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► **To cite this version:**

H. El Fadil, Fouad Giri, Fatima Zara Chaoui. Accounting for Input Limitation in the Control of Buck Power Converters. IEEE Transactions on Circuits and Systems I: Regular Papers, 2009, 56 (6), pp.1260-1271. 10.1109/TCSI.2009.2015171 . hal-01058666

HAL Id: hal-01058666

<https://hal.science/hal-01058666>

Submitted on 5 Oct 2021

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Accounting for Input Limitation in the Control of Buck Power Converters

Hassan El Fadil, Fouad Giri, Fatima-Zahra Chaoui, and Ouadia El Magueri

Abstract—We are considering the problem of controlling a DC–DC switched power converter of the Buck type. The converter involves an inherent control limitation; accordingly the control signal (duty ratio) can only take values in the interval $(0, 1)$. In the relevant literature, such a physical control limitation is generally not taken into account when designing the converter regulators. This is only dealt with in the control implementation stage, placing an isolated limiter between the (linear) controller and the controlled system. Furthermore, the presence of such a limiter is generally ignored when analyzing the closed-loop control system. In the present paper, the control signal limitation is dealt with using a nonlinear regulator that involves an internal limiter. The resulting closed-loop control system is shown to be equivalent to a nonlinear feedback involving a linear dynamics block in closed-loop with a nonlinear static element. Using absolute stability tools, sufficient conditions are established for the involved feedback to be L_2 -stable. If these conditions are respected when choosing the control design parameters then the regulator meets its objectives (closed-loop stability and output reference tracking). It is worth noting that, though the focus is made on a specific power converter, the paper includes an important theoretical dimension that may be of general interest.

Index Terms—Absolute stability, DC–DC power converters, input limitation, voltage regulation.

I. INTRODUCTION

THESE are three main types of switched power converters, namely boost, buck, and buck–boost. These have recently received an increasing deal of interest both in power electronics works and in automatic control applications. This is due to their wide applicability domain, e.g., domestic equipments, communication systems, computers, industrial electronics, battery-operating, embedded equipments, uninterruptible power sources. From an automatic control viewpoint, a switched power converter constitutes a challenging case study as it is variable-structure and nonlinear. Its rapid structure variation is generally coped with using averaged models [1], [2], [16]. Based on these average models, different nonlinear controllers have been developed using passivity techniques [2], feedback linearization, flatness methods [5], sliding mode control ([3], [4], [14]) and backstepping control technique [6]–[8]. In all works the proposed controllers are designed to achieve closed-loop global stability and voltage reference

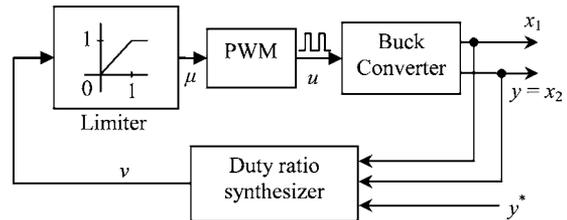


Fig. 1. Basic control setup of Buck converters.

tracking. However, in these studies it was never accounted for the limitation of the control signal (namely, the duty ratio μ). As a matter of fact, the control is not allowed to be outside the interval $(0, 1)$, due to the technological nature of the controlled circuits. Therefore, the usual practice consists, when it comes to implementing the controllers, in simply placing an isolated limiter between the designed controller and the controlled system (see Fig. 1). Unfortunately, the nonlinear effect of such a limiter is never taken into account when analyzing the closed-loop system ([2]–[8]). As a consequence, the aforementioned stability results may lose their global nature, the controller transient performances may deteriorate and the output-reference tracking objective may not be achieved.

On the other hand, the problem of controlling linear systems subject to input saturation constraint has received a great deal of interest over the last two decades [11]. Most theoretical results have focused on (local/global) stabilization. This was carried out following two main approaches. The first one consists in enforcing the control signal to stay all time within the allowed limits so that the closed-loop system stays in a region of linear behavior, see e.g., [12], [13] and reference list therein. It is proved that linear regulators can be designed so that to ensure the positive invariance of this linear region (or a subset of it including all admissible initial conditions). Such a positive invariant set is then a local region of stability. The second research direction to handle input constraints is one where the control signal is allowed to saturate. During the time intervals of control saturation, the closed-loop system is no longer linear and the controlled system is steered in open loop. Then, global asymptotic stabilization is only possible for stable systems [11]. The problem of output-reference (in presence of control input limitation) has not been so deeply investigated. When the controlled system is of type-1 and the reference signal is step-like, then the tracking problem can be transformed into a disturbance-free regulation problem and existing solutions can be applied, see e.g., [15]. For non-type-1 systems, an integrator should be incorporated in the regulator to make the tracking objective achievable. But, the presence of an integrator generally results in large control actions which, due to the control limitation, may lead to

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undesirable oscillatory behavior. To avoid such behavior, many authors have proposed linear regulators together with *ad hoc* “anti-windup” devices (examples of such devices can be found in [11]). Unfortunately, there is generally no formal proof that the proposed anti-windup regulators actually ensure the tracking objective for a specific class of reference signals.

The present paper is precisely focusing on the tracking issue in presence of control limitation. The focus is made on dc-to-dc Buck power converters operating according to the PWM principle. Our approach consists in first designing a linear control law that achieves the tracking objective in the absence of control limitation. Then, an adequate anti-windup device is incorporated, in the above control law, leading to a nonlinear regulator. It is shown that the resulting closed-loop control system is equivalent to a nonlinear feedback consisting of a linear dynamics block in closed-loop with a nonlinear static element. Sufficient conditions for L_2 -stability of this feedback are then established using tools from the absolute stability theory (circle criterion, Barbalat’s lemma . . .). The main condition is the real-positivity (RP) of a specific transfer function that involves, on one hand, the poles of the controlled system and, on the other hand, those of the underlying linear closed-loop system (i.e., the linear closed-loop system obtained when the controller never saturates). In fact, the RP condition defines a neighborhood of the controlled system poles in which should be assigned those of the closed loop. Such a requirement can always be satisfied through an adequate choice of the controller parameters. Finally, it is shown that if the reference signal is slowly varying (in a well defined sense) then the proposed nonlinear controller stops saturating after finite transient-periods (following the reference signal changes) and the system output tracks asymptotically its reference. Additional simulation results show that the developed regulator has quite interesting robustness capability e.g., it preserves a reasonable level of performances when facing uncertain load changes and transient discontinuous conduction mode operation.

The paper is organized as follows: in Section II, the Buck converter is described and the control objective is formulated; Sections III and IV are devoted to the regulator design and theoretical analysis; the closed-loop performances are illustrated in Section V. A conclusion and a reference list end the paper.

