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An indirect closed loop subspace identification method

Mathieu Pouliquen, Olivier Gehan, Eric Pigeon
Control Group, GREYC CNRS UMR 6072
ENSICAEN, 06 Bd du Marechal Juin
14050 Caen Cedex, France
mathieu.pouliquen@greyc.ensicaen.fr

Abstract—In this paper, we consider a closed-loop subspace identification problem. An indirect method is developed using exogenous input and knowledge of a part of the controller impulse response. The idea is to extract dynamic of the plant from dynamic of the closed loop system. Two main result allows this double estimation. Only the deterministic behavior of the plant is considered in this paper. A simulation example is given to illustrate the performances of the present method.

I. INTRODUCTION

The identification of closed loop systems has received much interest for the last decades ([33], [34] [26], [10], [8], [11]) because for many industrial processes open loop experiments are prohibited (safety, stability, efficiency of operation, etc.). Due to the feedback control, a main difficulty in the identification of system operating in closed loop is the existence of a correlation between plant inputs and disturbances, thus different specific methods have been developed.

Recently there has been also an increasing interest in subspace identification algorithm as pointed out by the available contributions ([35], [9], [18], [31], [32], [6]). These methods have been mainly motivated by a set of interesting properties: the simplicity, the intrinsic numerical robustness and their straightforward application to multivariable systems. However, it is a fact that classical subspace methods fail when data are collected in closed loop experiments: these methods require the noise sequence to be orthogonal to the input sequence which is not the case with a feedback.

To overcome this difficulty, some particular subspace methods devoted to closed loop identification have been proposed. These methods are divided into three groups namely the direct approaches, the indirect approaches and the joint input-output approaches:

- In the direct approaches the identification is performed as in an usual open loop context up to a suitable data processing ([25], [7], [30], [15], [23], [24], [16], [2], [3], [13], [14], [4], [39], [12]);
- The indirect approaches are mainly based on an open loop identification of the control system sensitivity function using the system output and an external excitation input ([36]-[37], [27], [29], [28]);
- The joint input-output approaches use the system input-output behavior together with an external excitation input ([38], [17], [19], [21], [22]).

Some very attractive direct approaches have been analyzed in [2], [3], [4], [5] and [6]. These methods "may be regarded (...) as a significant step towards a satisfactory subspace identification algorithm working with closed-loop data" ([4]). Analyzing the behavior of the closed loop system our indirect approach is very different: a characteristic of the closed loop system is first obtained using projections of subspace, as is the case in lot of subspace identification algorithms. Then model of the plant is extracted making use the knowledge of the controller dynamic. Notice that the deterministic part of the plant only is considered here.

The paper is organized as follows. In section II, the closed loop identification problem considered in the paper is formulated. Several notations and assumptions used throughout the paper are also given. Section III is the highlight of the paper: subsection III-A is devoted to two main results and the proposed indirect subspace identification method is explained in Subsection III-B. In Section IV, a numerical simulation study is reported and finally, some concluding remarks are presented in Section V.

II. PROBLEM FORMULATION AND NOTATION

A. Identification context

Consider the identification of the closed loop system shown in Fig. 1 where $u(t) \in \mathbb{R}^{n_u}$ and $y(t) \in \mathbb{R}^{n_y}$ are respectively the input and the output of the plant which is expressed as:

$$y(t) = G(q)u(t) + H(q)e(t) \quad (1)$$

$G(q)$ is a proper transfer matrix of the deterministic part of the plant, $H(q)$ is an inversely stable square transfer matrix and $e(t) \in \mathbb{R}^{n_y}$ a white noise representing the innovation. A minimal state space model for (1) can be written in an innovation form as

$$\begin{cases} x(t+1) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{cases} \quad (2)$$

with $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_u}$, $C \in \mathbb{R}^{n_y \times n}$, $D \in \mathbb{R}^{n_y \times n_u}$ and $K \in \mathbb{R}^{n \times n_y}$. (A, C) is assumed to be observable and (A, B) is assumed to be controllable. Note that A should not be stable, which it indeed often is not in a closed loop context.

