

Self-organization into quantized eigenstates of a classical wave driven particle

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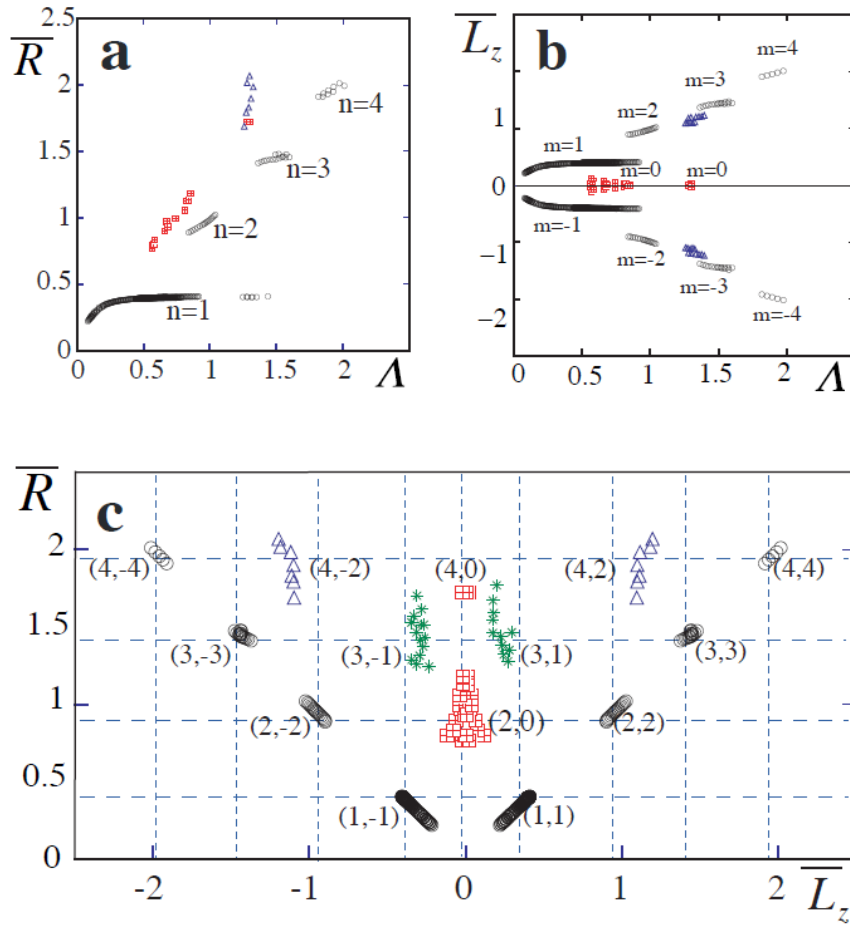
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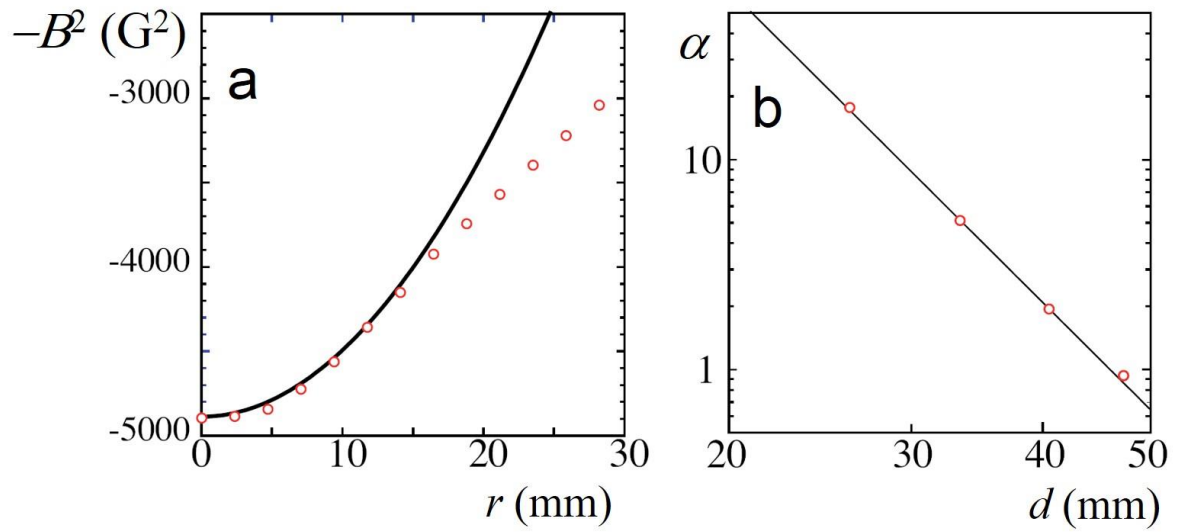
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Supplementary materials