II. CONVERTER MODELING AND CONTROL OBJECTIVE

A. Converter Modeling

A Buck converter is constituted of power electronic components connected together, as shown in Fig. 2. It operates according to the so-called Pulse Width Modulation (PWM) principle and is described by the following instantaneous model:

$$\begin{aligned} \dot{i}_L &= -\frac{1}{L}v_s + u\frac{E}{L} \\ \dot{v}_s &= \frac{1}{C}i_L - \frac{1}{RC}v_s \end{aligned} \quad (1a)$$

where $u \in \{0, 1\}$. Resorting to averaging techniques (see, e.g., [1], [2]), it is shown that the circuit, supposed to be operating

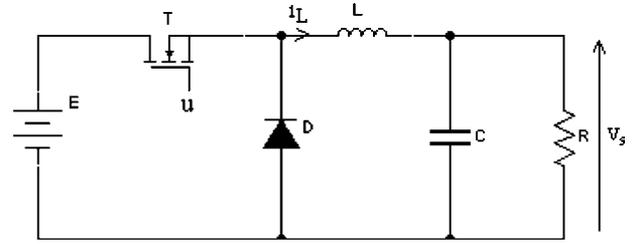


Fig. 2. Buck converter circuit.

in the continuous conduction mode, can be represented by the following averaged model:

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{L}x_2 + \mu\frac{E}{L} \\ \dot{x}_2 &= \frac{1}{C}x_1 - \frac{1}{RC}x_2. \end{aligned} \quad (2a)$$

As averaging is performed over cutting periods, the state variables $x_1(t)$ and $x_2(t)$ denote the average input current (i_L) and average output capacitor voltage (v_s), respectively. The control signal μ (said duty ratio) is in turn the average value of the actual binary-type control u . As a consequence of the binary nature of u , the duty ratio μ turns out to be a continuous function of time that only takes values in the real interval $[0, 1]$ i.e., $0 \leq \mu \leq 1$, all the time. By definition, the system output is:

$$y(t) = x_2(t). \quad (2b)$$

In the sequel, the symbol ‘ s ’ refers generally to the time-derivation operator, i.e., $s = d/dt$. Then, for a given signal x , one has:

$$sx = \frac{dx}{dt}. \quad (3)$$

Using this notation, the state-space model (2a) may be given the following input-output representation ¹

$$A(s)y = B(s)\mu \quad (4)$$

where

$$B(s) = b_0, \quad A(s) = s^2 + a_1s + a_0 \quad (5a)$$

$$b_0 = \frac{E}{LC}, \quad a_0 = \frac{1}{LC}, \quad a_1 = \frac{1}{RC}. \quad (5b)$$

The parameters a_0 , a_1 , b_0 are supposed to be constant and known during all stages of the regulator design (Section III) and closed-loop theoretical analysis (Section IV). Nevertheless, the effect of uncertain load changes and transient discontinuous conduction mode operation will be investigated when evaluating the regulator performances in Section V.

B. Control Objective

The control problem at hand is to design, for the considered circuit, a controller that meets the following.

- 1) The control action, i.e., the generated duty ratio function $\mu(t)$, should satisfy the following inequalities

$$\mu_m < \mu(t) < \mu_M \quad (6)$$

¹When clear from the context, the symbol ‘ s ’ will also used to designate the Laplace Transform variable. For instance, the transfer function of the system (4) is simply denoted $B(s)/A(s)$.

where μ_m, μ_M are any real constants such that $0 < \mu_m < \mu_M < 1$.

- 2) All the closed-loop signals should remain bounded.
- 3) The output tracking error $e_y = y - y^*$ should converge to zero, where y^* is a given bounded reference signal. This represents the desired trajectory of the output y . But, for this tracking objective to be achievable, the signal y^* has to be compatible with the control constraint (6), i.e.,:

$$\frac{B(0)}{A(0)}\mu_m < y^*(t) < \frac{B(0)}{A(0)}\mu_M. \quad (7)$$

Note that $B(0)/A(0) = b_0/a_0$ and this quantity is nothing other than the static gain of the system (4). The reference signal y^* is further supposed to be slowly varying in the sense that:

$$sy^*(t) \in L_2([0, +\infty]). \quad (8)$$

Remark 2.1:

- 1) Condition (7)–(8) will prove to be crucial in achieving the tracking objective in presence of the control limitation (6). In other words, a reference signal y^* that does not satisfy the double condition (7)–(8) is not admissible in presence of the constraint (6) (though it is admissible in the constraint-free case).
- 2) Theoretically, one may let $\mu_m = 0$ and $\mu_M = 1$ in (6). However, it is not recommended in practice to impose to the duty ratio μ the extreme values 0 and 1. Therefore μ_m and μ_M should be given values close to (but different from) 0 and 1, respectively.

III. REGULATOR DESIGN

The first step in the regulator design consists in transforming the initial tracking problem into a regulation problem. To this end, introduce the following tracking error:

$$e_y(t) = y(t) - y^*(t). \quad (9)$$

Operating $sA(s)$ on $e_y(t)$ and using (4) gives the following fictive system:

$$sA(s)e_y = B(s)(s\mu) - A(s)(sy^*), \quad (10)$$

The latter is viewed as a system of input $s\mu$ and output e_y . The quantity $sy^*(t)$, which belongs to $L_2([0, +\infty])$ due to (8), stands as a disturbance in the new system (10). Enforcing the output y of the initial system (4) to track a reference y^* , amounts to enforcing the output e_y of the new system (10) to vanish asymptotically. If the control input μ were not constrained, it would have been possible to achieve the new regulation objective using a linear regulator of the form $\mu = -S(s)/sR(s)e_y$. The differential operators $R(s)$ and $S(s)$ can be determined by linear design methods (e.g., pole-placement, linear-quadratic, see e.g., [10]). Since the control signal is constrained to stay in the interval $[\mu_m, \mu_M]$, we consider the following nonlinear controller:

$$v = \left(1 - \frac{sR(s)}{\Lambda(s)}\right)\mu - \frac{S(s)}{\Lambda(s)}e_y \quad (11a)$$

$$\mu = \text{sat}(v) \quad (11b)$$

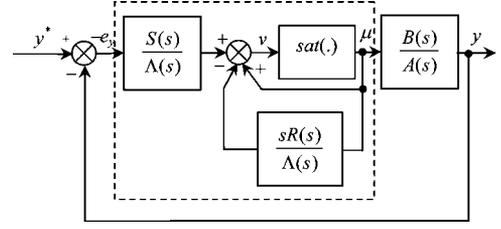


Fig. 3. Closed-loop system obtained with the nonlinear controller (11a)–(11b) that incorporates a control limiter.

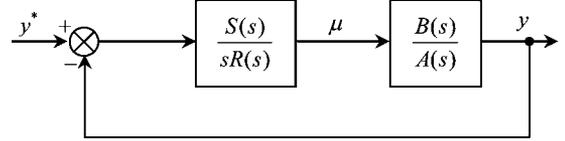


Fig. 4. Closed-loop system obtained with the standard linear controller (14) that involves no control limitation.

where $R(s)$ and $S(s)$ are differential operators of the form:

$$R(s) = s + \alpha_0 \quad (12a)$$

$$S(s) = \beta_2 s^2 + \beta_1 s + \beta_0 \quad (12b)$$

and $\text{sat}(\cdot)$ denotes the (nonsymmetrical) saturation function defined by

$$\text{sat}(v) = \begin{cases} \mu_M & \text{if } v > \mu_M \\ \nu & \text{if } \mu_m \leq v \leq \mu_M \\ \mu_m & \text{if } v < \mu_m \end{cases} . \quad (13)$$

The real coefficients $(\alpha_0, \beta_0, \beta_1, \beta_2)$ in (12a)–(12b) will be defined later. The closed-loop system thus obtained is represented by Fig. 3. Note the presence of the nonlinear element $\text{sat}(\cdot)$ in the controller block.

