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## FREE WAVE PROPAGATION IN TWO-DIMENSIONAL PERIODIC BEAM GRILLAGE

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Two-dimensional periodic structures are widely applied in the engineering domain, such as the orthogonally rib-stiffened panels used in the fuselage of an aircraft or the cellular structures like honeycombs used in sandwich construction. The dynamic behaviour of these structures has drawn a lot of attention among researchers for the last decades. In this paper, A beam grillage is studied as a particular example of a two-dimensional periodic structure. A single unit cell is modelled by the finite element method and periodic structure theory is employed to study the whole beam grillage. The method is known as the Wave Finite Element method (WFEM), which is usually used to study the one-dimensional waveguide or two-dimensional homogeneous waveguide. This work has extended the WFEM to the two-dimensional periodic beam grillage to calculate the wave dispersion characteristics. It also extended the model reduction based on Craig-Bampton method in WFEM to two-dimensional periodic structures. The simulation has been validated by the theoretical result in the literature. It is also compared with the dispersion relation by using the Fourier transformation of the displacement field under harmonic excitation. The WFEM shows a good agreement with little computational cost. The fixed frequency dispersion relation deduced from WFEM is used to predict the direction of wave propagation and to investigate the existence of direction where waves cannot propagate in the beam grillage.

### 1. Introduction

Periodic structures which can be obtained by repeating a single unit cell occur largely in engineering domain. For example an orthogonally stiffened plate or shell used in the fuselage of an aircraft or a ship hull. In this case the structure is called two-dimensional periodic because the whole structure can be formed by repeating the unit cell in two directions. This paper use the WFEM to analyse the two-dimensional periodic structure, in particular a two-dimensional periodic beam grillage. The method is usually used to model the one-dimensional waveguide or two-dimensional homogeneous waveguide [1, 2]. In this work, WFEM has been extended to study the two-dimensional periodic beam grillage. Similar to the one-dimensional waveguide, the internal nodes in an unit cell of two-dimensional periodic structures can be numerous, so it is interesting to apply the model reduction as

it is used in one-dimensional periodic case [3]. The result of WFEM on two-dimensional structures (WFE2DM) can be presented with the fixed frequency contour of dispersion relation, which can be used to determine the direction of wave propagation in the considered structures and in the investigation of the existence of directions within the structure, where waves do not propagate. It is called “wave beaming effect” where local vibrations are of primary concern [4, 5, 6]. The wave beaming effect of beam grillage is studied in this paper by both the dispersion relation and the displacement of an finite beam grillage under excitation.

## 2. Model reduction on the unit cell based on Craig-Bampton method

The structure to be modelled is periodic in both the  $x$  and  $y$  directions, and properties can also vary through its thickness in the  $z$  direction as in the laminate structures. The unit cell which forms the structures is discretized by the Finite Element (FE) method. The mass and stiffness matrices  $\mathbf{M}$  and  $\mathbf{K}$  of the unit cell are found using commercial FE package. All the boundary nodes are denoted  $\mathbf{q}_{bd}$ ,  $\mathbf{q}_I$  represents the internal DOFs. So the nodal DOFs of the unit cell become  $((\mathbf{q}_{bd})^T(\mathbf{q}_I)^T)^T$ . For free wave propagation, no external forces act on the structures apart from those on its boundaries from adjacent unit cells. So the equation of motion of the unit cell, assuming time-harmonic behaviour and neglecting damping, becomes:

$$[\mathbf{K} - \omega^2\mathbf{M}] \begin{pmatrix} \mathbf{q}_{bd} \\ \mathbf{q}_I \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ \mathbf{0} \end{pmatrix} \quad (1)$$

Matrix  $[\mathbf{K} - \omega^2\mathbf{M}]$  is the dynamic stiffness matrix of the unit cell, with the presence of the damping, it becomes complex and can be expressed by  $[\mathbf{K} + i\mathbf{K}' - \omega^2\mathbf{M}]$ . To model the dynamic behaviour in high frequency, the unit cell is often discretized with numerous number of internal DOFs. A reduced model of the unit cell is necessary so as to gain the computation time. In this approach, Craig-Bampton hybrid coordinates  $((\mathbf{q}_{bd})^T(\mathbf{P}_c)^T)^T$  are related to the physical coordinates  $((\mathbf{q}_{bd})^T(\mathbf{q}_I)^T)^T$  as follows:

$$\begin{pmatrix} \mathbf{q}_{bd} \\ \mathbf{q}_I \end{pmatrix} = \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{\Psi}_{bd} & \mathbf{\Psi}_c \end{bmatrix} \begin{pmatrix} \mathbf{q}_{bd} \\ \mathbf{P}_c \end{pmatrix} \quad (2)$$

With  $[\mathbf{\Psi}_{bd}]$  represent the static boundary modes,  $[\mathbf{\Psi}_c]$  represent the fixed base mode shapes. The approach has been used in one-dimensional WFEM, which shows that if the fixed base mode shapes under natural frequency lower than three time the maximum investigated frequency are retained in the modal basis, the WFEM on reduced model is able to give precise result [3]. The reduced model with less internal DOFs is easier to be applied dynamic condensation. The Eq. (1) can be represented by reduced dynamic stiffness matrix with only the boundary DOFs:

$$\mathbf{D}\mathbf{q}_{bd} = \mathbf{f} \quad (3)$$

Once the reduced dynamic stiffness matrix of an unit cell is obtained, the WFE2DM can be applied to study the wave propagation characteristics of the periodic structures.

