



**HAL**  
open science

## $H_\infty$ - based fault diagnosis for diesel engines

Boulaïd Boulkroune, Olivier Pages, Abdel Aitouche, Ali Zemouche, Ahmed El Hajjaji

► **To cite this version:**

Boulaïd Boulkroune, Olivier Pages, Abdel Aitouche, Ali Zemouche, Ahmed El Hajjaji.  $H_\infty$  - based fault diagnosis for diesel engines. IEEE Multi-Conference on Systems and Control, MSC 2014, Oct 2014, Antibes, France. 10.1109/CCA.2014.6981414 . hal-01059478

**HAL Id: hal-01059478**

**<https://hal.science/hal-01059478>**

Submitted on 24 Mar 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# $\mathcal{H}_\infty$ -based fault diagnosis for diesel engines

Boulaïd Boulkroune<sup>1</sup>, Olivier Pagès<sup>1</sup>, Abdel Aitouche<sup>2</sup>, Ali Zemouche<sup>3</sup> and Ahmed El Hajjaji<sup>1</sup>

**Abstract**—The problem of fault detection for diesel engines is addressed in this work. The proposed approach allows to detect and estimate actuator and sensor faults. The residual design procedure is proposed for nonlinear parameter-varying systems. Inspired by some works in literature, the maximization problem of fault effect and the minimization problem of disturbances effect on the residuals is transformed to one problem of  $\mathcal{L}_2$ -norm minimization. Sufficient conditions for the existence of residual generator are formulated in terms of linear matrix inequality (LMI) sets. Performances of the proposed approach are shown through the application to a diesel engine model.

## I. INTRODUCTION

In order to avoid process performance degradation, faults must be detected and located as quickly as possible in order to be able to take appropriate decisions before they turn into failure. This is why the early detection and isolation of faults is crucial for ensuring process safety and efficiency. Different fault detection and isolation (FDI) techniques are proposed in literature. The diversity of the proposed solutions was enriched by the growing interest of the industry. Among these solutions, model-based FDI systems have found extensive use due to its fast response to abrupt failure and to its implementation in real-time algorithms. This type of diagnosis method can be performed over a larger operating range. It allows also to isolate the faulty components since that a single fault in a process can propagate to several outputs. Furthermore, high diagnosis performance can be obtained despite the presence of disturbances.

Observer-based robust fault detection has received great attention in the last decades. For linear systems, various approaches have been proposed ([1],[2] and [3]). However, few methods are proposed for nonlinear systems and most of them are dedicated to a specific class of nonlinear systems. Besides, the extension of the existing results of linear case to nonlinear case is still a challenging task. The problem of model-based

fault detection for the class of Lipschitz systems has been covered by ([4], [5], [6] and [7]). Generally, a fault detection filter is designed in order to minimize the effect of external disturbances and to maximize its sensitivity to faults on the residuals ([8]). In order to achieve these objectives, many design criteria such as  $\mathcal{H}_\infty$  norm,  $\mathcal{H}_2$  norm, and  $\mathcal{H}_-$  index have been proposed to evaluate the effectiveness of fault detection filter design ([9], [10], [11], [12], [14] and [8]). These criteria are used for constructing optimal filters :  $\mathcal{H}_2/\mathcal{H}_\infty$ ,  $\mathcal{H}_\infty/\mathcal{H}_\infty$  and  $\mathcal{H}_-/ \mathcal{H}_\infty$ . The latter criterion is used to design robust fault detection filters for LPV systems (for instance [15] and [16]).  $\mathcal{H}_/\mathcal{H}_\infty$  is also used in fault diagnosis for Takagi-Sugeno nonlinear systems in [17]. The authors are extended the method of fault diagnosis based on  $\mathcal{H}_\infty$  control framework, developed for linear systems in [12] and [13] including a reference model shaping the residual signals in order to improve fault detection, isolation and estimation. Therefore, in this paper, this method will be extended to design residual generators for nonlinear parameter varying-systems with application to diesel engines.

On-board diagnosis of automotive engines has become increasingly important because of environmentally based legislative regulations such as OBDII (On-Board Diagnostics-II)[18]. On-board diagnosis is also needed to guarantee high-performance engine behavior. Today, due to the legislations, the majority of the code in modern engine management system is dedicated to diagnosis. Model-based diagnosis of automotive engines has been considered in earlier papers (see e.g. [19] and [?]), to name only a few. However, the engines investigated in these previous works were all gasoline-fueled and did not include Exhaust Gas Recirculation (EGR) and Variable Nozzle Turbine (VNT). Both these components make the diagnosis problem significantly more difficult since the air flow through the EGR-valve, and also the exhaust side of the engine have to be taken into account. An interesting approach to model-based air-path faults detection for an engine which includes EGR and VNT can be found in [20] and [21]. By using several models in parallel, where each one is sensitive to one kind of fault, predicted outputs are compared and a diagnosis is provided. In particular, the hypothesis test methodology proposed in [20] deals with the multi-fault detection in air-path system. In [21] the authors propose an extended adaptive Kalman filter to find which faulty model best matches with measured data, then a structured

<sup>1</sup> Modeling, Information and Systems (MIS) Laboratory, UFR sciences, University of Picardie Jules Verne, 33 Rue st Leu 80000, Amiens, France, e-mail : boulaïd.boulkroune@u-picardie.fr, opages@u-picardie.fr, ahmed.hajjaji@u-picardie.fr

<sup>2</sup> Hautes Etudes d'Ingénieur, 13 rue de Toul, 59046, Lille, France and Automatic Control Laboratory : LAGIS, UMR CNRS 8219, e-mail : abdel.aitouche@hei.fr