Remarks 3.1:

- a) Suppose that the computed control action $v(t)$ satisfy the constraint $0 \leq v(t) \leq \mu_M$ all the time. Then, it follows from (13) that: $\mu = \nu$ and, consequently, the control law (11a) reduces to a standard linear regulator of the form

$$\mu = -\frac{S(s)}{sR(s)}e_y. \quad (14)$$

The closed-loop control system then turns out to be a standard linear feedback as shown by (Fig. 4). Notice that (14) contains an integral action ($1/s$). Moreover, the above control law is nothing other than a PID controller. Indeed, using (12a)–(12b), it can be easily checked by direct substitution that the transfer function $S(s)/sR(s)$ may be given the following form

$$\frac{S(s)}{sR(s)} = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \tau s}\right)$$

with

$$\tau = \frac{1}{\alpha_0}; \quad T_i = \frac{\beta_1}{\beta_0} - \tau;$$

$$T_d = \frac{\beta_2}{\beta_0} \frac{1}{T_i} - \tau; \quad K_p = \beta_0 T_i \tau.$$

- b) Contrary to (14), the nonlinear controller (11a)–(11b) involves two control variables, namely ν and μ . The

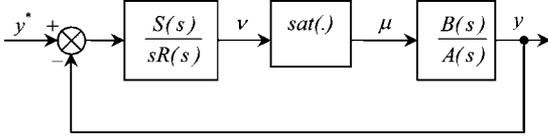


Fig. 5. Closed-loop system obtained with the standard controller (14) and an isolated control limiter.

first represents the control action that is suggested by the underlying linear controller (11a). The second is the control action that is actually applied to the controlled system. Equation (11a) shows that the suggested control action (ν) accounts for the actual control action (μ). Consequently, the nonlinear controller involves a feedback around the saturation element $\text{sat}(\cdot)$ (see Fig. 3). This contrasts with the usual practice (Fig. 1) that consists in just placing the saturation element $\text{sat}(\cdot)$ outside the loop, as this is illustrated by Fig. 5. In the later, there is no compensation of the disturbing effect due to the control limitation. It is clear that if the controller (11a)–(11b) never saturates (i.e., $0 \leq \nu \leq 1$ and $\mu = \nu$) it will coincide with the linear controller (14). When this happens, the closed loop systems (Figs. 3 and 4) will be identical.

- c) As this was mentioned in Section I, controllers like (11a)–(11b) that attempt to compensate the effect of control limitation are usually referred to anti-windup controllers (see e.g., [11]). In this respect, the controller of Fig. 5 involves no anti-windup device \square

Many design techniques may be used to obtain the operators $R(s)$ and $S(s)$ in (11a). Specifically, all linear design methods can be used e.g., pole-placement and linear-quadratic [10]. For the sake of simplicity, let us consider the pole placement technique. Accordingly, the above operators are uniquely obtained solving the following Bezout equation:

$$sA(s)R(s) + B(s)S(s) = C(s)\Lambda(s) \quad (15)$$

where

$$C(s) = s^2 + c_1s + c_0 \quad (16a)$$

$$\Lambda(s) = s^2 + \lambda_1s + \lambda_0 \quad (16b)$$

are any Hurwitz polynomials whose choice will be subject to conditions précised later². For now, let us just note that for these polynomials to be Hurwitz, their coefficients must be positive (because of the Routh criterion). That is, one has:

$$c_i > 0, \lambda_i > 0 \quad (i = 1, 2) \quad (16c)$$

IV. ANALYSIS OF THE CLOSED-LOOP CONTROL SYSTEM

The closed-loop control system of Fig. 3, consisting of the controlled system (4) and the regulator (11a)–(11b), will now be analyzed. The outcome of such an analysis will help making

²Recall that the operators $C(s)$, $\Lambda(s)$ are said Hurwitz if their zeroes have negative real parts. Then, the transfer functions $1/C(s)$, $1/\Lambda(s)$ are asymptotically stable, (see e.g., [11]).

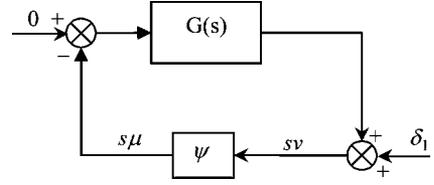


Fig. 6. Nonlinear feedback relating the signals sv and $s\mu$.

suitable choices of the design parameters, namely the operators $C(s)$ and $\Lambda(s)$, so that the control objective of Section II can be achieved. To this end, the closed-loop system will first be given a feedback representation that allows application of absolute stability theorems [9].

A. A Suitable Feedback Representation of the Closed-Loop System

Combining (10) with (11a) so that to eliminate e_y yields:

$$(sv) = -\frac{C(s) - A(s)}{A(s)}(s\mu) + \frac{S(s)}{\Lambda(s)}(sy^*). \quad (17)$$

Equation (17), together with the fact that $\mu = \text{sat}(\nu)$, can be given the feedback representation of Fig. 6, with

$$G(s) = \frac{C(s) - A(s)}{A(s)} \quad (18)$$

$$\delta_1 = \frac{S(s)}{\Lambda(s)}(sy^*) \quad (19)$$

and ψ is a (nonlinear) operator that maps sv into $s\mu$.

B. Conicity of the Nonlinear Element

The following analysis aims at establishing a sufficient condition (on the polynomial $C(s)$) that ensures the L_2 -stability of the above feedback. The first step is to show that ψ belongs to the sector $[0, 1]$.

Proposition 1: The nonlinear element ψ , in the feedback of Fig. 6, belongs to the sector $[0, 1]$ in the sense that: $0 \leq sv \cdot s\mu \leq (sv)^2$, where $s\mu = d\mu/dt$ and $sv = dv/dt$ \square

Proof: It follows from (13) that, for all $v \in (-\infty, +\infty)$:

$$0 \leq \frac{d \text{sat}(v)}{dv} \leq 1. \quad (20)$$

On the other hand, one has from (11b):

$$\frac{d\mu}{dt} = \frac{d \text{sat}(v)}{dv} \frac{dv}{dt}. \quad (21)$$

Multiplying both sides of (21) by (sv) gives:

$$(s\mu)(sv) = \frac{d \text{sat}(v)}{dv} (sv)^2 \quad (22)$$

which, together with (20), implies that

$$0 \leq (s\mu)(sv) \leq (sv)^2. \quad (23)$$

This proves Proposition 1. \blacksquare

C. L_2 -Stability of the Feedback of Fig. 6

Proposition 1 will now be used to establish a sufficient condition, on the operator $C(s)$, that ensures the L_2 -stability of the feedback system of Fig. 6.

Proposition 2: Let the polynomial $C(s)$, in (15) and (16a), be chosen so that:

$$\inf_{0 \leq \omega < +\infty} \operatorname{Re}(G(j\omega)) > -1 \quad (24)$$

which, in view of (18), amounts to

$$\inf_{0 \leq \omega < +\infty} \operatorname{Re} \left(\frac{C(j\omega)}{A(j\omega)} \right) > 0. \quad (25)$$

Then, the feedback of Fig. 6 is L_2 -stable and, consequently

$$s\mu \in L_2, \quad sv \in L_2, \quad se_y \in L_2, \quad sy \in L_2. \quad (26)$$

□

Proof: As ψ belongs to the sector $[0, 1]$ and $G(s)$ satisfies the positive real property (24), it follows from the circle criterion (see e.g., [9, p. 265]) that the feedback of Fig. 6 is L_2 -stable. On the other hand, as $\Lambda(s)$ is Hurwitz and $S(s)/\Lambda(s)$ is a proper transfer function³, one has from (8) and (19) that

$$\delta_1(t) \in L_2. \quad (27)$$

Then, it follows that all internal signals of the above feedback belong to L_2 . This particularly means that

$$s\mu \in L_2 \quad (28a)$$

$$sv \in L_2. \quad (28b)$$

Noting that $A(s)$ is Hurwitz, it follows from (10), (27), and (28a) that

$$se_y \in L_2. \quad (29)$$

Also, from (8) one gets

$$sy = se_y + sy^* \quad (30)$$

which together with (8) and (29), gives

$$sy \in L_2. \quad (31)$$

This ends the proof of Proposition 2 ■

In the light of Proposition 2, it is seen that all signals of the control system (Fig. 3) vary slowly, in the sense that their derivatives belong to L_2 . This will be used to establish a more crucial result, namely the fact that the deviation $(v(t) - \mu(t))$, between the computed and applied control, belongs to L_2 . This is the subject of the following proposition.