The control signal $u(t)$ is generated by $u(t) = r(t) - C(q)y(t)$. The controller state space equations are defined as

$$\begin{cases} x_c(t+1) &= A_c x_c(t) - B_c y(t) \\ u(t) &= C_c x_c(t) - D_c y(t) + r(t) \end{cases} \quad (3)$$

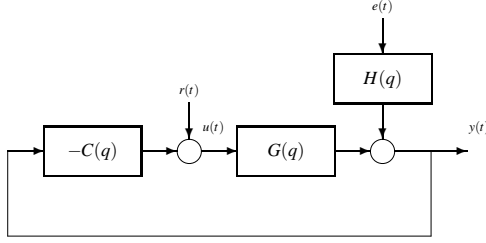


Fig. 1. Closed loop system

where $A_c \in \mathbb{R}^{n_c \times n_c}$, $B_c \in \mathbb{R}^{n_c \times n_y}$, $C_c \in \mathbb{R}^{n_u \times n_c}$ and $D_c \in \mathbb{R}^{n_u \times n_y}$. The controller should not be stable. Here $r(t) \in \mathbb{R}^{n_u}$ is an exogenous input for identification.

The identification problem treated in this paper is stated as: find the order n of the system and a realization of state space matrices (A, B, C, D) given input, output and external excitation measurements. The knowledge of the first Markov parameters of the controller is also required but they can easily be estimated (see end of subsection III-A).

We make some usual assumptions throughout the paper:

- The closed loop system is internally stable.
- $r(t)$ and $e(t)$ are wide sense stationary zero mean processes and are second order ergodic.
- $r(t)$ satisfies persistent excitation conditions.
- $r(t)$ and $e(t)$ are mutually uncorrelated.
- The control system of Fig. 1 is assumed to be well posed. The well posed assumption of the control system ensures that $(I_{n_u} + D_c D)$ and $(I_{n_y} + D D_c)$ are non singular.

B. Notations

Standard subspace algorithms split the available data into block matrices: the input Hankel matrices U_p and U_f are defined as

$$\begin{pmatrix} U_p \\ U_f \end{pmatrix} = \begin{pmatrix} u(t-p) & u(t-p+1) & \dots & u(t-p+j-1) \\ u(t-p+1) & u(t-p+2) & \dots & u(t-p+j) \\ \dots & \dots & \dots & \dots \\ u(t-1) & u(t) & \dots & u(t+j-2) \\ u(t) & u(t+1) & \dots & u(t+j-1) \\ u(t+1) & u(t+2) & \dots & u(t+j) \\ \dots & \dots & \dots & \dots \\ u(t+f-1) & u(t+f) & \dots & u(t+f+j-2) \end{pmatrix}$$

Similar definitions hold for R_f , R_p , Y_f , Y_p , E_f and E_p . Subscripts f and p respectively stand for "future" and "past" and are user-defined indexes such that $f, p > n + n_c$. j is the number of columns in the Hankel matrices. In subspace identification algorithm, it is assumed that there are long time series available i.e. $j \rightarrow \infty$. Due to the assumed ergodicity, the expected operator $\mathcal{E}\{\cdot\}$ will be replaced by the operator $\mathcal{E}_j\{\cdot\}$:

$$\mathcal{E}_j\{\cdot\} = \lim_{j \rightarrow \infty} \frac{1}{j} \{\cdot\}$$

(2) and (3) can be represented by the following basic subspace equations:

$$\begin{cases} X_t &= A^p X_{t-p} + \Delta_p^d U_p + \Delta_p^s E_p \\ Y_f &= \Gamma_f X_t + H_f^d U_f + H_f^s E_f \end{cases} \quad (4)$$

