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► **To cite this version:**

Emmanuel Friot. Time-domain versus frequency-domain effort weighting in active noise control. 2014.  
hal-01056910v1

**HAL Id: hal-01056910**

**<https://hal.science/hal-01056910v1>**

Preprint submitted on 20 Aug 2014 (v1), last revised 15 Oct 2015 (v2)

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# Time-domain versus frequency-domain effort weighting in active noise control

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## **Abstract**

Active Noise Control aims at reducing the noise at a set of error sensors, but is often designed by minimizing an error index which also includes a weighted penalty on the actuator inputs. In this way the control tends to be more robust and the effort-weighting parameter allows to monitor the maximum voltages which are applied to the control sources. Two similar effort-weighting techniques have been widely implemented in active control studies: optimal control can be computed using Tikhonov regularization in frequency-domain simulations and using the leaky Filtered-reference least mean square algorithm for real-time feed-forward control. This paper introduces the relationship between the two effort-weighting parameters which lead, in the case of a single-tone noise, to exactly the same error index both in the time and in the frequency domain; the best real-time leakage factor can then be computed from frequency-domain optimization. The paper also discusses numerical simulations of a single-channel set-up which show that, with these two related parameters, the control performances are indeed very close except for the case of a control filter with a very short

impulse response when control is slightly more conservative in the time domain than in the frequency-domain simulations.

## 1 Introduction

Loudspeakers or shakers can be destroyed when they are driven by a high voltage. To prevent this risk, when the primary noise to be reduced requires high voltages at the secondary actuators, the active noise control is usually designed so as to minimize an error index which combines a norm of the control signals (the actuator inputs) with a norm of the error signals (the residual noise at the error sensors) (*cf.* [1], [2], [3]). Adjusting a weighting coefficient in the error index allows to optimize the trade-off between control performances and voltage limitation.

From a theoretical point of view, including an effort-weighting parameter in the minimization index also allows to regularizes an ill-posed control problems (e.g. in the cases with more actuators than error sensors) [4]. It is also reported in [1] that effort weighting in active control may enlarge the area where noise is reduced because it makes noise minimization at a discrete set of sensors match better an underlying continuous-space global control problem. Last, but not least, leakage has the virtue of increasing the robustness of real-time adaptive algorithm to errors in the secondary path estimate which is required for on-line computations (see [2], section 3.4.7). Therefore, besides the monitoring of the control inputs in the case of a loud primary noise, it is generally a good idea to include an effort weighting parameter in the design of an active control set-up.

Most active control studies address the weighting of the control inputs using two different techniques:

1. at the design stage, the control signals and performance are usually computed off-line in the frequency domain. In this case the actuator inputs can be limited by minimizing, at each frequency, a quadratic index which weights the norm of the sensor signal vector and the norm of the actuator input vector (see [2], section 4.2.6). The optimal control can then be determined in practice by performing the regularized inversion of the theoretical or the measured actuator-to-sensor response matrix [4].
2. For effective real-time control, the actuator inputs are usually computed on-line in the time domain using an adaptive algorithm such as the Filtered Reference Least Mean Square algorithm (FxLMS). The FxLMS constantly adapts the coefficients of transverse Finite Impulse Response (FIR) control filters which generate the control signals. In this case, effort weighting is easily taken into account by including the norm of the FIR filter coefficients in the error index, which leads to the introduction of a so-called leakage factor (*cf.* [2], section 3.4.7) in the FxLMS updating formula.

These approaches are similar since they both include a quadratic vector norm in the error index which must be minimized by the active control. In the time domain the leakage factor drives the trade-off between control inputs and performance, as does the regularization parameter in the frequency domain. In the case of a single-tone noise, these approaches are exactly equivalent because a change in magnitude in the control

filter coefficients leads to the same change in the control signals. However no obvious relationship had been established between the two approaches even though it would be interesting to be able to deduce the leakage factor that has to be implemented in real-time from preliminary frequency-domain simulations. Indeed a trial-and-error procedure can be used in the frequency domain simulations to determine which effort weighting parameter is best, but optimizing the leakage factor in the time domain cannot be performed without taking the risk of damaging the actuators.

This paper gives, in the case of a single-tone noise, the theoretical relationship between the regularization parameter, in the frequency domain, and the leakage factor, in the time domain, which lead to the minimization of the same error index. Using this relationship, the leakage factor which has to be used on-line can be computed from a regularization parameter selected using frequency-domain simulations.

The paper also presents numerical simulations of FxLMS control for a simple acoustic set-up in which the leakage coefficient has been computed from the frequency-domain regularization parameter. The simulations show that the real-time control leads to slightly lower control signals and performance than predicted in the frequency domain. The gap between the respective control results narrows as the length of the control FIR is increased.