Proposition 3: Consider the control system of Fig. 3 where:

- i) $C(s)$ is any Hurwitz operator satisfying inequality (25)
- ii) $R(s)$ and $S(s)$ are solutions of (15)
- iii) $\Lambda(s)$ is any Hurwitz operator of the form (16b)

³Recall that a transfer function is proper if the degree of its numerator is equal or less than the degree of the denominator. An improper transfer function is not physically realizable.

Then, one has

$$(v - \mu) \in L_2 \quad (32)$$

□

Proof: In view of (12b), (11a) can be rewritten as follows:

$$v = \mu - \frac{s_0}{\Lambda(s)} e_y + \delta_2 = \mu - \frac{s_0}{\Lambda(s)} y + \frac{s_0}{\Lambda(s)} y^* + \delta_2 \quad (33)$$

where

$$\delta_2 = -\frac{R(s)}{\Lambda(s)} s\mu - \frac{(s_2s + s_1)}{\Lambda(s)} se_y. \quad (34)$$

Using (28a) and (29) and the fact that $\Lambda(s)$ is Hurwitz, it follows from (34) that

$$\delta_2 \in L_2. \quad (35)$$

On the other hand, (4) can be transformed as follows:

$$a_0y = b_0\mu + \delta_3 \quad (36)$$

with

$$\delta_3 = -(s + a_1) sy. \quad (37)$$

Furthermore, one readily gets from (4) that

$$s^2y = \frac{sB(s)}{A(s)} s\mu. \quad (38)$$

Since $s\mu \in L_2$ (by (28a)) and the transfer function $sB(s)/A(s)$ is Hurwitz and strictly proper, it follows from (38) that $s^2y \in L_2$. Then, using (31), it follows from (37) that

$$\delta_3 \in L_2. \quad (39)$$

On the other hand, multiplying (36) by $\lambda_0 c_0 / b_0 a_0$ gives

$$\frac{\lambda_0 c_0}{b_0} y = \frac{\lambda_0 c_0}{a_0} \mu + \frac{\lambda_0 c_0}{b_0 a_0} \delta_3. \quad (40a)$$

Also, letting $s = 0$ in (15) yields $b_0 s_0 = \lambda_0 c_0$. Using this in (33), one gets

$$v = \mu - \frac{c_0 \lambda_0}{b_0} \frac{1}{\Lambda(s)} y + \frac{c_0 \lambda_0}{b_0} \frac{1}{\Lambda(s)} y^* + \delta_2. \quad (40b)$$

Combining (40a) and (40b) gives

$$v = \mu - \frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0} \mu + \frac{1}{\Lambda} \frac{\lambda_0 c_0}{b_0} y^* - \frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0 b_0} \delta_3 + \delta_2. \quad (41)$$

Now, let us introduce the notation

$$\mu^* = \frac{a_0}{b_0} y^*. \quad (42)$$

It is readily seen from (7) that

$$\mu_m < \mu^* < \mu_M \quad (43a)$$

and from (8) that

$$s\mu^* \in L_2. \quad (43b)$$

With the above notation, (41) can be rewritten

$$v - \mu = -\frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0} \mu + \frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0} \mu^* + \delta_4 \quad (44)$$

with

$$\delta_4 = -\frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0 b_0} \delta_3 + \delta_2. \quad (45)$$

Using (35), (39) and the fact that $\Lambda(s)$ is Hurwitz, it follows from (45) that

$$\delta_4 \in L_2. \quad (46)$$

Now, adding $c_0/a_0\mu - c_0/a_0\mu + c_0/a_0\mu^* - c_0/a_0\mu^* = 0$ to the right-hand side of (44), one gets (rearranging terms)

$$v - \mu = -\frac{c_0}{a_0} \mu + \frac{c_0}{a_0} \mu^* + \left[-\frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0} \mu + \frac{c_0}{a_0} \mu \right] + \left[\frac{1}{\Lambda(s)} \frac{\lambda_0 c_0}{a_0} \mu^* - \frac{c_0}{a_0} \mu^* \right] + \delta_4. \quad (47)$$

In view of (15b), (47) simplifies to

$$\begin{aligned} v - \mu &= -\frac{c_0}{a_0} \mu + \frac{c_0}{a_0} \mu^* + \frac{c_0}{a_0} \left[\frac{(s + \lambda_1) s}{\Lambda(s)} \right] \mu \\ &\quad + \frac{-c_0}{a_0} \left[\frac{(s + \lambda_1) s}{\Lambda(s)} \right] \mu^* + \delta_4 \\ &= \mu - \frac{c_0}{a_0} \mu + \frac{c_0}{a_0} \mu^* + \frac{c_0}{a_0} \left[\frac{(s + \lambda_1) s}{\Lambda(s)} \right] (s\mu - s\mu^*) + \delta_4 \end{aligned} \quad (48)$$

The last equation is given the more compact form

$$v - \mu = -\frac{c_0}{a_0} \mu + \frac{c_0}{a_0} \mu^* + \delta_5 \quad (49)$$

with

$$\delta_5 = \frac{c_0}{a_0} \left[\frac{(s + \lambda_1) s}{\Lambda(s)} \right] (s\mu - s\mu^*) + \delta_4. \quad (50)$$

Since $\Lambda(s)$ is Hurwitz, it follows from (50) [using (43b), (28a) and (46)] that

$$\delta_5 \in L_2. \quad (51)$$

The rest of the proof consists in showing that $|v(t) - \mu(t)| \leq |\delta_5(t)|$. To this end let us consider the three cases suggested by (13).

Case 1 $v(t) < \mu_m$. Equation (11b) and (13) imply that $\mu = \mu_m$. Then, as $\mu^*(t) \geq \mu_m$ and c_0, a_0 are positives constants, one has

$$-\frac{c_0}{a_0} \mu + \frac{c_0}{a_0} \mu^* \geq 0.$$

This together with (49) yield $0 > v - \mu \geq \delta_5$, which implies

$$|v(t) - \mu(t)| \leq |\delta_5(t)|. \quad (52)$$

Case 2 $\mu_m \leq v(t) \leq \mu_M$. Equations (11b) and (13) yield $\mu(t) = v(t)$ and, consequently

$$|v(t) - \mu(t)| = 0 \quad (53)$$

Case 3 $v(t) > \mu_M$. Equation (11b) and (13) give that $\mu(t) = \mu_M$ and $v(t) - \mu(t) > 0$. Also, (43a) implies that

$$-\frac{c_0}{a_0} \mu(t) + \frac{c_0}{a_0} \mu^*(t) \leq 0. \quad (54)$$

Using the above observations one gets from (54)

$$0 < v(t) - \mu(t) \leq \delta_5(t) \quad (55)$$

which immediately gives

$$|v(t) - \mu(t)| \leq |\delta_5(t)|. \quad (56)$$

In view of inequalities (52), (53) and (56) it follows that, whatever the value of $v(t)$, the following inequality holds:

$$|v(t) - \mu(t)| \leq |\delta_5(t)|. \quad (57)$$

Then, it follows from (51) that

$$(v(t) - \mu(t)) \in L_2. \quad (58)$$

This establishes Proposition 3 ■

Proposition 3 is in turn used to demonstrate a new crucial result stated in the following proposition.

