θ such that $(1 - \nu)\theta > k + c/2$, which is always possible since $\nu < 1$. This ends the proof. ■

IV. Example: Engine transient fuel identification

This example deals with the identification of the transient fuel in a port fuel injected internal combustion engine which can be described by the following second order linear time-delay system [9]:

$$\left\{\begin{array}{lcl}\dot{z}_1(t) &=& z_2(t) \\ \dot{z}_2(t) &=& \rho_1\dot{u}(t - \rho_5) + \rho_2 u(t - \rho_5) \\ & & -\rho_3 z_2(t) - \rho_4 z_1(t) \\ y(t) &=& z_1(t)\end{array}\right.$$

where the output $y(t)$ is the measured fuel-to-air ratio and the input $u(t)$ is the injected fuel over air ratio. It is assumed that this system is internally asymptotically stable whereas the unknown parameters $\{\rho_i\}_{i\in[1,5]}$ are constant and positive. This system has been considered in [9] where the authors proposed an on-line identification procedure to estimate the model parameters. The main drawback of the previously proposed procedure lies in the fact that the delay, i.e. ρ_5, can be estimated provided that its possible value belongs to a set of finite numbers of known values. Such a prerequisite is not necessary with the approach proposed in this paper since system (28) is under form (1) with

$$A \triangleq \left(\begin{array}{cc}0 & 1 \\ 0 & 0\end{array}\right) \tag{28}$$

and

$$\varphi(u,z,\rho) \triangleq \begin{pmatrix} 0 \\ \rho_1\dot{u}(t-\rho_5)+\rho_2 u(t-\rho_5)-\rho_3 z_2(t)-\rho_4 z_1(t) \end{pmatrix} \quad (29)$$

and one can then use an observer of the form (15) for the estimation of the parameters $\{\rho_i\}_{i\in[1,5]}$ as well as the missing state z_2.

In this case, all the unknown parameters appear for the first time in the second equation and hence all the characteristic indices are equal to 2. So, one has $\Omega_\theta = \theta^{-2} I_5$ and the matrix $\Phi(u,\hat{z},\hat{\rho})$ coincides with $\dfrac{\partial\varphi}{\partial\rho}(u,\hat{z},\hat{\rho})$, i.e.

$$\Phi(u,\hat{z},\hat{\rho}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \dot{u}(t-\hat{\rho}_5) & u(t-\hat{\rho}_5) & -\hat{z}_2 & -\hat{z}_1 \\ 0 \\ -\left(\hat{\rho}_1\ddot{u}(t-\hat{\rho}_5)+\hat{\rho}_2\dot{u}(t-\hat{\rho}_5)\right) \end{pmatrix} \quad (30)$$

The input $u(t)$ was particularly generated as the output of a second order filter with a double pole $p_1 = p_2 = -5$ and an input $v(t)$ chosen as a normally distributed white noise. This allowed to simultaneously obtain the time derivatives $\dot{u}(t)$ and $\ddot{u}(t)$. The simulation was carried out under MATLAB environment and the input of the second order filter was chosen through the following call of MATLAB standard function

$$v(t) = 0.012(4 + randn([0:0.1:t_f],1)); \quad (31)$$

where t_f denotes the final time of simulation ($t_f = 40s$ in simulation). This input generator allowed to obtain an output varying in the normal engine operating range, i.e. about 0.08 (see figure 1).

The true values of the parameters used in the model simulation are those given in [9], namely

$$\rho_1 = 4, \quad \rho_2 = 12.5, \quad \rho_3 = 12.5, \quad \rho_4 = 7.5 \quad and \quad \rho_5 = 0.3$$

The state variables of the engine and observer were initialized as follows:

$$z_1(0) = \hat{z}_1(0) = 0.068, \quad z_2(0) = 0.1, \quad and \quad \hat{z}_2(0) = 0$$

while the parameter estimates were arbitrarily initialized to zero.

The choice of the tuning parameter θ is achieved through a trial-and-error approach. This value has been fixed to 3 when simulating observer (15).

The parameter estimates $\{\hat{\rho}_i\}_{i\in[1,5]}$ as well as those the state estimates $\{\hat{z}_i\}_{i\in[1,2]}$ provided by the observer are compared with their true values (issued from the model simulation) in figures 2 and 3. Again, these results clearly show the good performance of the observer which provides satisfactory estimates of the states as well as of the unknown parameters.