$$\begin{cases} X_t^c &= A_c^p X_{t-p}^c - \Delta_p^c Y_p \\ U_f &= \Gamma_f^c X_t^c - H_f^c Y_f + R_f \end{cases} \quad (5)$$

where X_{t-p} and X_t are respectively the past and future state matrices of the plant:

$$X_{t-p} = \begin{pmatrix} x(t-p) & x(t-p+1) & \dots & x(t-p+j-1) \end{pmatrix}$$

$$X_t = \begin{pmatrix} x(t) & x(t+1) & \dots & x(t+j-1) \end{pmatrix}$$

and X_{t-p}^c and X_t^c are respectively the past and future state matrices of the controller.

$$\Gamma_f = \begin{pmatrix} C \\ CA \\ \dots \\ CA^{f-1} \end{pmatrix} \quad \text{and} \quad \Delta_p^d = \begin{pmatrix} A^{p-1}B & \dots & AB & B \end{pmatrix}$$

are respectively the extended observability matrix and the reversed extended controllability matrix of the deterministic part of the plant. H_f^d is defined as

$$H_f^d = \begin{pmatrix} D & 0 & \dots & 0 \\ CB & D & 0 & \dots \\ \dots & \dots & \dots & \dots \\ CA^{f-2}B & \dots & CB & D \end{pmatrix}$$

and it corresponds to a lower triangular Toeplitz matrix containing the first f Markov parameters of the deterministic part of the plant. Similar notations hold for the stochastic part i.e. Δ_p^s and H_f^s and for the controller i.e. Γ_f^c , Δ_p^c and H_f^c .

Combinaisons of (2) and (3) yields

$$\begin{cases} x^{cl}(t+1) &= A_{cl} x^{cl}(t) + B_{cl/r} r(t) + B_{cl/e} e(t) \\ y(t) &= C_{cl} x^{cl}(t) + D_{cl/r} r(t) + D_{cl/e} e(t) \end{cases}$$

with

$$x^{cl}(t) = \begin{pmatrix} x(t) \\ x_c(t) \end{pmatrix}$$

A_{cl} , C_{cl} , $B_{cl/r}$, $D_{cl/r}$, $B_{cl/e}$ and $D_{cl/e}$ are determined by the state space matrices (A, B, C, D, K) and (A_c, B_c, C_c, D_c) . The order of this closed loop state space model is $n_{cl} = n + n_c$. This order should not be the minimal order n_{cl}^{min} i.e. $n_{cl}^{min} \leq n_{cl}$.

Similarly, a combinaison of (4) and (5) leads to an extended state space model of the closed loop:

$$\begin{cases} X_t^{cl} &= A_{cl}^p X_{t-p}^{cl} + \Delta_p^{cl/r} R_p + \Delta_p^{cl/e} E_p \\ Y_f &= \Gamma_f^{cl} X_t^{cl} + H_f^{cl/r} R_f + H_f^{cl/e} E_f \end{cases} \quad (6)$$

with

$$X_t^{cl} = \begin{pmatrix} X_t \\ X_t^c \end{pmatrix} \quad \Gamma_f^{cl} = \begin{pmatrix} T_f \Gamma_f & T_f H_f^d \Gamma_f^c \end{pmatrix}$$

$$\Delta_p^{cl/r} = \begin{pmatrix} \Delta_p^d (I_{p n_u} - H_p^c T_p H_p^d) \\ -\Delta_p^c T_p H_p^d \end{pmatrix} \quad H_f^{cl/r} = T_f H_f^d$$

$$\Delta_p^{cl/e} = \begin{pmatrix} -\Delta_p^d H_p^c T_p H_p^s + \Delta_p^s \\ -\Delta_p^c T_p H_p^s \end{pmatrix} \quad H_f^{cl/e} = T_f H_f^s$$

where $T_f = (I_{f n_y} + H_f^d H_f^c)^{-1}$. Notice that the well posed assumption of the closed loop ensures $(I_{f n_y} + H_f^d H_f^c)$ to be invertible. A_{cl}^p can easily be expressed with matrices A^p , A_c^p , H , Δ and Γ .