Proposition 4: Consider the control system of Fig. 3 where the different differential operators are chosen as in Proposition 3. Then, one has $e_y \in L_2$.

Proof: Let us operate $R(s)$ on both sides of (10) and $\Lambda(s)B(s)$ on both sides of (11a). Adding the resulting equations one get

$$e_y = \frac{B(s)}{C(s)} (\mu - \nu) - \frac{A(s)R(s)}{C(s)\Lambda(s)} s y^* \quad (59)$$

where (15) has been used. Since the transfer functions on the right side of (59) are L_2 -stable and $(\mu - \nu) \in L_2, s y^* \in L_2$ it follows that

$$e_y \in L_2. \quad (60)$$

This establishes Proposition 4 ■

Propositions 3 and 4 are key ingredients to get the main results of this paper i.e., the tracking objective and the fact that the control signal stops saturating asymptotically. This is properly formulated in the following theorem.

Theorem 1: Consider the closed-loop system sketched by Fig. 3 and composed of:

- i) the Buck power converter of Fig. 1, represented by (4) and subject to the control saturation constraint (6);
- ii) the nonlinear controller (11a)–(11b), where $y^*(t)$ satisfies conditions (7)–(8) and the operator $C(s)$ satisfies the real positive condition (25).

Then, the tracking error $e_y(t)$ and the deviation $v(t) - \mu(t)$, between the computed and the applied controls, both vanish asymptotically \square

Proof: Let us first show that $v(t) - \mu(t)$ vanishes asymptotically. As $v(t) - \mu(t) \in L_2$, it is sufficient to show that $v(t)$ and $\mu(t)$ are continuous. From (17) one gets

$$v = -G(s)\mu + \frac{S(s)}{\Lambda(s)}y^* + \delta_6 \quad (61)$$

where $\delta_6(t)$ is an exponentially vanishing term due to initial conditions. Using (11b), (61) can be rewritten as follows:

$$v = -G(s)\text{sat}(v) + \frac{S(s)}{\Lambda(s)}y^* + \delta_6. \quad (62)$$

Let (F, H, T) denote any state-space realization of the (strictly proper) transfer function $G(s) = (C(s) - A(s))/A(s)$. Then (62) can be given the following state-space representation:

$$\dot{X} = FX - H\text{sat}(v) \quad (63a)$$

$$v = TX + z \quad (63b)$$

with

$$z = \frac{S(s)}{\Lambda(s)}y^* + \delta_6. \quad (63c)$$

Equations (63a)–(63c) imply

$$\dot{X} = FX - H\text{sat}(TX + z) \stackrel{\text{def}}{=} g(X, z). \quad (64)$$

The function $g(\cdot)$ is Lipschitz because the operator $\text{sat}(\cdot)$ is so. Furthermore, z is piecewise continuous as y^* and δ_6 are so. Then, it follows from the existence theorem (see, e.g., [9, p. 93]) that, for any initial conditions, the differential equation (64) has a unique solution X . Furthermore, X and its derivative \dot{X} are both continuous. Then, it follows from (63b) that the signal v is in turn continuous. The continuity of $\mu = \text{sat}(v)$ is an obvious consequence of the continuity of v . Since the signal $v(t) - \mu(t)$ is continuous and belongs to L_2 it converges to zero (by Barbalat's Lemma, see, e.g [9, p.323]). Finally, it follows from (4) that y is continuous and in view of (10) so is e_y . But this also belongs to L_2 . Then, using again Barbalat's Lemma, the signal e_y in turn vanishes asymptotically. Theorem 1 is established. \blacksquare

Remarks 4.1:

- 1) The fact that $e_y(t)$ converges to zero means that the tracking objective is perfectly reached asymptotically.
- 2) As $(v(t) - \mu(t))$ vanishes asymptotically, the nonlinear controller (11a)–(11b) stops saturating (for sufficiently large t). Consequently, the controller behaves asymptotically like the underlying linear controller (defined by (16)). In practice, the transient period (before the nonlinear controller stops saturating) will last a finite time after each change in the output reference signal.
- 3) Conditions (7), (8), and (25) turned out to be crucial in achieving the above two results. Condition (7) ensures that the tracking objective is achievable with a control signal that takes its steady-state values in the interval $[\mu_m, \mu_M]$. Condition (8) stipulates that the output reference signal should be slowly time-varying. This is necessary to limit

the control activity. Finally, the positive real condition (25) defines a neighborhood of the controlled system poles (i.e., the zeros of $A(s)$) in which those of the closed-loop system (i.e., the zeros of $C(s)$) should be assigned. This means that the closed-loop system is not allowed to be too much faster than the controlled system. Doing so, one limits the involved control activity and avoids long saturation periods.

- 4) In practice, an appropriate choice of the polynomial $C(s)$ consists in using the following general rule:

$$C(s) = A(s + \gamma) \quad \text{with} \quad \gamma \geq 0.$$

It is clear that if $\gamma = 0$ then $C(s) = A(s)$ and condition (25) is satisfied because then one has:

$$\text{Re} \left(\frac{C(j\omega)}{A(j\omega)} \right) = 1 > 0.$$

But, letting $\gamma = 0$ means that the closed-loop system is not allowed to be faster than the controlled system. For the closed-loop to be faster, the parameter γ should be positive because then the zeros of $C(s)$ are obtained left shifting those of $A(s)$ with λ . More specifically, if $A(s)$ has a zero at p_i then $C(s)$ will have a zero at $p_i - \gamma$. The larger γ is the faster the closed-loop system (provided that condition (25) holds).

- 5) $\Lambda(s)$ can also be chosen of the form $\Lambda(s) = A(s + \gamma')$ with $\gamma' \geq \gamma$. This choice is compatible with the fact that $\Lambda(s)$ corresponds (in the state-space interpretation of the controller (11a)–(11b)) to the poles of the observer (see e.g., [11]). Indeed, it is well known that the observer poles (zeros of $\Lambda(s)$) should preferably be faster than the regulation poles (zeros of $C(s)$).
- 6) It is worth noticing that in the usual control practice the input constraints (6) are essentially carried out limiting the time-variation of the output reference signal and slowing down the closed-loop system rapidity. However, these requirements are generally met using heuristic rules (generally try-an-error). One major motivation of the present work is the development of a theoretical background to that usual practice and formulates rigorously accurate requirements on the allowed reference time-variation and the closed-loop rapidity. \square

V. REGULATOR EVALUATION

A. Circuit Simulation

The controlled Buck circuit is simulated by its instantaneous model defined by (1a) with the following characteristics: $R = 1.5 \Omega$, $L = 100 \mu\text{H}$, $C = 560 \mu\text{F}$, $E = 24 \text{ V}$. The switching frequency f_s (used in the PWM process) is fixed bearing in mind several considerations. First, the averaged model (2a) is obtained supposing that f_s is sufficiently large compared to the cut-off frequency of the underlying low-pass filter see e.g., [16]. In the present case, the underlying low-pass filter is a second order whose cut-off frequency is equal to $1/2\pi\sqrt{LC}$. Therefore, the above averaging requirement amounts to let $f_s \gg 1/2\pi\sqrt{LC}$ which numerically yields $f_s \gg 672.55 \text{ Hz}$. Furthermore, in practical applications where highly silent circuits are

required, the switching frequency should be above the audible frequency i.e., $f_s \geq 20$ kHz. On the other hand, to minimize the current ripple in the coil, the switching frequency must be chosen large enough. In the light of the above discussion, it is seen that a suitable choice of the switching frequency can be 200 kHz.