In section 2 of the paper, the usual two techniques for effort weighting in active noise control are briefly recalled. Notations are introduced for the subsequent derivation of the correspondance between the two techniques. The relationship between the regularization parameter and the leakage factor is derived in section 3. Section 4 presents

numerical simulations of a simple control set-up which show that, using corresponding minimization indexes, control with a leaky FxLMS algorithm is slightly more conservative regarding control input limitations than the direct frequency domain optimization.

## 2 Weighted error indexes and optimal control

Figure 1 sketches a typical feedforward active noise control set-up. Reference signal  $x$ , which is correlated with the which active control has to reduce, drives a set of linear filters whose  $\mathbf{w}(\omega)$  is the vector of Frequency Response Functions (FRFs) at angular frequency  $\omega$ .  $\mathbf{u}(\omega)$  is the vector of the control filter outputs, which are the actuators inputs, and  $\mathbf{e}$  is the vector of the noise at the set of error sensors.  $\mathbf{e}_p$  is the so-called primary noise at the sensors, i.e. the noise without control. Matrix  $\mathbf{H}(\omega)$  denotes the so-called secondary path matrix of FRFs between the actuator inputs and the error sensors at angular frequency  $\omega$ . In a similar way, vector  $\mathbf{f}(\omega)$  denotes the primary path between the reference signal  $x$  and the primary noise  $\mathbf{e}_p$ .

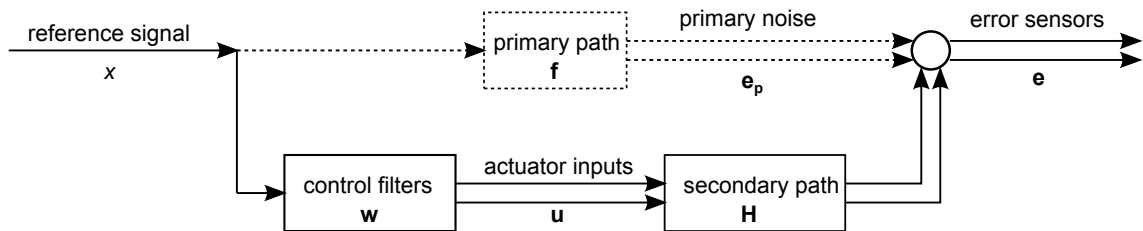


Figure 1: A feedforward active noise control set-up

## 2.1 Optimal control in the frequency domain

Active control at angular frequency  $\omega$  can easily take effort weighting into account by using the minimization index (see [2], section 4.2):

$$J_\omega = \|\mathbf{e}(\omega)\|^2 + \gamma^2 \|\mathbf{u}(\omega)\|^2 \quad (1)$$

where  $\|\bullet\|$  is the usual vector 2-norm and  $\gamma^2$  is the effort-weighting parameter. Index 1 can be expanded as:

$$J_\omega = \|\mathbf{H}(\omega)\mathbf{u}(\omega) + \mathbf{e}_p(\omega)\|^2 + \gamma^2 \|\mathbf{u}(\omega)\|^2 \quad (2)$$

The vector of input signals which minimizes this index is then given by (*cf.* [4]):

$$\hat{\mathbf{u}}(\omega) = \mathbf{H}^\dagger \mathbf{e}_p(\omega) \quad (3)$$

where  $\mathbf{H}^\dagger$  is the so-called Tikhonov-regularized pseudo-inverse of matrix  $\mathbf{H}(\omega)$ .  $\mathbf{H}^\dagger$  can be computed by inverting matrix  $\mathbf{H}(\omega)$  through Singular Value Decomposition and by substituting  $\frac{\sigma_i}{\sigma_i^2 + \gamma^2}$  to every singular value  $\frac{1}{\sigma_i}$  of the matrix  $\mathbf{H}$  inverse. The Matlab<sup>®</sup> function *pinv* directly performs such a regularized inversion.

## 2.2 Optimal control in the time-domain

Following [2], section 3.4.7, effort weighting can be taken into account in FxLMS control by minimizing the error index:

$$J_n = \|\mathbf{e}(n)\|_n^2 + \nu \|\mathbf{w}(n)\|_n^2 \quad (4)$$

where

- $\mathbf{e}(n)$  is the vector of the instantaneous error signals at discrete time  $n$  and  $\|\mathbf{e}(n)\|_t$  its mean square value over time,
- $\mathbf{w}(n)$  is the stacked vector of all the control FIR at time  $n$ ,
- $\nu$  is an arbitrary effort weighting coefficient.