## 3. Wave Finite Element method applied on two-dimensional periodic beam grillage

A beam grillage is a simple form of continuum two-dimensional periodic structure. This section explains how WFE2DM can be applied to study the beam grillage.

### 3.1 WFE2DM on periodic beam grillage

The beam grillage consists of beams in two directions which intersect on the joints. An unit cell can be used to represent the beam grillage. The one used in this paper is as shown in Fig. 1, which contains two beams and 1/4 mass at the joints and 1/2 mass at the other end of the beams. So the whole beam grillage with  $(2*1/4+1/2)=1$  mass at the joints can be obtained by repeating this unit cell. The unit cell consists of  $[q_1, q_2, q_3, q_I]$ , so after the dynamic condensation of the internal DOFs, the

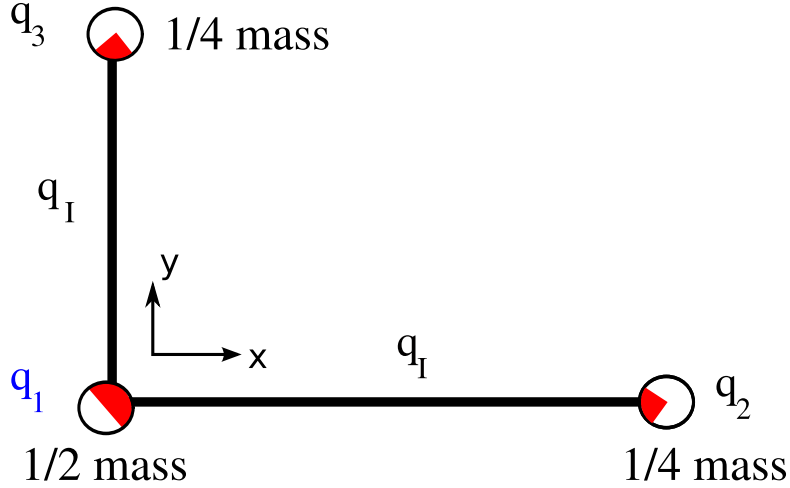


Figure 1. Unit cell used in WFE2DM

equilibrium equation becomes:

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \quad (4)$$

Using the periodic structure theory, the displacement in the unit cell can be related as follows:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \mathbf{I}_s \\ \lambda_x \mathbf{I}_s \\ \lambda_y \mathbf{I}_s \end{pmatrix} q_1 = \mathbf{\Lambda}_R q_1 \quad (5)$$

The relation between  $\lambda$  and wavenumber  $k$  is as follows:

$$\lambda_x = e^{-ik_x L_x}, \lambda_y = e^{-ik_y L_y} \quad (6)$$

The sum of the nodal forces of all elements connected to the nodes 1 is zero, so

$$\begin{pmatrix} \mathbf{I}_s & \lambda_x^{-1} \mathbf{I}_s & \lambda_y^{-1} \mathbf{I}_s \end{pmatrix} \mathbf{f}_1 = \mathbf{\Lambda}_L \mathbf{f}_1 = \mathbf{0} \quad (7)$$

So the equilibrium equation becomes:

$$\mathbf{\Lambda}_L * \mathbf{D} * \mathbf{\Lambda}_R = \mathbf{0} \quad (8)$$

Suppose the frequency  $\omega$  and one between  $(\lambda_x, \lambda_y)$  is known, for example  $\lambda_y$ . This may represent the situation where a known wave is incident on a straight boundary so that  $\lambda_y$  is given and all possible solutions for  $\lambda_x$  are sought which correspond to evanescent or propagation waves.  $\lambda_x$  can be calculated by solving the following eigenvalue problem

$$\frac{1}{\lambda_x} (\mathbf{A}_2 \lambda_x^2 + \mathbf{A}_1 \lambda_x + \mathbf{A}_0) = \mathbf{0} \quad (9)$$