B. Regulator Design

The regulator design is based on the average model equations (2a)–(2b) and the associated transfer function which, in view of the above circuit characteristics, turns out to be the following:

$$\frac{B(s)}{A(s)} = \frac{4.28 \times 10^8}{s^2 + 1190s + 1.78 \times 10^7}. \quad (65)$$

The next step in the regulator design is to solve the Bezout (15) to get the operator $R(s)$ and $S(s)$. In view of (5a), (12), and (16a)–(16c), it can be easily checked that the solution of (15) is defined by

$$\alpha_0 = \lambda_1 + c_1 - a_1 \quad (66a)$$

$$\beta_0 = \frac{1}{b_0} \lambda_0 c_0 \quad (66b)$$

$$\beta_1 = \frac{1}{b_0} [\lambda_0 c_1 + \lambda_1 c_0 - a_0 (\lambda_1 + c_1 - a_1)] \quad (66c)$$

$$\beta_2 = \frac{1}{b_0} [\lambda_0 + c_0 + \lambda_1 c_1 - a_0 - a_1 (\lambda_1 + c_1 - a_1)]. \quad (66d)$$

In the above equations the parameters $\lambda_0 > 0$ and $\lambda_1 > 0$ are freely chosen. But, the parameters c_0 and c_1 should be chosen bearing in mind the positive real condition (25). To this end, one gets from (5a) and (15a)

$$\begin{aligned} \operatorname{Re} \left(\frac{C(j\omega)}{A(j\omega)} \right) &= \frac{(c_0 - \omega^2)(a_0 - \omega^2) + a_1 c_1 \omega^2}{(a_0 - \omega^2)^2 + a_1^2 \omega^2} \\ &= \frac{a_0 c_0 + (a_1 c_1 - a_0 - c_0) \omega^2 + \omega^4}{(a_0 - \omega^2)^2 + a_1^2 \omega^2}. \end{aligned} \quad (67)$$

For the positive real condition (25) to hold, it is sufficient to choose c_0 and c_1 such that: $c_0 > 0$, $c_1 > 0$ and either

$$a_1 c_1 - a_0 - c_0 > 0 \quad (68a)$$

or $a_1 c_1 - a_0 - c_0 < 0$ and $(a_1 c_1 - a_0 - c_0)^2 - 4a_0 c_0 < 0$. But, the last two conditions are in turn equivalent to the following single inequality:

$$a_1 c_1 - a_0 - c_0 + 2\sqrt{a_0 c_0} > 0. \quad (68b)$$

In summary, condition (25) is satisfied if $c_0 > 0$, $c_1 > 0$ and either (68a) or (68b) are fulfilled. Now, following Remark 4.1 (Part 4), the polynomial $C(s)$ may be chosen of the form

$$C(s) = A(s + \gamma) = s^2 + (2\gamma + a_1)s + \gamma^2 + a_1\gamma + a_0 \quad (69a)$$

where $\gamma > 0$ is any real number such that either (68a) or (68b) hold with:

$$c_0 = \gamma^2 + a_1\gamma + a_0; \quad c_1 = 2\gamma + a_1. \quad (69b)$$

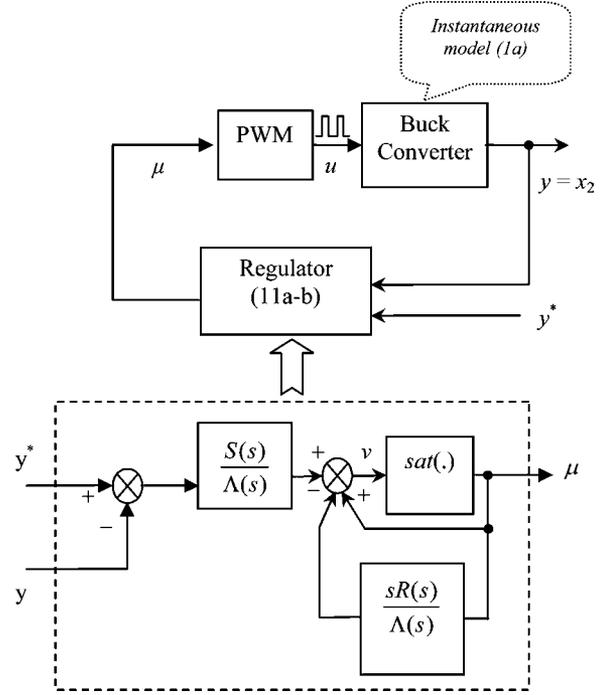


Fig. 7. Experimental setup used for regulator evaluation.

Substituting (69b) in (68a) and (68b), it follows that $\gamma > 0$ must satisfy either

$$-\gamma^2 + a_1\gamma + a_1^2 - 2a_0 > 0 \quad (70a)$$

or

$$-\gamma^2 + a_1\gamma + a_1^2 - 2a_0 + 2\sqrt{a_0(\gamma^2 + a_1\gamma + a_0)} > 0. \quad (70b)$$

C. Experimental Setup

The experimental control setup is described by Fig. 7 and is implemented using the MATLAB/SIMULINK software.

D. Simulation Results

The behavior of the closed-loop control system of (Fig. 7) is illustrated by Figs. 8–16, where the parameters μ_m , μ_M that characterizes the saturation function (13) are given the values 0.05 and 0.95, respectively.

1) *Tracking Performance in Presence of Time-Varying Output Reference Signal (Figs. 8 and 9)*: Bearing in mind (70a)–(70b) and Remark 4.1 (Parts 4 and 5), the following (non-unique) choice is made for the design parameters λ and λ'

$$\lambda = 6.5 \times 10^3, \quad \lambda' = 6 \times 10^4.$$

Then, it follows from (69) that

$$c_0 = 6.78 \times 10^7, \quad c_1 = 1.42 \times 10^4. \quad (71a)$$

Similarly, letting $\Lambda(s) = A(s + \gamma')$ one gets

$$\lambda_0 = 3.69 \times 10^9, \quad \lambda_1 = 1.21 \times 10^5 \quad (71b)$$

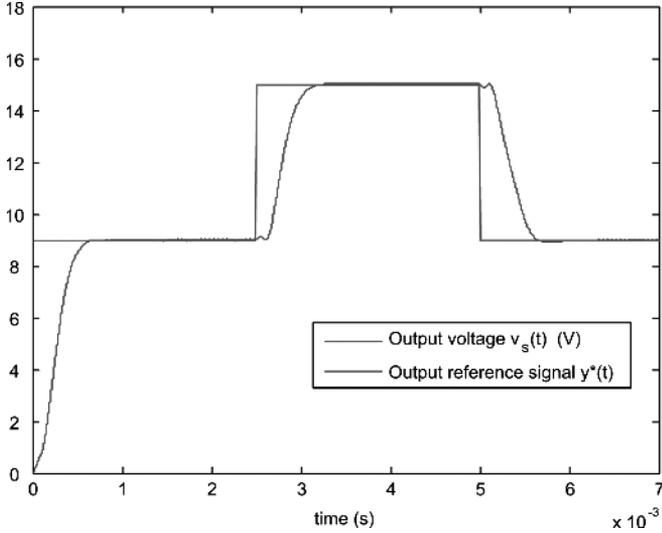


Fig. 8. Tracking behavior of the controller in presence of a time-varying output reference signal $y^*(t)$ switching between 9 and 15 V.