The filter responses which minimize index  $J_t$  can be computed using the adaptive form:

$$\mathbf{w}(n+1) = (1 - \nu\beta)\mathbf{w}(n) - \beta\mathbf{R}(n)e(n) \quad (5)$$

where

- $\beta$  is a proper convergence coefficient
- $\mathbf{R}(n)$  is the so-called *filtered reference*, filtering of the reference signal  $x(n)$  through FIR estimates of the secondary path.

The updating formula 5 is constitutive of the so-called leaky FxLMS algorithm.

### 3 Linking the effort-weighting parameters

In this section, the error indexes 1 and 4 are manipulated in order to determine which weighting parameters  $\gamma$  and  $\nu$  lead to the same minimization problem.

At first, because of Parseval's theorem, the error signal contributions have the same form in the two indexes as soon as the FxLMS has converged to the steady-state optimal control. Therefore the minimization index in the time domain can be rewritten:

$$J_n = \|\mathbf{e}(\omega)\|^2 + \nu\|\mathbf{w}(n)\|_n^2 = \|\mathbf{e}(\omega)\|^2 + \nu\frac{\|\mathbf{w}(n)\|_n^2}{\|\mathbf{u}(\omega)\|^2}\|\mathbf{u}(\omega)\|^2 \quad (6)$$



Comparing equations 4 and 6 shows that both minization indexes will lead to the same solution if and only if:

$$\nu = \frac{\gamma^2 \|\hat{\mathbf{u}}(\omega)\|^2}{\|\hat{\mathbf{w}}\|^2} \quad (7)$$

where  $\hat{\mathbf{w}}$  is the stacked vector of the *optimal* control filters FIRs.

Equation 7 formally connects the two effort weighting factor, but the computation of optimal control in the frequency domain does not directly provide the  $\|\hat{\mathbf{w}}\|$  factor in it. However in the frequency domain the FRFs  $\hat{\mathbf{w}}(\omega)$  of the optimal filters are easy to compute because the optimal control inputs  $\hat{\mathbf{u}}$  are the filtering of the reference signal:

$$\|\hat{\mathbf{w}}(\omega)\| = \frac{\|\hat{\mathbf{u}}(\omega)\|}{\sqrt{(2)}\|x(n)\|_n} \quad (8)$$

(take notice of the  $\sqrt{(2)}$  factor in this formula which arises from the definition of  $\|x(n)\|_n$  as the quadratic mean square value of the reference signal  $x(n)$ ).

The FIRs of the optimal control filters can now be determined from the FRFs by using the fact that, amongst all the FIR filters with frequency responses  $\hat{\mathbf{w}}(\omega)$ , the optimal control FIR vector has minimum vector 2-norm because it minimizes index 4.

In the single actuator case, optimal FRF  $\hat{\mathbf{w}}(\omega)$  is a single complex number and  $\hat{\mathbf{w}}$  is as a single FIR  $[\hat{w}_0 \ \hat{w}_1 \ \dots \ \hat{w}_{N-1}]^t$ , where  $N$  is the FIR length, such as:

$$\hat{\mathbf{w}}(\omega) = \hat{\mathbf{w}}_0 + \hat{w}_1 e^{-i\omega/f_s} + \dots + \hat{w}_{N-1} e^{-i(N-1)\omega/f_s} = [1 \ e^{-i\omega/f_s} \ \dots \ e^{-i(N-1)\omega/f_s}]^t \hat{\mathbf{w}} \quad (9)$$

where  $f_s$  is the sampling frequency of the real-time control. Vector  $\hat{\mathbf{w}}$  is therefore the least square solution of the linear equation:

$$\begin{bmatrix} \Re(\hat{\mathbf{w}}(\omega)) \\ \Im(\hat{\mathbf{w}}(\omega)) \end{bmatrix} = \begin{bmatrix} 1 & \dots & \cos((N-1)\omega/f_s) \\ 0 & \dots & \sin((N-1)\omega/f_s) \end{bmatrix}^t \hat{\mathbf{w}} \quad (10)$$

whose solution e.g. in Matlab<sup>®</sup> code is simply given by a matrix division using symbol  $\backslash$ .

It must be noted that the FIRs  $\hat{\mathbf{w}}$  and therefore the computed leakage factor  $\nu$  depend strongly, but not in an simple way, on the length  $N$  of the control filter FIRs. Therefore in real-time a change in the control filter length requires a change in the leakage factor if effort weighting has to be maintained at the same level.

In the multiple-actuator case, the stacked vector  $\hat{\mathbf{w}}$  of optimal FIRs is obtained in the same way by stacking the optimal control FRFs in the left hand side of equation 10.