To end this section let define the following projection operators as in [35]:

- The orthogonal projection A/B of the row space of the matrix A on the row space of the matrix B :

$$A/B = \mathcal{E}_j \{ A B^T \} (\mathcal{E}_j \{ B B^T \})^{-1}$$

The projection of A onto the orthogonal complement of the row space of B is defined as $A \Pi_B^\perp$

$$A \Pi_B^\perp = A - A/B$$

- The oblique projection A/C of the row space of the matrix A on the row space of the matrix B along the row space of C :

$$A/C = \mathcal{E}_j \{ A \begin{pmatrix} B^T & C^T \end{pmatrix} \} \left(\mathcal{E}_j \left\{ \begin{pmatrix} B B^T & B C^T \\ C B^T & C C^T \end{pmatrix} \right\} \right)^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix}$$

$$A/C = \mathcal{E}_j \{ A \Pi_C^\perp B^T \} \left(\mathcal{E}_j \{ B \Pi_C^\perp B^T \} \right)^{-1}$$

III. PROPOSITION OF CLOSED LOOP IDENTIFICATION ALGORITHM

Most of classical open loop subspace identification methods are based on the estimation of the extended observability matrix Γ_f . Whereas in a closed loop context the direct application of these methods gives biased estimates, the proposed indirect method allows an unbiased estimation of Γ_f : this is the aim of the first subsection. A possible application algorithm for the estimation of a state matrices realization (A, B, C, D) is given in the second subsection.

A. Main results

The first steps of the indirect algorithms by ([36]-[37]) and ([27], [29], [28]) are to estimate respectively $T_f \Gamma_f$ and Γ_f . Here the estimation of Γ_f (or $T_f \Gamma_f$) will be done in a next step. We first concentrate on the estimation of a matrix describing the closed loop behavior i.e. the extended observability matrix of the closed loop Γ_f^{cl} . To this end the following result uses the standard procedure of subspace approach (see [35]) on (6).

Result 1:

Let define the following oblique projection matrix \mathcal{O}_f^{cl} :

$$\mathcal{O}_f^{cl} = Y_f /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp \quad (7)$$

where Ψ is an instrumental variable matrix such that:

- $E_f /_{R_f}^\Psi = 0$
- the matrix $\mathcal{E}_j \left\{ \begin{pmatrix} X_f^{cl} \\ R_f \end{pmatrix} \begin{pmatrix} \Psi^T & R_f^T \end{pmatrix} \right\}$ has rank $(n_{cl}^{min} + f n_u)$

Under assumptions stated in section II-A we have:

- 1) \mathcal{O}_f^{cl} can be expressed with the extended observability matrix of the closed loop Γ_f^{cl} and the state matrix X_t^{cl}

$$\mathcal{O}_f^{cl} = \Gamma_f^{cl} X_t^{cl} /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp$$

- 2) The singular value decomposition (SVD) of \mathcal{O}_f^{cl} is given by

$$\mathcal{O}_f^{cl} = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \quad (8)$$

$$\mathcal{O}_f^{cl} = U_1 S_1 V_1^T$$

where $S_1 \in \mathbb{R}^{n_{cl}^{min} \times n_{cl}^{min}}$,

- 3) Γ_f^{cl} can be taken equal to $\Gamma_f^{cl} = U_1 T$ where T is a similarity transformation.

Remark 1: The algorithm by [36]-[37] uses a similar oblique projection matrix: $Y_f /_{U_f}^\Psi \Psi \Pi_{U_f}^\perp$. However this choice does not allow the estimation of the closed loop behavior.