<sup>3</sup> Research Center for Automatic Control of Nancy, CNRS-UMR 7039, University of Lorraine, 54400 Cosnes et Romain, France, e-mail : Ali.Zemouche@iut-longwy.uhp-nancy.fr

hypothesis allows going back to the faults. A structural analysis for air path of an automotive diesel engine has been developed in order to study the monitorability of the system [22]. Other approaches to detect intake leakages in diesel engines based on adaptive observers are proposed in [23] and [24] and recently in [25]. Another approach based on a nonlinear unknown input observer (NIUO) for intake leakage detection is proposed in [26]. For sensor fault diagnosis, few results have been proposed in the literature although the use of healthy measurements is strongly required in such systems. In [27], on-line sensor fault detection, isolation, and accommodation in automotive engines was proposed. In [28], fault detection for diesel engines using signal and process model-based methods was proposed. An expert system for fault diagnosis system in internal combustion engines using wavelet packet transform (WPT) and artificial neural networks (ANNs) techniques has been proposed by [29]. Recently, a sensor fault detection approach for diesel engines is proposed in [30]. This approach is based on nonlinear unknown input observers which are used to estimate jointly the original states and the sensor fault.

This work deals with the problem of fault detection for diesel engines. The proposed approach can be used for detecting, isolating and estimating actuator and sensor faults in nonlinear parameter-varying systems. First, the nonlinear structure of the system is used in the residual generation to keep the system dynamics. Based on the the modified mean value theorem (MMVT) presented in [31], the nonlinear term in the error dynamics is expressed as a convex combination of known matrices with time varying coefficients. Then, the problem of disturbance effect minimization and fault effect maximization on residuals is reduced to a problem of  $\mathcal{L}_2$ -norm minimization. This was possible by building the residual generators as a difference between real residuals, i. e. difference between real output and its estimate, and a reference model virtually fed by the fault signal. The reference model is chosen as first order low-pass filter. Finally, the  $\mathcal{L}_2$ -norm minimization problem is solved by applying the bounded real lemma (BRL). The main advantage of this method is its applicability to a wide large operating points and the residual generator gains are determined off-line which making its real online application very easy.

## II. PRELIMINARIES AND DEFINITIONS

We introduce some notation and definitions that will be used throughout this paper.  $\mathfrak{R}$  denotes the set of real numbers. The set of  $p$  by  $q$  real matrices is denoted as  $\mathfrak{R}^{p \times q}$ .  $A^T$  and  $A^{-1}$  represent the transpose of  $A$  and its left inverse (assuming  $A$  has full column rank), respectively.  $I_r$  represents the identity matrix of dimension  $r$ .  $(\star)$  is used for the blocks induced by

symmetry.  $\|\cdot\|$  represents the usual Euclidean norm.

Let us consider the following class of nonlinear systems

$$\begin{aligned} \dot{x} &= A(\rho_j)x + B_g g(v, y, u) + B_f f(x, u) + B_d d + B_w w & (1a) \\ y &= Cx + D_d d + D_w w & (1b) \end{aligned}$$

with  $A(\rho_j) = \sum_{j=1}^{n_\rho} \rho_j A_j$ , where  $x \in \mathfrak{R}^{n_x}$  is the state vector,  $u \in \mathfrak{R}^{n_u}$  is the control input vector,  $y \in \mathfrak{R}^{n_y}$  is the output vector,  $v \in \mathfrak{R}^{n_v}$  is the vector of measurable signals,  $d \in \mathfrak{R}^{n_d}$  represents the actuator and sensor fault vector and  $w \in \mathfrak{R}^{n_w}$  is the disturbances vector.  $A_j$ , with  $j = 1, \dots, n_\rho$ ,  $B_g$ ,  $B_f$ ,  $B_d$ ,  $B_w$ ,  $D_d$ ,  $D_w$  and  $C$  are constant matrices with appropriate dimensions. All the pairs  $(A_i, C)$ , with  $i = 1, \dots, n_\rho$ , are assumed detectable. Functions  $g(v, y, u)$  and  $f(x, u)$  are nonlinear. Besides, function  $f(x, u)$  is assumed to be once differentiable. The weighting functions  $\rho_j$  are assumed known and depend on measurable variables, and verify

$$\sum_{j=1}^{n_\rho} \rho_j = 1, \rho_j \geq 0, \quad \forall j \in \{1, \dots, n_\rho\} \quad (2)$$

It is important to note that the structure of system (1) is different from Takagi-Sugeno (TS) models. In fact, sometimes it is not important to write nonlinear models in a TS form especially when the number of nonlinearities is large as the case of diesel engine models (see section IV). Consequently, rewrite nonlinear models as in (1) becomes an interesting solution to overcome these difficulties. Besides, most of nonlinear systems can be easily written as in (1) without any transformation.

For manipulating the nonlinear function  $f$ , the MMVT theorem presented for a general vector function  $f$  in [31] is applied in this work. It is used to express the nonlinear error dynamics as a convex combination of known matrices with time varying coefficients. The main principle of MMVT is presented in the following Lemma.