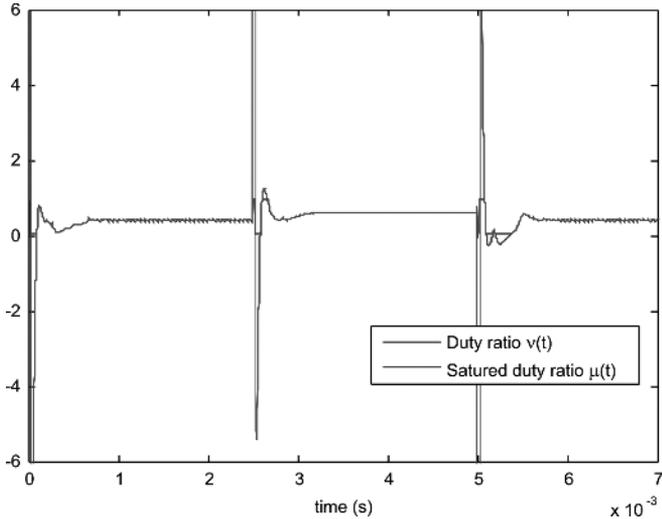


Fig. 9. Comparison between the computed duty ratio $\nu(t)$ and the applied value $\mu(t)$ in presence of a time-varying input reference.

Fig. 8 shows the closed-loop behavior in ideal conditions (no model uncertainty) when the output reference trajectory $y^*(t)$ is a signal switching between 9 V and 15 V. It is clearly seen that the output voltage (v_s) tracks perfectly its reference after transient periods (following the reference signal changes).

Fig. 9 shows the computed and applied control signals, $\nu(t)$ and $\mu(t)$. It is seen that the former tracks the latter, after finite transient periods. That is, the controller stops saturating after these periods, confirming thus Theorem 1.

Remark 5.1: It is readily seen from Fig. 8 that the closed-loop settling time is nearly 0.5 ms. This is much smaller than the settling time of the (open loop) controlled converter. Indeed, step responses of the (average) transfer function (65) show that the converter settling time is nearly equal to 10 ms. That is, the closed loop control system is much more rapid than the (open loop) controlled circuit. It may be possible to further increase the closed-loop rapidity using different polynomials $C(s)$ and $\Lambda(s)$. However, there is a limit imposed by conditions (68a)–(68b).

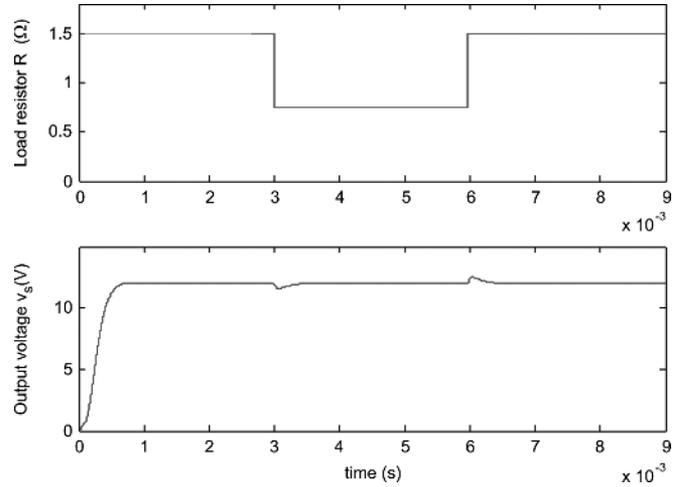


Fig. 10. Robustness of the controller with respect to load resistance changes.

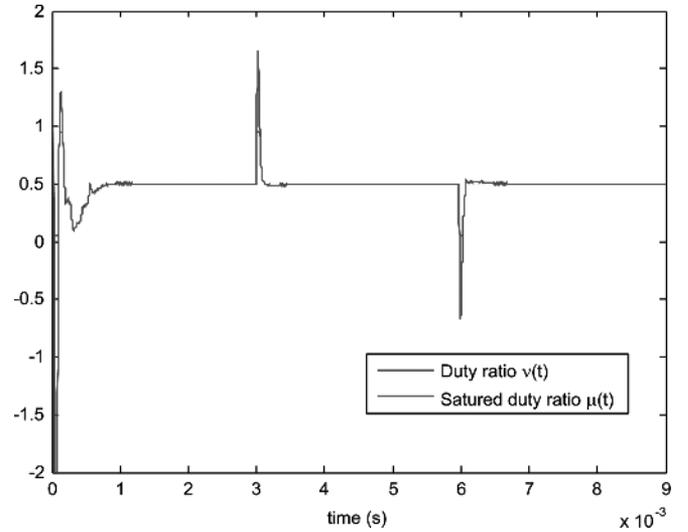


Fig. 11. Comparison between the computed duty ratio $\nu(t)$ and the applied value $\mu(t)$ in presence of load resistance changes.

Indeed, if these are violated the regulator will be too rapid and may generate an excessive control activity. The control limitation (13) will then cause the saturation of the regulator, if not all time, at least over long periods. Consequently, there will be no guarantee that the tracking performances of Theorem 1 will be preserved. Hence, a suitable choice of the design parameters is one which leads to the best compromise between the involved control activity and the achievement of perfect asymptotic tracking. \square

2) *Regulator Sensitivity to Load Uncertainty (Figs. 10 and 11):* In practical situations the load resistance is changing and sometimes its value may not be precisely known. The point is that such features are not accounted for in the control model (2a). Therefore, it is important to see whether the controller (11a)–(11b) is robust with respect to such load uncertainty. Fig. 10 illustrates the response of the closed-loop system to a reference step of 12 V, in presence of a load change of 50% of the nominal value. Specifically, the load switches from 1.5 Ω to 0.75 Ω and vice-versa. While the load resistance is changing,

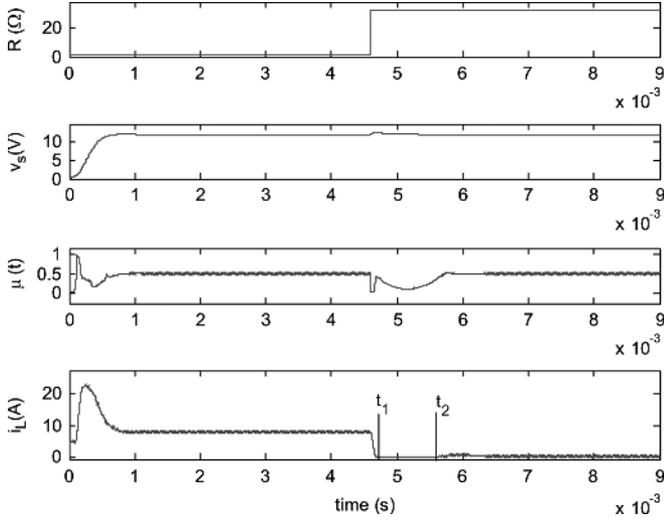


Fig. 12. Regulator behaviour in presence of discontinuous conduction mode.

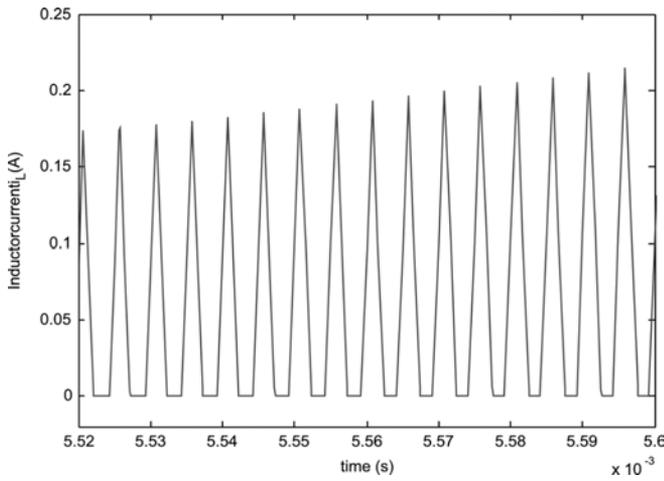


Fig. 13. Inductor current i_L during the discontinuous conduction period.

its value in the controller design is kept constant all the time (it is set to the nominal value 1.5Ω). It is seen from Fig. 10 that the proposed regulator compensates very well the effect of load uncertainty and variations. Fig. 11 shows that the involved control activity remains acceptable and, again, the controller stops saturating after these transient periods.