The first condition on the instrumental variable matrix Ψ must be guaranteed in order to obtain consistent estimation under the noise-corrupted context. Let remark that A should not be stable which yields $E_f /_{U_p} \neq 0$ as discussed in [2], thus U_p don't have to be directly introduced in Ψ . Two possible choices are $\Psi = R_p$ or $\Psi = \begin{pmatrix} R_p \\ Y_p \end{pmatrix}$.

The second condition on Ψ (the rank condition) involves some conditions on the choice of the indexes f and p ($f, p > n_{cl}^{min}$), the degree of persistence excitation and the closed loop complexity. According to [1], if $\Psi = R_p$ or $\Psi = \begin{pmatrix} R_p \\ Y_p \end{pmatrix}$ this rank condition is generically satisfied provided that the exogenous input is persistently exciting enough.

Remark 2: Some weighting matrices are often added in (7) in order to include some well known subspace methods such that N4SID, PO-MOESP, CVA, IVM, etc. The MOESP type algorithms make use of the two previous instrumental variable matrices in (7).

Once the extended observability matrix known, we have to find Γ_f . From (6) it is easy to rewrite \mathcal{O}_f^{cl} as

$$\mathcal{O}_f^{cl} = T_f \Gamma_f X_t /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp + T_f H_f^d \Gamma_f^c X_t^c /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp$$

which gives

$$\Gamma_f X_t /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp = T_f^{-1} \left(\mathcal{O}_f^{cl} - T_f H_f^d \Gamma_f^c X_t^c /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp \right)$$

with

$$\Gamma_f^c X_t^c = U_f + H_f^c Y_f - R_f \quad (9)$$

To compute Γ_f we need to determine matrices $T_f H_f^d$ and T_f . From (6) we get

$$Y_f /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp = \Gamma_f^{cl} X_t^{cl} /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp + T_f H_f^d R_f$$

where $Z = \begin{pmatrix} R_f \\ \Psi \end{pmatrix}$. With $(\widehat{\Gamma_f^{cl}})^\perp$ computed from the previous result we find:

$$(\widehat{\Gamma_f^{cl}})^\perp Y_f /_{R_f}^\Psi \Psi \Pi_{R_f}^\perp = (\widehat{\Gamma_f^{cl}})^\perp T_f H_f^d R_f$$

with

$$T_f H_f^d = \begin{pmatrix} L_0 & 0 & \cdots & 0 \\ L_1 & L_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ L_{f-1} & \cdots & L_1 & L_0 \end{pmatrix}$$

where $L_0 = D_{cl}/r$ and $L_i = C_{cl}A_{cl}^{i-1}B_{cl}/r$. These f elements $L_i \in \mathbb{R}^{n_y \times n_u}$ can easily be extracted using the procedure given in [36].

Once $\widehat{T_f H_f^d}$ is known, it is easy to get an estimated of T_f :

$$\widehat{T_f} = I_{fn_y} - \widehat{T_f H_f^d} H_f^c \quad (10)$$

By using previous estimations, the following result allows the computation of the extended observability matrix Γ_f of the plant model.

Result 2:

Let define the following matrix \mathcal{O}_f :

$$\mathcal{O}_f = T_f^{-1} \left(\mathcal{O}_f^{cl} - T_f H_f^d \Gamma_f^c X_t^c / \Psi \Psi \Pi_{R_f}^\perp \right)$$

Under the assumption stated in part II-A and in the previous result we have:

- 1) The matrix \mathcal{O}_f can be expressed with the extended observability matrix of the plant Γ_f and the state matrix X_t

$$\mathcal{O}_f = \Gamma_f X_t / \Psi \Psi \Pi_{R_f}^\perp$$

- 2) The SVD of \mathcal{O}_f is given by

$$\mathcal{O}_f = \begin{pmatrix} \mathbb{U}_1 & \mathbb{U}_2 \end{pmatrix} \begin{pmatrix} \mathbb{S}'_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbb{V}_1^T \\ \mathbb{V}_2^T \end{pmatrix} \quad (11)$$

$$\mathcal{O}_f = \mathbb{U}'_1 \mathbb{S}'_1 \mathbb{V}_1^T$$

where $\mathbb{S}'_1 \in \mathbb{R}^{n \times n}$,

- 3) Γ_f can be taken equal to $\Gamma_f = \mathbb{U}'_1 T$ where T is a similarity transformation.