*Lemma 1:* [[31]] Let the canonical basis of the vectorial space  $\mathfrak{R}^s$  for all  $s \geq 1$  be defined by

$$E_s = \{e_s(i) \mid e_s(i) = \underbrace{(0, \dots, 0, \overset{i \text{ th}}{1}, 0, \dots, 0)}_{s \text{ components}}^T, i = 1 \dots s\}$$

Let  $f(x) : \mathfrak{R}^n \rightarrow \mathfrak{R}^n$  be a vector function continuous on  $[a, b] \in \mathfrak{R}^n$  and differentiable on convex hull of the set  $(a, b)$ . For  $s_1, s_2 \in [a, b]$ , there exist  $\delta_{ij}^{\max}$  and  $\delta_{ij}^{\min}$  for  $i = 1 \dots n$  and  $j = 1 \dots n$  such that

$$\begin{aligned} f(s_2) - f(s_1) &= \left[ \left( \sum_{i,j=1}^{n,n} H_{ij}^{\max} \delta_{ij}^{\max} \right) + \left( \sum_{i,j=1}^{n,n} H_{ij}^{\min} \delta_{ij}^{\min} \right) \right] (s_2 - s_1) \\ \delta_{ij}^{\max}, \delta_{ij}^{\min} &\geq 0, \delta_{ij}^{\max} + \delta_{ij}^{\min} = 1 & (3) \\ \bullet \quad h_{ij}^{\max} &\geq \max \left( \frac{\partial f_i}{\partial x_j} \right) \text{ and } h_{ij}^{\min} \leq \min \left( \frac{\partial f_i}{\partial x_j} \right) \\ \bullet \quad H_{ij}^{\max} &= e_n(i) e_n^T(j) h_{ij}^{\max} \text{ and } H_{ij}^{\min} = e_n(i) e_n^T(j) h_{ij}^{\min}. \end{aligned}$$

## III. OBSERVER-BASED RESIDUAL GENERATION DESIGN

The aim of this work is to design a residual generator for nonlinear parameter-varying systems described by (1). Indeed, the system (1) represents a large class of

nonlinear systems. Let's consider the following residual filter :

$$\dot{\hat{x}} = A(\rho)\hat{x} + B_G g(v, y, u) + B_f f(\hat{x}, u) + L(\rho)(y - \hat{y}) \quad (4a)$$

$$\hat{y} = C\hat{x} \quad (4b)$$

$$r = K(y - \hat{y}) \quad (4c)$$

with  $L(\rho) = \sum_{j=1}^{n_p} \rho_j L_j$ .  $\hat{x} \in \mathfrak{R}^{n_x}$  and  $r(t) \in \mathfrak{R}^{n_r}$  are respectively the estimated state vector and the residual vector. The residual generator gains are matrices  $L$  and  $K$ . These matrices should be determined such that the asymptotic stability of residuals is guaranteed. Notice that the index  $\rho$  is omitted where it is not necessary in order to simplify the notations.

First, let define the state estimation error  $e$  as  $e(t) = x(t) - \hat{x}(t)$ . Then, the error dynamics and residual expression can be expressed as:

$$\dot{e} = (A - LC)e + B_f \tilde{f} + (B_d - LD_d)d + (B_w - LD_w)w \quad (5a)$$

$$r = KCe + KD_d d + KD_w w \quad (5b)$$

with  $\tilde{f} = f(\hat{x}, u) - f(x, u)$ .

Now, by using Lemma 1, the nonlinear term  $\tilde{f}(\hat{x}, x)$  can be written as

$$\tilde{f}(\hat{x}, x) = \left[ \left( \sum_{i,j=1}^{n,n} H_{ij}^{\max} \delta_{ij}^{\max} \right) + \left( \sum_{i,j=1}^{n,n} H_{ij}^{\min} \delta_{ij}^{\min} \right) \right] e \quad (6a)$$

$$\delta_{ij}^{\max}, \delta_{ij}^{\min} \geq 0, \delta_{ij}^{\max} + \delta_{ij}^{\min} = 1 \quad (6b)$$

To simplify the form of the final result as in [31], the terms  $\sum_{i,j=1}^{n,n} (\delta_{ij}^{\max} + \delta_{ij}^{\min})$  must be scaled to one. If all the terms in  $\frac{\partial f_i}{\partial x_j}$  are not zero, then the scaling factor  $\beta$  can be computed by  $\sum_{i,j=1}^{n,n} (\delta_{ij}^{\max} + \delta_{ij}^{\min}) = n^2 = \beta$ .

Then, (6a) are rewritten as  $\tilde{f} = \alpha e$ , where  $\alpha = \left[ \left( \sum_{i,j=1}^{n,n} \bar{H}_{ij}^{\max} \bar{\delta}_{ij}^{\max} \right) + \left( \sum_{i,j=1}^{n,n} \bar{H}_{ij}^{\min} \bar{\delta}_{ij}^{\min} \right) \right]$ , with

$$\bar{\delta}_{ij}^{\max}, \bar{\delta}_{ij}^{\min} \geq 0, \bar{\delta}_{ij}^{\max} + \bar{\delta}_{ij}^{\min} = \frac{1}{\beta}, \sum_{i,j=1}^{n,n} (\bar{\delta}_{ij}^{\max} + \bar{\delta}_{ij}^{\min}) = 1$$

where  $\bar{H}_{ij}^{\max} = \beta H_{ij}^{\max}$ ,  $\bar{\delta}_{ij}^{\max} = \delta_{ij}^{\max} / \beta$  and  $\bar{\delta}_{ij}^{\min} = \delta_{ij}^{\min} / \beta$ .

Then, (5) becomes :

$$\dot{e} = A_e e + B_{ed} d + B_{ew} w \quad (7a)$$

$$r = C_e e + D_{rd} d + D_{rw} w \quad (7b)$$

with  $A_e = A - LC + B_f \alpha$ ,  $B_{ed} = B_d - LD_d$ ,  $B_{ew} = B_w - LD_w$ ,  $C_e = KC$ ,  $D_{rd} = KD_d$  and  $D_{rw} = KD_w$ .