In summary, the leakage factor corresponding to a given regularization parameter  $\gamma$  can be computed with the following procedure:

- compute the optimal control inputs  $\hat{u}(\omega)$  using a pseudo-inversion, as discussed in section 2.1
- compute the FRFs of the optimal filters  $\hat{\mathbf{w}}(\omega)$  using equation 9
- compute the FIRs of the optimal filters  $\hat{\mathbf{w}}$  using least-square inversion (equation 10)
- compute the leakage factor from equation 7:

$$\nu = \frac{\gamma^2 \|\hat{\mathbf{u}}(\omega)\|^2}{\|\hat{\mathbf{w}}\|^2} \quad (11)$$

## 4 Time-domain vs. frequency-domain simulations

The above procedure was implemented for the numerical simulation of a very simple single-channel control case.

The primary noise was a 300Hz pure-tone  $e_p(t) = \pi \sin(2\pi 300t)$ . The reference signal was taken as  $x = 3\sqrt{2} \sin(2\pi 300t)$ , which was perfectly correlated with the primary noise. FxLMS was simulated at the sampling frequency  $f_s = 1000Hz$ . The secondary path was chosen as a pure two-sample delay with FIR  $\mathbf{h} = [0 \ 0 \ 1]$ .

For the results given below, the FxLMS convergence coefficient ( $\beta$  in equation 5), normalized by the control filter FRF length and the mean square value of the reference (*cf.* [2], section 3.4.4), was assigned to 0.4. The mean-square value of the error and control signals were computed over 2 seconds after running the FxLMS for 8 seconds, which was much longer than the apparent control convergence time.

Table 1 shows the control performances for a various set of regularization parameters  $\gamma$  and of control FIR lengths  $N$ .  $u_\omega$  and  $u_n$  are the mean-square value of the control signal, respectively in the frequency and in the time domain, and  $\mathcal{A}_\omega$  and  $\mathcal{A}_n$  are the corresponding noise attenuation.

At first table 1 shows that, as expected, the regularization factor and the leakage factor monitor the control signal; the larger the effort weighting, the smaller the control signal and the noise attenuation.

Secondly table 1 shows that the regularization parameter  $\gamma$  and the leakage factor  $\nu$ , computed as discussed in section 3, lead to a control signal magnitude and a noise attenuation that are very close. This result confirms, if necessary, that the procedure discussed in section 3 gives effort-weighting parameters that lead to the minimization problem. However, the control in the time-domain is slightly more conservative when the control filter FIR is very short length. This is probably due to the lack of accuracy

$\gamma$	$n_w$	$u_{freq}$	$u_{temp}$	$\mathcal{A}_\omega$ (dB)	$\mathcal{A}_n$ (dB)
0	2	0.7405	0.7407	$+\infty$	223
0.05	2	0.7386	0.7099	52.1	27.2
0.05	10	0.7386	0.7385	52.1	50.7
0.05	50	0.7386	0.7385	52.1	52.1
0.2	2	0.7120	0.4357	28.3	7.20
0.2	10	0.712	0.7076	28.3	27.0
0.5	2	0.5924	0.1818	14.0	1.1
0.5	10	0.5924	0.5666	14.0	12.6
1	10	0.3702	0.2920	6.0	4.3

of the FxLMS when the selected control filter FIR length is too short.

## 5 Concluding remarks

In the case of a single-tone primary noise, Tykhonov regularization in the frequency domain and leaky-FxLMS active control are exactly equivalent. A simple numerical procedure has been introduced in this paper to compute the leakage factor corresponding to a given regularization parameter; simple numerical simulations have confirmed that the two effort-weighting approaches lead to almost the same control results.

In the case of a primary noise involving several tones, the convergence of the FxLMS algorithm can be considered independently at each frequency. Therefore one leakage

factor  $\nu(\omega_k)$  could be computed for each frequency from the regularization parameters  $\gamma(\omega_k)$ , but, because equation 10 is frequency dependent, a single global leakage factor cannot be computed even if the frequency-domain parameter is the same for all frequencies. In the case of a broadband noise, control cannot be computed independently at each frequency because of the causality constraint (*cf.* [2]); in this case even optimal control in the frequency domain cannot be computed as discussed in section 2.1.

Finally, although this paper has focused on active noise control, real-time effort-weighting is of interest for other problems in Acoustics. For example the so-called Adaptive Wave-Field Synthesis, which has been introduced for 3D sound field generation, involves in the on-line minimization of an index including a quadratic penalty [5]. In this case the time-domain regularization parameter can also be computed from the frequency-domain parameter, as discussed in section 2, in the case of single-tone synthesis.

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