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# A NITSCHKE FINITE ELEMENT METHOD FOR DYNAMIC CONTACT

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We present a new approximation of elastodynamic frictionless contact problems based both on the finite element method and on an adaptation of Nitsche's method which was initially designed for Dirichlet's condition [4]. The corresponding space semi-discretized weak form reads:

$$\left\{ \begin{array}{l} \text{Find a displacement } \mathbf{u}^h : [0, T] \rightarrow \mathbf{V}^h \text{ such that for } t \in [0, T] : \\ \langle \rho \ddot{\mathbf{u}}^h(t), \mathbf{v}^h \rangle + A_{\Theta\gamma_h}(\mathbf{u}^h(t), \mathbf{v}^h) + \int_{\Gamma_C} \frac{1}{\gamma_h} [P_{\gamma_h}(\mathbf{u}^h(t))]_+ P_{\Theta\gamma_h}(\mathbf{v}^h) d\Gamma = L(t)(\mathbf{v}^h), \\ \forall \mathbf{v}^h \in \mathbf{V}^h, \\ \mathbf{u}^h(0, \cdot) = \mathbf{u}_0^h, \quad \dot{\mathbf{u}}^h(0, \cdot) = \dot{\mathbf{u}}_0^h. \end{array} \right. \quad (1)$$

In the above formulation  $\mathbf{V}^h$  is a finite element space built from standard Lagrange finite elements, piecewise linear or quadratic,  $A_{\Theta\gamma_h}(\mathbf{u}^h, \mathbf{v}^h) := \int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}^h) : \boldsymbol{\varepsilon}(\mathbf{v}^h) d\Omega - \int_{\Gamma_C} \Theta\gamma_h \sigma_n(\mathbf{u}^h)\sigma_n(\mathbf{v}^h) d\Gamma$  and  $P_{\Theta\gamma_h}(\mathbf{v}^h) := v_n^h - \Theta\gamma_h \sigma_n(\mathbf{v}^h)$ . The linear form  $L(\cdot)$  stands for prescribed body and boundary forces. The domain of the elastic body is denoted by  $\Omega$  and the contact boundary by  $\Gamma_C$ ;  $T$  is the final time of simulation;  $\rho$  is the density of the elastic material; the notation  $v_n^h$  stands for the normal component on  $\Gamma_C$  of  $\mathbf{v}^h$ , and  $\sigma_n(\mathbf{v}^h)$  is the normal stress on  $\Gamma_C$ ;  $\mathbf{u}_0^h$  (resp.  $\dot{\mathbf{u}}_0^h$ ) is an approximation of the initial displacement  $\mathbf{u}_0$  (resp. the initial velocity  $\dot{\mathbf{u}}_0$ ). The notation  $[\cdot]_+$  stands for the positive part of a scalar quantity, and  $\langle \cdot, \cdot \rangle$  stands for the  $L^2(\Omega)$  inner product. The parameter  $\gamma_h$

is a positive piecewise constant function on the contact interface  $\Gamma_C$  ( $\gamma_h(x) = \gamma_0 h_T(x)$ , where  $\gamma_0$  is a positive constant and  $h_T(x)$  is the size of the mesh element  $T$ ). Note the additional numerical parameter  $\Theta$  which can be freely chosen in  $\mathbb{R}$ . As in Nitsche's method for (static) unilateral contact, values of interest for  $\Theta$  are  $-1, 0, 1$  [1].

A main interesting characteristic is that this approximation produces well-posed space semi-discretizations, for any value of  $\gamma_0 > 0$ , contrary to standard finite element discretizations. We study associated energy conservation properties and manage to prove that :

$$\frac{d}{dt} E_{\Theta}^h(t) = (\Theta - 1) \int_{\Gamma_C} \frac{1}{\gamma_h} [P_{\gamma_h}(\mathbf{u}^h(t))]_+ \dot{u}_n^h(t) d\Gamma$$

where  $E_{\Theta}^h(t) := E^h(t) - \Theta R^h(t)$  is an augmented energy associated to the semi-discrete solution  $\mathbf{u}^h(t)$ . The mechanical energy is given by  $E^h(t) := \frac{1}{2} [\rho \|\dot{\mathbf{u}}^h(t)\|_{0,\Omega}^2 + \frac{1}{2} a(\mathbf{u}^h(t), \mathbf{u}^h(t))]$  and  $R^h(t) := \frac{1}{2} [\|\gamma_h^{\frac{1}{2}} \sigma_n(\mathbf{u}^h(t))\|_{0,\Gamma_C}^2 - \|\gamma_h^{-\frac{1}{2}} [P_{\gamma_h}(\mathbf{u}^h(t))]_+\|_{0,\Gamma_C}^2]$  is an extra term, which represents, roughly speaking, the non-fulfillment of the contact conditions at the semi-discrete level. Note in particular that the symmetric variant ( $\Theta = 1$ ) conserves the discrete augmented energy  $E_{\Theta}^h$ .

Various time-discretizations for (1) are then considered, with the families of  $\theta$ -schemes and Newmark schemes. We also introduce a new hybrid scheme which is second-order accurate and numerically stable without any restriction on the time-step in the case  $\Theta = 1$ . This new scheme is inspired from [2, 3], and introduces much less dissipation than unconditionally stable variants of  $\theta$ -schemes and Newmark schemes. We study theoretically the well-posedness of each discrete scheme as well as its energy conservation properties. We finally achieve the corresponding numerical experiments.

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