As seen in equation (7), the error dynamics and the residual  $r$  depend only on the fault  $d$  and the disturbances  $w$ . Now, the fault detection problem is reduced to minimize the effect of disturbances and to maximize the effect of the fault on the residual simultaneously. Indeed, one solution is to use the well known  $H_- / H_\infty$  approach. Thus, the effect of disturbances on the residuals is minimized using the  $H_\infty$  norm while the effect of the fault on the residual is maximized using the  $H_-$  index. However, using the two performances simultaneously increases the problem complexity. In this work, the  $H / H_\infty$  optimization

problem is reduced to one  $H_\infty$  optimization problem by introducing the following variable  $r_e$  expressed as :

$$r_e = r - W_f d \quad (8)$$

where  $W_f$  is a transfer function representing the desired transfer from the fault  $d$  to residual  $r$  and is called reference model. The idea of a reference model has successfully used in controller design, adaptive control and diagnosis. Therefore, the maximization of the effect of the fault on the residual  $r$  is equivalent to minimization of the effect of the fault on the residual error  $r_e$ .

As explained in [12], structure of  $W_f$  can be chosen depending on the fault diagnosis purpose. Thus, the fault estimation problem can be obtained by choosing  $W_f$  equal to the identity matrix,  $W_f = I$  and the fault detection problem can be obtained when  $W_f$  is a  $1 \times n_d$  matrix. Also,  $W_f$  is not required to be a static matrix. In this work,  $W_f$  is chosen as

$$W_f = \begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix} \quad (9)$$

Now, from (7) and (8), the following augmented system is obtained :

$$\dot{\xi} = A_\xi \xi + B_\xi v \quad (10a)$$

$$r_e = \tilde{C}_\xi \xi + \tilde{D}_\xi v \quad (10b)$$

where  $\xi = \begin{bmatrix} e \\ x_f \end{bmatrix}$ ,  $v = \begin{bmatrix} d \\ w \end{bmatrix}$ ,

$$A_\xi = \begin{bmatrix} A_e & 0 \\ 0 & A_f \end{bmatrix}, B_\xi = \begin{bmatrix} B_{ed} & B_{ew} \\ B_f & 0 \end{bmatrix} \quad (11b)$$

$$\tilde{C}_\xi = [ C_e \quad -C_f ], \tilde{D}_\xi = [ D_{rd} - D_f \quad D_{rw} ] \quad (11c)$$

The problem now is reduced to determine the gains  $L$  and  $K$  that provides convergence of  $\xi$  and minimizes the influence of the fault and disturbances on the residual error ( $r_e$ ) using the standard  $\mathcal{L}_2$ -control problem. In this case,  $r_e$  represents the controlled output. It is worth to note that in the fault estimation problem, the transfer function from the vector  $v$  to the residual  $r_e$  includes a direct term  $D_\xi$  which is not null. Consequently, the lower bound ( $\gamma_{\min}$ ) of  $H_\infty$  norm of the closed loop transfer function from internal input to external output will be greater than or equal to  $\|D_\xi\|_\infty$ . When  $D_\xi = I$ ,  $\gamma_{\min}$  will be greater than or equal to 1 which leads to an over estimation of the fault estimation error. To overcome this difficulty, two solutions are proposed in [12]. The first solution is to filter the estimation error using a low-pass filter by taking :

$$r_{ef} = W_{re} r_e \quad (12)$$

where  $W_{re}$  is a low pass weighting matrix. The second solution is to model the fault signal as the output signal from a low pass dynamic system by taking

$$d = W_d \tilde{d} \quad (13)$$

where  $W_d$  is a diagonal weighting matrix with first-order low pass transfer functions in the diagonal. The first approach is used in this work with a constant matrix  $W_{re}$ .

Based on (13), the augmented system (10) is rewritten as follows

$$\dot{\xi} = A_\xi \xi + B_\xi v \quad (14a)$$

$$r_{ef} = C_\xi \xi + D_\xi v \quad (14b)$$

where

$$C_\xi = W_{re} [ C_e \quad -C_f ], \quad (15a)$$

$$D_\xi = W_{re} [ D_{rd} - D_f \quad D_{rw} ] \quad (15b)$$

The sufficient condition of the existence of the residual generator  $r$  is given in theorem 1.

*Theorem 1:* System (14a)-(14b) is asymptotically stable if there exist a symmetric positive definite matrix  $P > 0$  and a positive scalar  $\gamma > 0$  such that

$$\begin{bmatrix} \Lambda_{ijk}^{\max} & P\bar{B} - R_k\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix} < 0 \quad (16a)$$

$$\begin{bmatrix} \Lambda_{ijk}^{\min} & P\bar{B} - R_k\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix} < 0 \quad (16b)$$

$\forall i = 1, \dots, n, j = 1, \dots, n$  and  $k = 1, \dots, n_\rho$ , with

- $\Lambda_{ijk}^{\max} = (A_k + B_f \bar{H}_{ij}^{\max})^T P + P(A_k + B_f \bar{H}_{ij}^{\max}) - \bar{C}^T R_k^T - R_k \bar{C}$ ;
- $\Lambda_{ijk}^{\min} = (A_k + B_f \bar{H}_{ij}^{\min})^T P + P(A_k + B_f \bar{H}_{ij}^{\min}) - \bar{C}^T R_k^T - R_k \bar{C}$ ;

where  $\bar{H}_{ij}^{\max} = Z_H H_{ij}^{\max}$ ,  $\bar{H}_{ij}^{\min} = Z_H H_{ij}^{\min}$ , with  $Z_H = n \times n$ . Matrices  $\bar{B}$ ,  $\bar{C}$ ,  $\bar{D}$ ,  $\bar{C}_f$ ,  $\bar{D}_f$  and  $\bar{K}$  are given as

$$\bar{B} = \begin{bmatrix} B_d & B_w \\ B_f & 0 \end{bmatrix}, \bar{C} = [ C \quad 0 ], \quad (17a)$$

$$\bar{D} = [ D_f \quad D_w ], \bar{C}_f = W_{re} [ 0 \quad -C_f ], \quad (17b)$$

$$\bar{D}_f = W_{re} [ -D_f \quad 0 ], \bar{K} = W_{re} K \quad (17c)$$

Matrices  $P$ ,  $R_k$  and  $K$  are obtained by solving LMIs (16a)-(16b). The matrices  $L_k$  can be obtained from  $L_k = [ I \quad 0 ] P^{-1} R_k$ .