Remark 3: It is possible to estimate a matrix $\widehat{X}_t = X_t / \Psi \Psi \Pi_{R_f}^\perp = T^{-1} \mathbb{S}'_1 \mathbb{V}_1^T$. This can be interpreted as the state sequence estimated by a bank of non steady state Kalman filters ([35]).

Remark 4: It should be noted that results 1 and 2 are based on classical subspace procedures ([35]) and algebraic computations applicable to MIMO processes, thus these results are also applicable to this type of processes.

Remark 5: In the previous result the Markov parameters of the controller are needed: in (9) and (10). It should be noted that (5) can be rewritten as $(U_f - R_f) = \Gamma_f^c X_t^c - H_f^c Y_f$. This equation is free noise, thus the matrix H_f^c can easily be estimated without bias making use an open loop subspace algorithm.

B. Closed loop identification algorithm

In this section we propose the estimation of a realization of state space matrices (A, B, C, D) . This can be realized with the estimated state sequence \widehat{X}_t , however we present the solution making use of the shift invariance structure of the extended observability matrix Γ_f .

The basic steps of the proposed algorithm are as follows:

step 1: Compute the LP decomposition

$$\begin{pmatrix} R_f \\ \Psi \\ Y_f \end{pmatrix} = \begin{pmatrix} M_{11} & 0 & 0 \\ M_{21} & M_{22} & 0 \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} N_1^T \\ N_2^T \\ N_3^T \end{pmatrix}$$

which gives the first oblique projection

$$\mathcal{O}_f^{cl} = M_{32} N_2^T$$

step 2: Compute the SVD (8) and extract Γ_f^{cl}

$$\Gamma_f^{cl} = \mathbb{U}_1$$

step 3: Estimate $T_f H_f^d$ and T_f

step 4: Compute the LP decomposition

$$\begin{pmatrix} R_f \\ \Psi \\ T_f H_f^d \Gamma_f^c X_t^c \end{pmatrix} = \begin{pmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{pmatrix}$$

which gives the second oblique projection

$$\mathcal{O}_f = T_f^{-1} (\mathbb{U}_1 \mathbb{S}_1 \mathbb{V}_1^T - R_{32} Q_2^T)$$

step 5: Compute the SVD (11) and extract Γ_f

$$\Gamma_f = \mathbb{U}'_1$$

step 6: The system matrices A and C can be estimated by

$$C = \Gamma_f(1 : n_y, :)$$

$$A = (\underline{\Gamma}_f^T \underline{\Gamma}_f)^{-1} \underline{\Gamma}_f^T \bar{\Gamma}_f$$

with $\underline{\Gamma}_f = \Gamma_f(1 : (f-1)n_y, :)$ and $\bar{\Gamma}_f = \Gamma_f(n_y + 1 : fn_y, :)$.

B and D can be estimated from the following equation

$$\left(T_f^{-1} \Gamma_f^{cl} \right)^\perp T_f^{-1} Y_f / Z Z = \left(T_f^{-1} \Gamma_f^{cl} \right)^\perp H_f^d (B, D) R_f$$

using the procedure given in [35].

Remark 6: Note that the absence of feedback corresponds to the case $H_f^c = 0$, $\Gamma_f^c = 0$, $R_f = U_f$ and $R_p = U_p$. As a consequence, in an open loop context the proposed algorithm can be reduced to steps 1, 2 and 6. This corresponds to the open loop MOESP type algorithms: with $\Psi = U_p$ our algorithm is equivalent to the PI-MOESP algorithm, with $\Psi = \begin{pmatrix} U_p \\ Y_p \end{pmatrix}$ our algorithm is equivalent to the PO-MOESP algorithm.

