Toward alternative bounding techniques for robust goal-oriented error estimation applied to linear problems

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Abstract. *In this work, we propose appealing bounding techniques that enable to derive accurate and strict error bounds on outputs of interest computed from numerical approximation methods such as the finite element method. These techniques are based on Saint-Venant's principle and exploit specific homotheticity properties in order to improve the quality of the bounds computed from the classical bounding technique. The capabilities of the proposed approaches are illustrated through two-dimensional numerical experiments carried out on a linear elasticity problem.*

Keywords: Verification; finite element method; goal-oriented error estimation; constitutive relation error; guaranteed error bounds; Saint-Venant's principle; homotheticity properties.

During the last forty years, numerical simulation has received a growing interest for both academic and industrial applications. This has intensively spurred the development of practical and effective tools allowing to control the quality of approximate solutions obtained through current (and suited to modern computational hardware) numerical methods, such as the finite element method (FEM). Early works tackling this research issue, henceforth denoted *model verification*, appeared in the 1980s and were dedicated to the development of effective methods, known as *global error estimation methods*, providing an assessment of the global measure of the discretization error by means of global energy norms [1, 2]. However, such global methods do not provide a useful and valuable information regarding the error in specific outputs, which constitutes a serious drawback for decision making and certification in engineering activities. Hence, most of research works in model verification currently deal with the development of robust *goal-oriented error estimation methods* designed to achieve strict and high-quality error bounds on specific quantities of interest.

A general and well-established method [3] relies on the use of both extraction techniques and robust global error estimation methods. Extraction (or adjoint-based) techniques consist of expressing the local quantity in a global form by means of extraction operators, called extractors, and leads to the definition of an auxiliary problem, that is the so-called dual or adjoint problem. A sufficiently fine solution of this adjoint problem is required to obtain precise local error bounds. Nonetheless, the classical bounding technique may leads to very coarse error bounds on specific quantities of interest in some particular cases, e.g. when the global estimated errors related to both reference (primal) and adjoint (dual) problems are mainly concentrated in distant regions. Such situations may arise when the zone of interest, i.e. the support of the given quantity of interest, is located far from the predominant contributions of the global estimate associated to the reference problem. The classical bounding technique actually suffers from the use of the Cauchy-Schwarz inequality, which seems to be the root cause of overestimation. This point has motivated the development of novel bounding techniques able to circumvent, or at least alleviate, this major drawback by improving the sharpness and practical relevance of the original computed bounds [4].

In the present work, we revisit the main features of the new improved bounding techniques based on non-classical mathematical tools, such as homotheticity properties [4]. These techniques are carefully tailored to derive specific inequalities between appropriate quantities over two homothetic domains centered around the zone of interest and contained in the whole structure. Such inequalities can then be interpreted as promising alternatives to the well-known Cauchy-Schwarz inequality. These relations are based on Saint-Venant's principle and seem to be limited to solely linear problems. The classical and enhanced techniques can be combined with an intrusive approach (local refinement techniques) or a non-intrusive one (handbook techniques [5]) to get a reliable solution of the adjoint problem.

Several two-dimensional numerical examples are provided with comparative results between conventional and alternative bounding techniques within the linear elasticity framework. Various linear quantities of interest (such as the local average of a stress component, the pointwise value of a displacement component or a stress intensity factor) are considered to illustrate the behavior of the different bounding techniques. Those numerical experiments demonstrate the efficiency of these methods to produce strict and precise bounds on the errors in linear quantities of interest in cases where the classical bounding technique fails, especially when the discretization error related to the reference problem is not concentrated in the local zone of interest. Moreover, the second proposed technique seems to achieve sharper local error estimates than the first one in this case. However, in situations where the zone of interest coincides with the most concentrated error regions associated to the reference problem, the novel bounding techniques do not seem to be capable of sharpening the local error bounds obtained from the classical one. Further investigations are necessary in order to determine the influence of the geometrical features of homothetic domains and associated parameters on the accuracy of the resulting error bounds. Such innovative methods may provide concrete guidelines for future research in the field of robust goal-oriented error estimation methods.

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