*Proof 1:* Based on linear bounded lemma [32], the transfer from  $\xi$  to  $r_e$  is minimized using  $H_\infty$  criterion if a matrix  $P$  and a positive constant  $\gamma$  exist such that the following condition holds:

$$\Xi = \begin{bmatrix} A_\xi^T P + P A_\xi & P B_\xi & C_\xi^T \\ (*) & -\gamma I & D_\xi \\ (*) & (*) & -\gamma I \end{bmatrix} < 0 \quad (18)$$

where  $P$  is positive symmetric matrix.

From (11b), (15a) and (15b), (18) can be expressed as

$$\Xi = \begin{bmatrix} \bar{A}^T P + P \bar{A} - \bar{C}^T R^T - R \bar{C} & P\bar{B} - R\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix}$$

where  $\bar{A} = A + B_f \alpha$ ,  $R = P \begin{bmatrix} L \\ 0 \end{bmatrix}$ . Matrices  $\bar{B}$ ,  $\bar{C}$ ,  $\bar{D}$ ,  $\bar{C}_f$ ,  $\bar{D}_f$  and  $\bar{K}$  are given in (17).

By substituting  $\alpha$  by its expression, we obtain :

$$\begin{aligned} \Xi = & \sum_{i,j=1}^{n,n} \delta_{ij}^{\max} \underbrace{\begin{bmatrix} \Lambda_{ij}^{\max} & P\bar{B} - R\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix}}_{M_{ij}^{\max}} \\ & + \sum_{i,j=1}^{n,n} \delta_{ij}^{\min} \underbrace{\begin{bmatrix} \Lambda_{ij}^{\min} & P\bar{B} - R\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix}}_{M_{ij}^{\min}} \end{aligned}$$

with  $\Lambda_{ij}^{\max} = (A + B_f \bar{H}_{ij}^{\max})^T P + P(A + B_f \bar{H}_{ij}^{\max}) - \bar{C}^T R^T - R \bar{C}$ ,  $\Lambda_{ij}^{\min} = (A + B_f \bar{H}_{ij}^{\min})^T P + P(A + B_f \bar{H}_{ij}^{\min}) - \bar{C}^T R^T - R \bar{C}$ . Since we have  $\delta_{ij}^{\max}, \delta_{ij}^{\min} \geq 0$ , the negativity of  $\Xi$  is ensured if

$$M_{ij}^{\max} < 0 \quad \text{and} \quad M_{ij}^{\min} < 0 \quad (19)$$

By substituting  $A(\rho)$  and  $L(\rho)$  by their expressions, (19) can be written as follows:

$$\sum_{k=1}^{n_\rho} \rho_k M_{ijk}^{\max} < 0, \quad \sum_{k=1}^{n_\rho} \rho_k M_{ijk}^{\min} < 0 \quad \text{for all } k = 1, \dots, n_\rho \quad (20)$$

where

$$M_{ijk}^{\max} = \begin{bmatrix} \Lambda_{ijk}^{\max} & P\bar{B} - R_k\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix}$$

$$M_{ijk}^{\min} = \begin{bmatrix} \Lambda_{ijk}^{\min} & P\bar{B} - R_k\bar{D} & (\bar{K}\bar{C} + \bar{C}_f)^T \\ (*) & -\gamma I & (\bar{K}\bar{D} + \bar{D}_f) \\ (*) & (*) & -\gamma I \end{bmatrix}$$

with

- $\Lambda_{ijk}^{\max} = (A_k + B_f \bar{H}_{ij}^{\max})^T P + P(A_k + B_f \bar{H}_{ij}^{\max}) - \bar{C}^T R_k^T - R_k \bar{C}$ ;
- $\Lambda_{ijk}^{\min} = (A_k + B_f \bar{H}_{ij}^{\min})^T P + P(A_k + B_f \bar{H}_{ij}^{\min}) - \bar{C}^T R_k^T - R_k \bar{C}$ ;

Since  $\rho_k > 0$ ,  $\forall k = 1, \dots, n_\rho$ , the condition (20) holds if  $M_{ijk}^{\max} < 0$  and  $M_{ijk}^{\min} < 0, \forall i = 1, \dots, n, j = 1, \dots, n$  and  $k = 1, \dots, n_\rho$ . This ends the proof.

#### IV. ENGINE MODEL

The diesel engine considered in this paper is a four-cylinder engine with a EGR and a VNT. Due to space limitation, we present directly the air-path model writing in state space form as (1). For more details we refer the reader to [30]. The engine is equipped with sensors measuring in-flowing air  $W_{HFM}$ , the temperature after CAC  $T_{CAC}$ , inlet-manifold pressure  $P_{Inlet}$ , exhaust-pressure  $P_{Exh}$  and exhaust-temperature  $T_{Exh}$ . The control inputs are the injected fuel  $W_{Fuel}$ , the turbine vane position  $X_{VNT}$ , and the EGR-valve position determining the valve opening-area  $A_{EGR}$ . Notice that the following variables :  $T_{CAC}$ ,  $W_{HFM}$ , and  $N_{Eng}$  are considered as measurable signals.