IV. NUMERICAL EXAMPLE

In this section we consider a fifth order plant presented in [38] and used as a benchmark problem by [36] for closed loop subspace identification. This example is also adopted in [24], [20], [13], [12], [21] for comparison.

The transfert functions of the plant and controller are respectively given by

$$G(q) = 10^{-3} q^{-1} \frac{0.98 + 12.99q^{-1} + 18.59q^{-2} + 3.3q^{-3} - 0.02q^{-4}}{1 - 4.4q^{-1} + 8.09q^{-2} - 7.83q^{-3} + 4q^{-4} - 0.86q^{-5}}$$

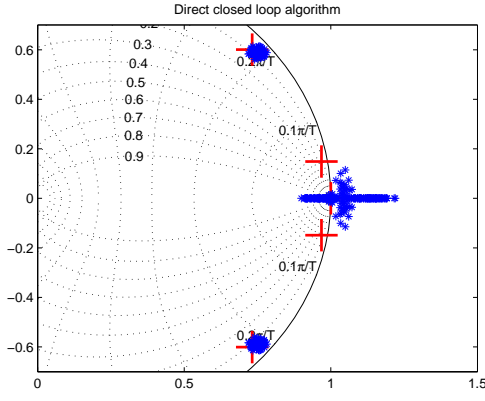


Fig. 2. The eigenvalues of estimated A matrix with direct approach: * estimated pole, + system pole.

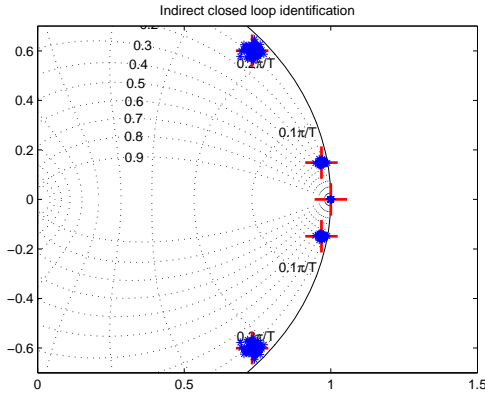


Fig. 3. The eigenvalues of estimated A matrix with our indirect approach: * estimated pole, + system pole.

$$C(q) = \frac{0.63 - 2.083q^{-1} + 2.8222q^{-2} - 1.865q^{-3} + 0.4978q^{-4}}{1 - 2.65q^{-1} + 3.11q^{-2} - 1.75q^{-3} + 0.39q^{-4}}$$

The exogenous input $r(t)$ is a gaussian white noise sequence with variance 1. The innovation $e(t)$ is a gaussian white noise with variance 1/9 and the noise model is given by

$$H(q) = \frac{1 - 2.1q^{-1} + 1.45q^{-2} - 0.315q^{-3} - 0.0146q^{-4} + 0.0034q^{-5}}{1 - 4.4q^{-1} + 8.09q^{-2} - 7.83q^{-3} + 4q^{-4} - 0.86q^{-5}}$$

The number of columns in Hankel matrices is $j = 1200$ and we generate 100 data sets, each time with the same exogenous input but with different noise sequences. We choose $f = 20$ and the following instrumental variable Ψ

$$\Psi = \begin{pmatrix} R_p \\ Y_p \end{pmatrix}$$

where R_p and Y_p are built with $p = f = 20$. The Markov parameters of the controller are supposed to be known.

Some estimates of the poles are shown in Figs. 2 and 3: Fig. 2 corresponds to the N4SID algorithm proposed by MATLAB, Fig. 3 corresponds to our indirect algorithm. We can see that our indirect method looks better than the other in spite of one remark: poles $0.7319 \pm 0.6007i$ are difficult to estimate, the estimation variance is higher (Fig 3).