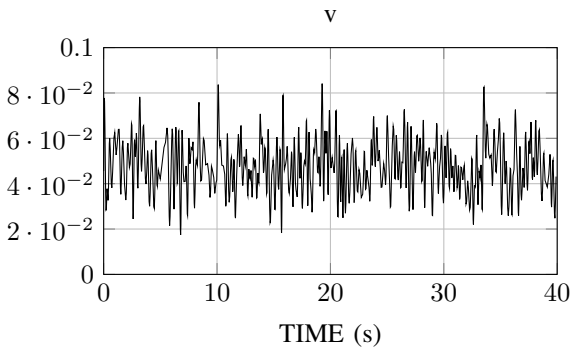

Fig. 1. Exemple - Time evolution of the input generator, $v(t)$

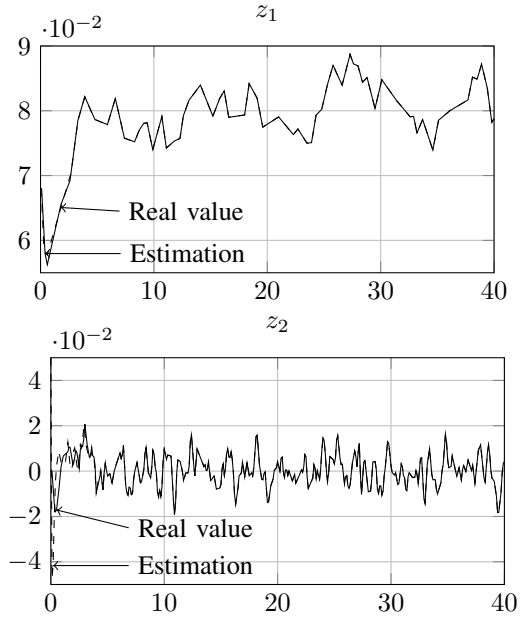

Fig. 2. Exemple - State and estimates of the system

V. Conclusion

In this paper, we have improved the adaptive high gain observer proposed in [7] thanks to the introduction of some characteristic indices. Indeed, these indices allowed to formulate a new persistent excitation condition which is more admissible than that condidered in [7], providing thereby a natural construction of the gain matrix. The performance of the proposed observer and its main properties have been highlighted in simulation through an example dealing with the identification of a delay input linear system.

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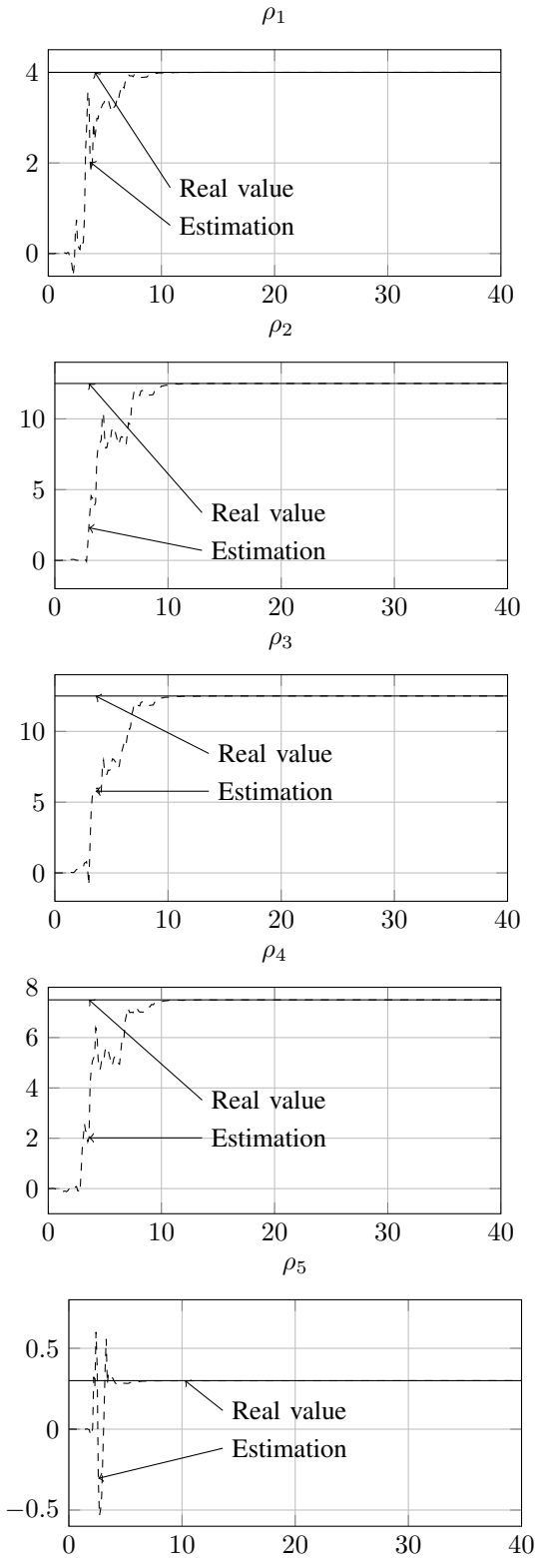

Fig. 3. Exemple - Estimation of the unknown parameters

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