The considered air path model is expressed as follows (1), where the state, known input and output vectors and the variables  $\rho$  and  $v$  are defined as

$$x = [ P_{Inlet} \quad m_{Air} \quad m_{EGR} \quad m_{Exh} ]^T \quad (21)$$

$$u = [ A_{EGR} \quad X_{VNT} \quad W_{Fuel} ]^T \quad (22)$$

$$y = [ P_{Inlet} \quad P_{Exh} ]^T \quad (23)$$

$$\rho = N_{Eng} \quad (24)$$

$$v = [ T_{CAC} \quad W_{HFM} ]^T \quad (25)$$

where  $m_{Air}$  and  $m_{EGR}$  are respectively the mass of air and EGR-gaz in intake manifold.  $m_{Exh}$  represents the mass of exhaust gas in exhaust manifold. The matrices  $A_1$  and  $A_2$  are given by :

$$A_1 = \begin{bmatrix} -a_1 & 0 & 0 & 0 \\ 0 & -a_1 & 0 & 0 \\ 0 & 0 & -a_1 & 0 \\ 0 & a_1 & a_1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -a_2 & 0 & 0 & 0 \\ 0 & -a_2 & 0 & 0 \\ 0 & 0 & -a_2 & 0 \\ 0 & a_2 & a_2 & 0 \end{bmatrix}$$

with  $a_1 = \frac{f_{vol} \underline{N}_{Eng} V_{Eng}}{120V_{Inlet}}$  and  $a_2 = \frac{f_{vol} \overline{N}_{Eng} V_{Eng}}{120V_{Inlet}}$ , where  $\underline{N}_{Eng}$  and  $\overline{N}_{Eng}$  are, respectively, the minimum and maximum value of the measurable variable  $N_{Eng}$ .  $\rho_1$  and  $\rho_2$  are defined as

$$\rho_1 = \frac{\overline{N}_{Eng} - N_{Eng}}{\overline{N}_{Eng} - \underline{N}_{Eng}} \quad \text{and} \quad \rho_2 = \frac{N_{Eng} - \underline{N}_{Eng}}{\overline{N}_{Eng} - \underline{N}_{Eng}} \quad (26)$$

Matrices  $B_g$ ,  $B_f$ ,  $B_d$ ,  $B_w$ ,  $D_w$ ,  $C$  and  $D_d$  are expressed as :

$$B_g = \begin{bmatrix} \frac{R_{Air} c_{p,Air}}{c_{v,Air} V_{Inlet}} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_f = \begin{bmatrix} -\frac{f_{vol} V_{Eng} N_{Eng}}{120V_{Inlet}} & \frac{R_{Exh}^2 T_{EGRCp,Exh}}{V_{Inlet} c_{v,Exh} V_{Exh}} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{R_{Exh}}{V_{Exh}} & 0 \\ 0 & -\frac{R_{Exh}}{V_{Exh}} & 1 \end{bmatrix}, \quad (27)$$

$$B_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 13 \times 10^{-4} \end{bmatrix}, B_w = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, D_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (28)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{R_{Exh} T_{Exh}}{V_{Exh}} \end{bmatrix}, D_d = \begin{bmatrix} 2 \times 10^4 & 0 & 0 \\ 0 & 2.5 \times 10^4 & 0 \end{bmatrix} \quad (29)$$

Functions  $g$  and  $f$  are given by :

$$g = \begin{bmatrix} W_{HFM} T_{CAC} \\ W_{HFM} \end{bmatrix}, f = \begin{bmatrix} \frac{R_{Inlet} P_{Inlet}}{c_{v,Inlet}} \\ \frac{A_{EGRM_{Exh}} \Psi_{K_{Exh}} T_{Exh}}{\sqrt{R_{Exh} T_{Exh}}} \\ W_{Fuel} - \frac{R_{Exh} T_{Exh} m_{Exh} \tau \left( \frac{P_{Exh}}{P_{Atm}}, X_{VNT} \right)}{V_{Exh} \sqrt{T_{Exh}}} \end{bmatrix}$$

All the parameters and variables used in this model are listed in Table in appendix ???. The temperature  $T_{EGR}$  and  $T_{Exh}$  are assumed to be constant. The static functions :  $f_{vol}$ ,  $h$  and  $\tau$  are represented as interpolation in lookup tables.

## V. SIMULATION RESULTS

The proposed approach has been tested on a four-cylinder diesel engine model running on Mathworks Matlab®/Simulink. The aim is to detect two types of faults : actuator and sensor faults. The two first components of the vector  $d$  are, respectively, the inlet and exhaust manifold sensor faults. Two step-like faults with a magnitude equal to 10% of the mean values of inlet and exhaust pressures are considered. The last component on the fault vector is an actuator fault represented by a step-like fault with a magnitude equal to 40% of the mass-flow of the injected fuel. This fault is due either to a drop in the pressure in the common rail or a dysfunction of one or more injectors.

Due to space limitations, the numerical values of vectors and matrices, in Theorem 1, are omitted. Notice that the LMIs (16a)-(16b) are solved using YALMIP, a toolbox for modeling and optimization in Matlab®. In this work, a dedicated residual for each fault is constructed. Besides, this structure helps us to isolate easily each fault. In addition, in each case  $W_f$  is chosen as a stable first order low-pass filter. By solving the LMIs (16a)-(16b), the following optimal values of  $\gamma$  are obtained:  $\gamma_1^* = 0.8665$ ,  $\gamma_2^* = 0.9083$  and  $\gamma_2^* = 0.7141$ .