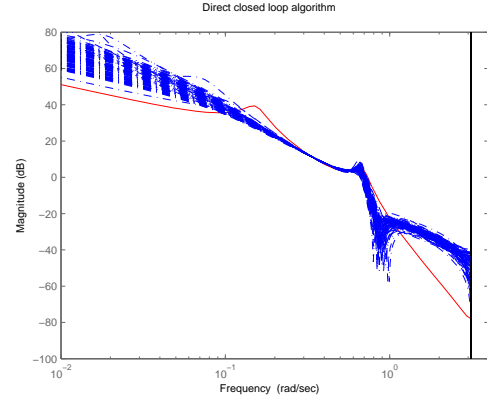


Fig. 4. The Bode magnitude plot of direct approach.

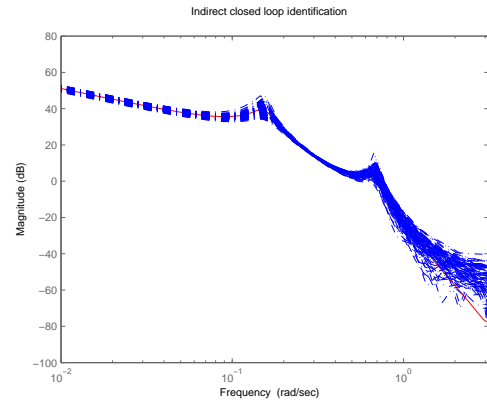


Fig. 5. The Bode magnitude plot of our indirect approach.

Estimates of the frequency response are shown in Fig. 4 to 7: Fig. 4 corresponds to the N4SID algorithm proposed by MATLAB, Fig. 5 corresponds to our indirect method, Fig. 6 corresponds to the method proposed in [36]-[37] (with $p = f = 10$ – as proposed in these papers) and Fig. 7 corresponds to the method proposed in [29] (with $p = f = 20$ – the best choice after several tests). Here again, our indirect method works well: all the methods match with the frequency response of the real system at low and medium frequency.

V. CONCLUSION

In this paper, an indirect closed-loop identification algorithm is developed using the subspace identification culture. The key idea was to estimate a characteristic matrix of the closed loop behavior and then to extract a characteristic matrix of the plant. Notice that two LQ decompositions and two SVD are needed and the algorithm allows the estimation of the closed loop complexity and plant complexity. In this paper only the deterministic part of the plant is identified, follow-up paper will concentrate on the stochastic part.

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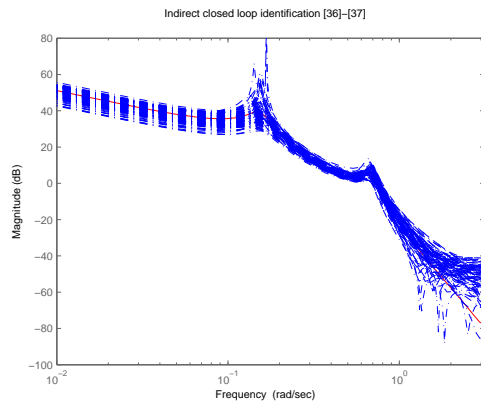


Fig. 6. The Bode magnitude plot of indirect approach of [36]-[37].

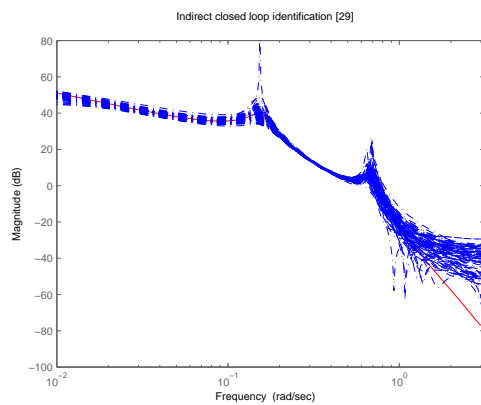


Fig. 7. The Bode magnitude plot of indirect approach of [29].

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