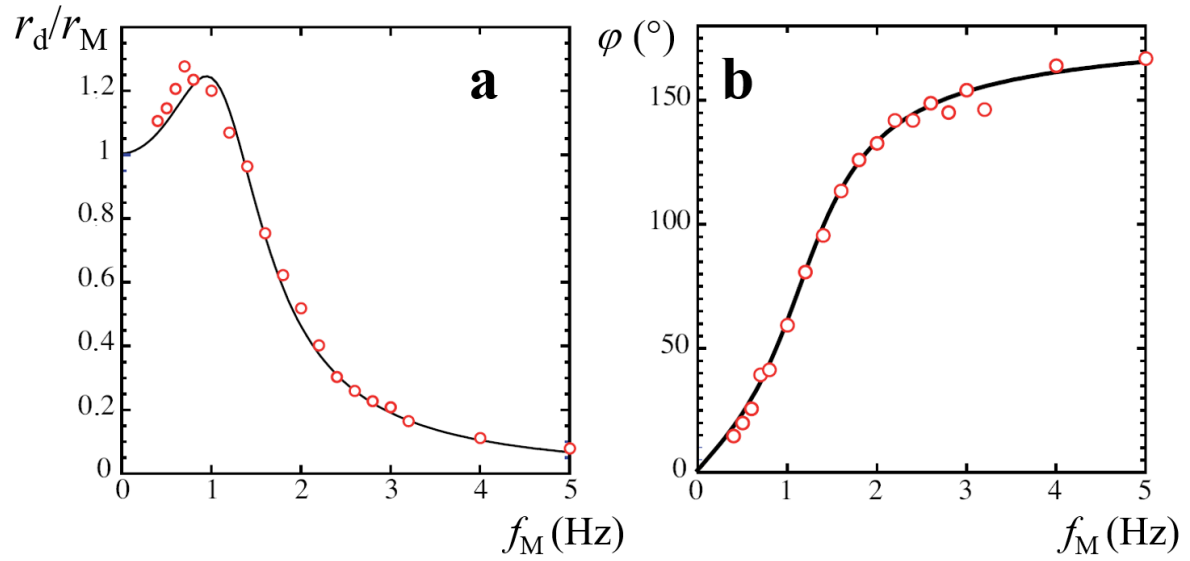
Supplementary Figures:



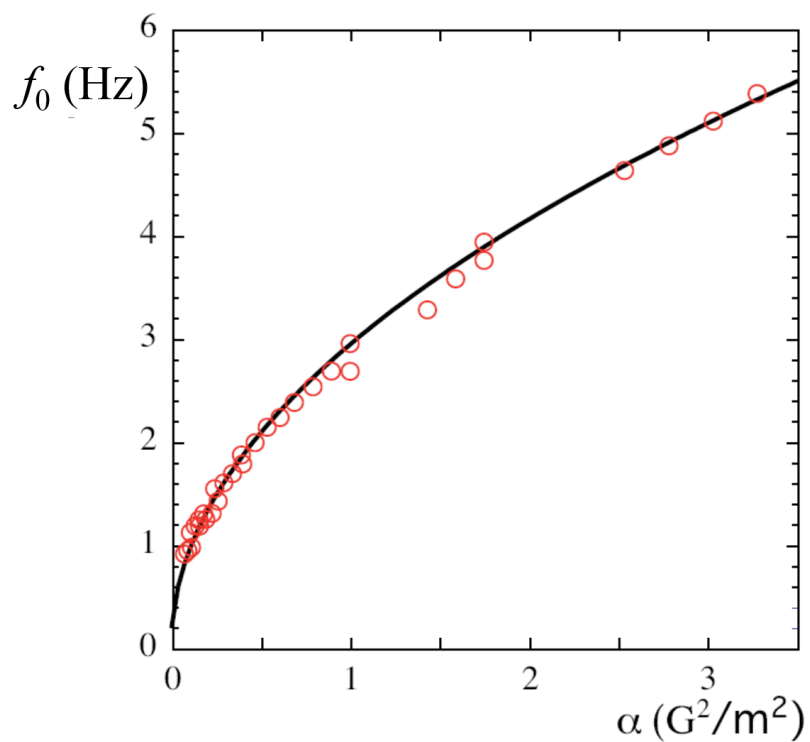
Supplementary Figure 1. The double quantization as obtained with the numerical simulations using the path-memory model. **a**, evolution of the mean spatial extent \overline{R} as a function of the control parameter Λ . The black circles correspond to the circular orbits, the red squares represent the lemniscates and the blue triangles represent the trefoils. **b**, evolution of the mean angular momentum \overline{L}_z as a function of Λ for the same set of trajectories. **c**, the same set of trajectories is now characterized by plotting their angular momentum \overline{L}_z as a function of their mean spatial extent \overline{R} . The symbols are the same as those in **a**. The trajectories are labelled by their position (n, m) on the lattice. The type of orbits corresponding to the $(n=3, m=1)$ mode is not observed as a stable mode. A good candidate (green stars) for this mode shows up in the decomposition of complex modes and takes the form of small loops of radius $0.37 \lambda_F$ distributed at a distance $1.4 \lambda_F$ from the centre. The simulations show a similar double quantization as the experiments (see Fig. 4).



Supplementary Figure 2. Magnetic trap characterization. **a**, plot of $-B^2$ as a function of r for a distance of the magnet $d=40$ mm (red circles) and parabolic fit (black line). **b**, curvature α at the vertex of the parabola fit of $-B^2$ as a function of the distance d of the magnet (red circles). The continuous function (black line) is obtained by the whole fitted magnetic field of the magnet.



Supplementary Figure 3. Frequency response of the bouncing droplet submitted to a forced oscillation. The magnet located at a distance $d=30$ mm oscillates horizontally with an amplitude of 10 mm. **a**, frequency response of the normalized amplitude of the drop motion to that of the magnet. **b**, frequency response of the relative phase shift between the droplet and the magnet motion. The two continuous curves are fits by the linear response of forced oscillator with damping. We find a quality factor of 1.3 and a resonant frequency of 1.1 Hz.



Supplementary Figure 4. Resonant frequency of the droplet as a function of the curvature α of the potential well. This latter parameter has been computed from the measurement of the magnetic field (see Supplementary Fig. 2b). The experimental data (red circles) are fitted accurately with the expected square root dependence (black line).