The simulation is performed with engine average speed  $N_{Eng} = 3000\text{rpm}$  (see Fig. 1). In this experience, the minimum and maximum value of  $N_{Eng}$  ( $\underline{N}_{eng}$  and  $\overline{N}_{eng}$ ) are chosen as : 2000 and 4000rpm respectively. For simulation purpose, we consider that the measurements are perturbed by a gaussian white noise with a standard deviation equal to  $5 \times 10^3$ .

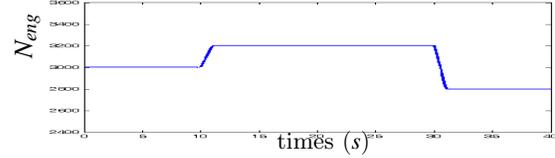


Fig. 1. Engine speed behavior

The obtained residuals are illustrated in figure 5. In this figure, each fault effect is well shown in the equivalent residual. Besides, all residuals are not affected by regime change.

## VI. CONCLUSION

The design of residual generators for nonlinear parameter-varying systems is treated in this paper. The proposed approach aims to detect actuator and sensor faults in diesel engines. The disturbance effect minimization and fault effect maximization problem is transformed to a problem of  $\mathcal{L}_2$ -norm minimization. The residual generator gains are obtained by solving a set of LMIs. The efficiency of the proposed method is shown through the application to a diesel engine model where the effect of each fault is well shown on the equivalent residual.

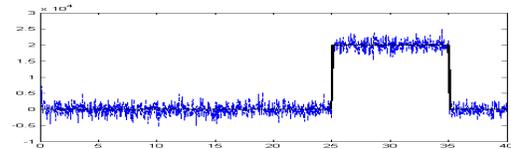


Fig. 2. Inlet manifold pressure sensor fault

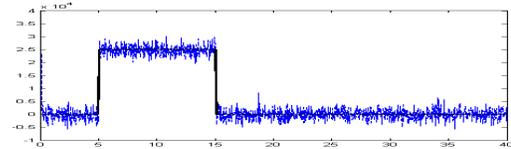


Fig. 3. Exhaust manifold pressure sensor fault

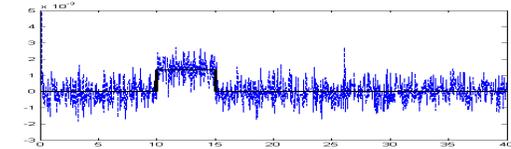


Fig. 4. Actuator fault

Fig. 5. Real fault (continuous line) and its equivalent residual (dashed line)

## ACKNOWLEDGMENTS

This work was produced in the framework of CEREEV (Combustion Engine for Range-Extended Electric Vehicle), a European territorial cooperation project part-funded by the European Regional Development Fund (ERDF) through the INTERREG IVA France (Channel)-England program, and the research department of the Picardie, France.

## REFERENCES

- [1] R.J. Patton and J. Chen. Observer-based fault detection and isolation: Robustness and applications. *Control Eng. Practice*, 5(5):671–682, 1997.
- [2] X. Ding and L. Guo. On observer based fault detection In *Proc. SAFEPROCESS'97*, volume 1, pages 112–120, 1997.
- [3] A. Saberi, A.A. Stoorvogel, Sannuti P., and H. Niemann. Fundamental problems in fault detection and identification. *Int. J. Robust Nonlinear Control*, 10:1209–1236, 2000.
- [4] C. De Persis and A. Isidori. A geometric approach to nonlinear fault detection and isolation. *IEEE Trans. on Automatic Control*, 46(6):853–865, 2001.
- [5] X.D. Zhang, M.M. Polycarpou, and T. Parisini. A robust detection and isolation scheme for abrupt and incipient faults in nonlinear systems. *IEEE Trans. Automat. Contr.*, 47:576–593, 2002.
- [6] Y. Chen, Z. Weng, and S. Shi. Robust fault diagnosis for nonlinear difference-algebraic systems. *Int. J. of Control*, 76(15):1560–1569, 2003.
- [7] A.M. Pertew, H.J. Marquez, and Q. Zhao. Lmi based sensor fault diagnosis for nonlinear lipschitz systems. *Automatica*, 43:1464–1469, 2007.
- [8] X. Li and K. Zhou. A time domain approach to robust fault detection of linear time-varying systems. *Automatica*, 45(1):94–102, 2009.
- [9] S. Grenaille, D. Henry, and A. Zolghadri. In *Fault diagnosis in linear parameter varying systems*, Toulouse, France, 2006.
- [10] S.X. Ding, T. Jeinsch, P.M. Frank, and E.L. Ding. A unified approach to the optimization of fault detection systems. *International Journal of Adaptive Control and Signal Processing*, 14(7):725–745, 2000.
- [11] R.J. Hou, and M. Patton. An LMI approach to  $\mathcal{H}_2/\mathcal{H}_\infty$  fault detection observers In *Control '96, UKACC International Conference*, pages 305–310, 1996.
- [12] J. Stoustrup and H. Niemann. Application of an H-infinity based FDI and control scheme for the three tank system. In *SAFEPROCESS'2000, Budapest, Hungary*, 2000.
- [13] J. Stoustrup and H. Niemann. Fault estimation—a standard problem approach. In *International Journal of Robust and Nonlinear Control*, 12(8), pages 649–673, 2002.
- [14] I.M. Jaimoukha, L. Zhenhai, and V. Papakos. A matrix factorization solution to the  $\mathcal{H}_2/\mathcal{H}_\infty$  fault detection problem. *International Journal of Adaptive Control and Signal Processing*, 42(11):1907–1912, 2006.
- [15] X. Wei and M. Verhaegen. Mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  Fault Detection Observer Design for LPV Systems. In *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 1073–1078, 2008.
- [16] B. Boulkroune, S. Halabi, and A. Zemouche.  $\mathcal{H}_2/\mathcal{H}_\infty$  fault detection filter for a class of nonlinear descriptor systems. *International Journal of Control*, 86(2):253–262, 2013.
- [17] D. Ichalal, B. Marx, J. Ragotb, and D. Maquinb. Fault detection, isolation and estimation for Takagi-Sugeno nonlinear systems. *Journal of the Franklin Institute*, In press, 2013.
- [18] CARB. California's obd-ii regulation (section 1968.1 title 13, california code of regulations), resolution 93-40, july 9. pages 220.7-220.12(h). 1993.
- [19] J. Gertler, M. Costin, X. Fang, and R. Hira. Mode based diagnosis for automotive engines - algorithm development and testing on a production vehicle. *IEEE Transactions on Control Systems Technology*, 3(1):61–69, 1995.
- [20] M. Nyberg. Model-based diagnosis of an automotive engine using several types of fault models. *IEEE Transactions on control systems technology*, 10(5), pp. 679–689, 2002.