3) *Regulator Behavior in Presence of Discontinuous Conduction Mode Operation (Figs. 12 and 13)*: In practice, dc-dc converters may enter into a discontinuous conduction mode operation. This means that, in each switching period, the current $\Lambda(s)$ vanishes during a very short time interval. The point is that such phenomenon is not accounted for in the control model (2a), based on in the regulator design. Therefore, it is of interest to check whether the regulator (11a)–(11b) preserves its performances when it faces such converter behavior. To push the converter into discontinuous mode operation, a sudden and drastic change of the load is produced at time instant $t = 4.6 \times 10^{-3}$ s (Fig. 12). This results in a drastic decrease of the current i_L that makes the converter operate in discontinuous mode during an interval $[t_1, t_2]$ following the sudden load change. This is illustrated making a zoom on the inductor current i_L during the

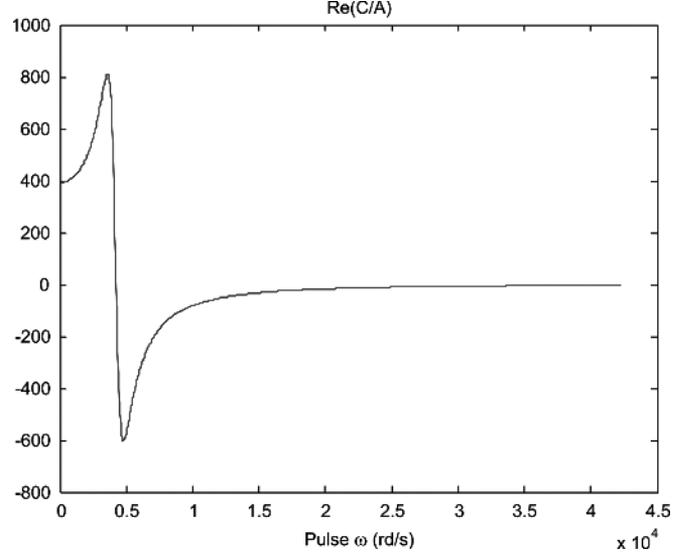


Fig. 14. Real part of $C(j\omega)/A(j\omega)$ versus ω .

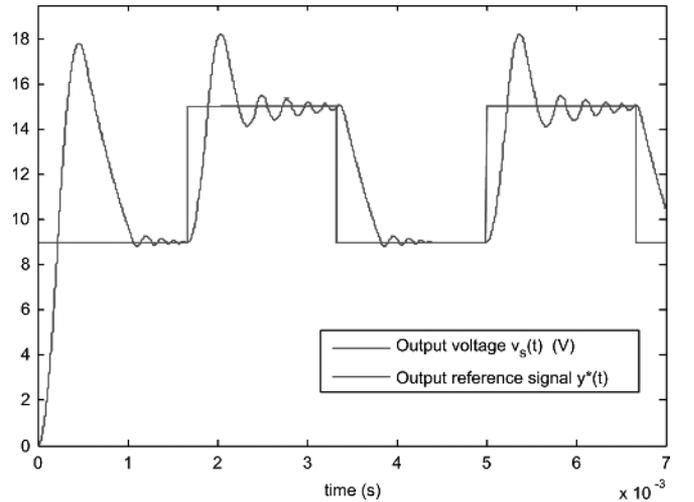


Fig. 15. Deterioration of tracking performances when condition (25) is not satisfied.

interval $[t_1, t_2]$ Fig. 13. Fig. 12 shows that such disturbing phenomenon produces only a temporary and slight deviation of the output voltage from its reference value.

4) *Regulator Tracking Behavior When Condition (25) is Not Satisfied (see Figs. 14–16)*: Here, the values of the coefficients c_0 and c_1 have been chosen in such a way that the real positive condition (25) is not satisfied. Then the regulator performances may deteriorate drastically. Figs. 15 and 16 illustrate such deterioration when

$$c_0 = 7 \times 10^9, \quad c_1 = 10^4. \quad (72)$$

The simulation conditions are similar to those of Sections V-A–C. It is seen from Fig. 14 that $\text{Re}(C(j\omega)/A(j\omega))$ is actually negative for some values of ω . Figs. 15 and 16 show that the regulator behaviour deteriorates significantly (compared to Figs. 8 and 9). In particular, it is observed that: (i) the reference-output tracking performance is not satisfactory; (ii) the controller never stops saturating.

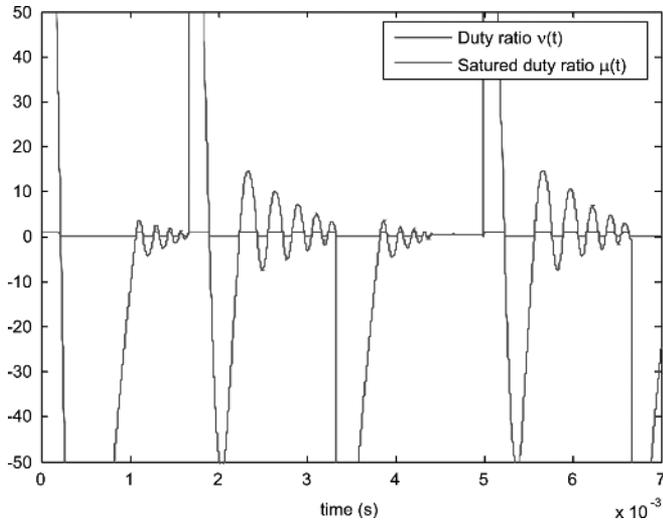


Fig. 16. Control activity when condition (25) is not satisfied: $\nu(t)$ computed value of the duty ratio, $\mu(t)$ the applied value.

VI. CONCLUSION

The control limitation (6) is inherent to all power converters operating according to the PWM principle, whatever the control design approach. In the present paper, such issue has been dealt with, for converters of the Buck type, using the nonlinear controller (11a)–(11b). This involves an internal loop around the saturation function $sat(\cdot)$, see Fig. 3. It contrasts with previous solutions (Fig. 5) that consist in first designing a (linear) controller as if there was no control limitation and then incorporating an isolated limiter, placed between the controller and the controlled system. The point is that the effect of such a limiter is also ignored, in previous works, when analyzing the closed-loop control system.

The control solution developed in the present paper has been analyzed using tools from the absolute stability theory. The key step is to have shown that the resulting (nonlinear) closed loop system (Fig. 3) gives rise to the feedback system of Fig. 6. This is composed of a linear direct path and a nonlinear feedback path of the conic type. Then, L_2 -stability of this feedback system is guaranteed provided that $C(s)$ satisfies the real positive condition (25). In fact, this defines a neighborhood of the controlled systems poles (zeros of $A(s)$) in which the closed-loop poles (zeros of $C(s)$) should be assigned. For the sake of simplicity, the pole placement design technique has been resorted to meet such requirement. But other linear design techniques could as well be used. The L_2 stability of the feedback of Fig. 6 makes it possible to establish the tracking objective for a class of slowly varying reference signals defined by (7)–(8). The proposed regulator shows quite interesting robustness properties especially against uncertain load changes and discontinuous conduction mode operation.

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