- [21] M. Nyberg and T. Sutte. Model based diagnosis of the air path of an automotive diesel engine. *Control Engineering Practice* 12., pages 513–525, 2004.
- [22] I. Djemili, A. Aitouche, and V. Cocquempot. Structural analysis for air path of an automotive diesel engine. In *IEEE-International Conference on Communications, Computing and Control Applications (CCCA'11)*, Hammamet, Tunisia, 2011.
- [23] R. Ceccarelli, C. Canudas-de Wit, P. Moulin, and A. Sciarretta. Model-based adaptive observers for intake leakage detection in diesel engines. In *IEEE American Control Conference (ACC 09)*, St. Louis, Missouri, USA, 2009.
- [24] R. Ceccarelli, P. Moulin, and C. Canudas-de Wit. Robust strategy for intake leakage detection in diesel engines. In *IEEE Application Control Conference, MSC (2009)*, Saint Petersburg, Russia, 2009.
- [25] I. Djemili, A. Aitouche, and V. Cocquempot. Adaptive observer for intake leakage detection in diesel engines described by takagi-sugeno model. In *19th Mediterranean Conference on Control & Automation (MED 2011)*, Corfu, Greece, 2011.
- [26] B. Boulkroune, I. Djemili, A. Aitouche, and V. Cocquempot. Nonlinear unknown input observer design for diesel engines. In *IEEE American Control Conference*, Washington, DC, 2013.
- [27] D. Capriglione, C. Liguori, C. Pianese, and A. Pietrosanto. On-line sensor fault detection, isolation, and accommodation in automotive engines. *IEEE Transactions on Instrumentation and Measurement*, 52(4):1182–1189, 2004.
- [28] F. Kimmich, A. Schwarte, and R. Isermann. Fault detection for modern diesel engines using signal- and process modelbased methods. *Control Engineering Practice*, 13:189–203, 2005.
- [29] J.D. Wu and C.H. Liu. An expert system for fault diagnosis in internal combustion engines using wavelet packet transform and neural network. *Expert Systems with Applications*, 36:4278–4286, 2009.
- [30] B. Boulkroune, A. Aitouche, and V. Cocquempot. Sensor fault detection for diesel engines. In *Proceedings of CoDIT'13*, Hammamet, Tunisie, 2013.
- [31] G. Phanomchoeng, R. Rajamani, and D. Piyabongkarn. Non-linear observer for bounded jacobian systems, with applications to automotive slip angle estimation. *IEEE Trans. Automat. Contr.*, 56(5):1163–1170, 2011.
- [32] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. Linear matrix inequalities in system and control theory. *SIAM Studies in Applied Mathematics*.

## APPENDIX NOMENCLATURE

Symb.	Quantity	Value/Unit
$P_{im}$	Pressure in intake manifold	$Pa$
$P_{em}$	Pressure in exhaust manifold	$Pa$
$p_a$	Atmospheric pressure	$1.013^5 Pa$
$R_a$	Gas constant of air	$287 J / (Kg.K)$
$c_p$	Specific heat at const. pres.	$1014.4 J / (Kg.K)$
$c_v$	Specific heat at const. vol.	$727.4 J / (Kg.K)$
$\kappa$	Ratio of specific heats	$c_p / c_v$
$\tau$	Time constant	0.11s
$W_c$	Compressor mass flow rate	$kg.s^{-1}$
$W_{egr}$	EGR mass-flow into int. mani.	$kg.s^{-1}$
$W_{ei}$	Total mass flow rate into cylinders	$kg.s^{-1}$
$W_t$	Turbine mass flow rate	$kg.s^{-1}$
$W_f$	Fuel mass flow rate	$kg.s^{-1}$
$T_{im}$	Intake manifold temperature	313K
$T_{em}$	Exhaust manifold temperature	509K
$T_a$	Ambient temperature	298K
$N$	Engine speed	$tr/min$
$V_d$	Displacement volume	$0.002 m^3$
$n_{cyl}$	Number of cylinders	4
$V_{im}$	Intake manifold volume	$0.006 m^3$
$V_{em}$	Exhaust manifold volume	$0.001 m^3$
$\eta_{vol}$	Volumetric efficiency	0.87
$\eta_t$	Turbine efficiency	0.76
$\eta_c$	Compressor efficiency	0.61