with

$$\mathbf{A}_0 = \mathbf{D}_{23}\lambda_y + \mathbf{D}_{21} \quad (10a)$$

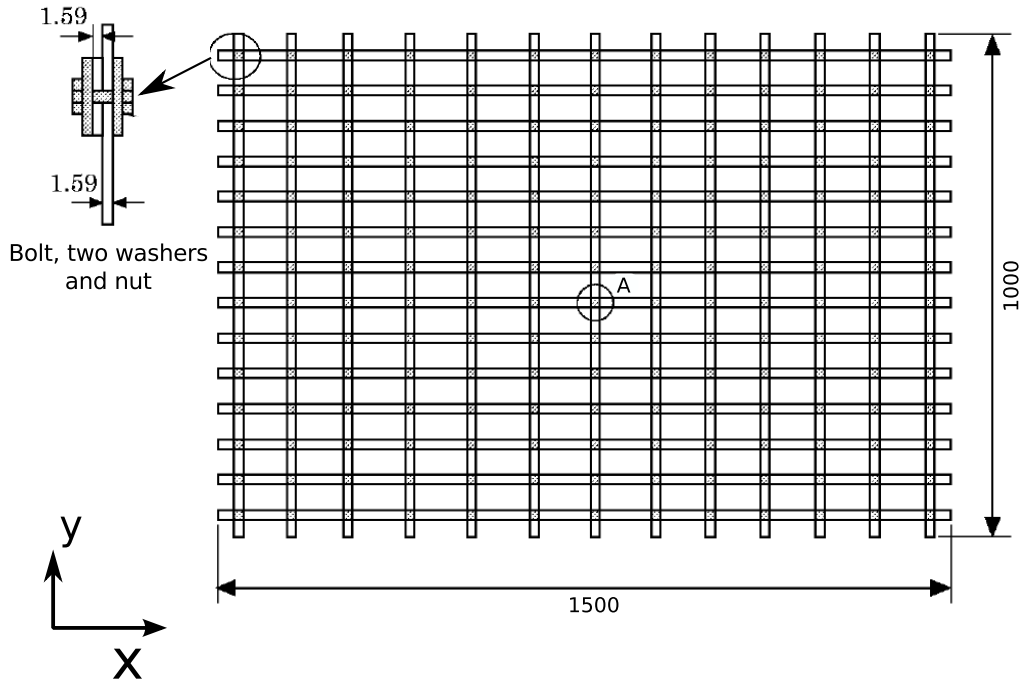
$$\mathbf{A}_1 = \mathbf{D}_{11} + \mathbf{D}_{22} + \mathbf{D}_{33} + \mathbf{D}_{13}\lambda_y + \mathbf{D}_{31}\lambda_y^{-1} \quad (10b)$$

$$\mathbf{A}_2 = \mathbf{D}_{12} + \mathbf{D}_{32}\lambda_y^{-1} \quad (10c)$$

Then the quadratic eigenvalue problem on  $\lambda_x$  can be solved, which contain  $2s$  solutions,  $s$  is the number of the DOFs of  $\mathbf{q}_1$ . The dispersion relation is presented as an iso-frequency contour in coordinate system  $(k_x, k_y)$ , then the Poynting vector (i.e., the energy flow vector) is normal to the contour[7].

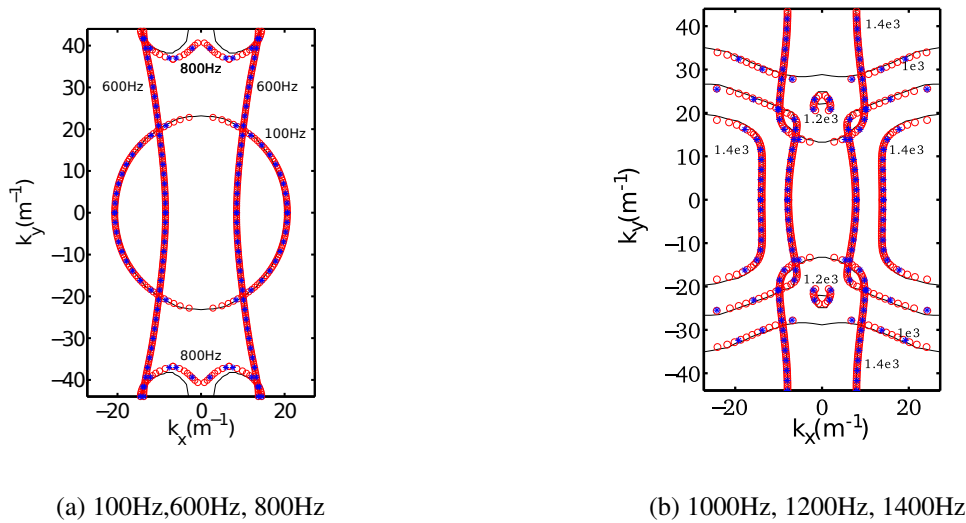
### 3.2 Validation of the WFE2DM by the result in the reference

Langley and Bardell have done the experimental test on a beam grillage constructed from strips of aluminium with bolted joints[4]. The same structure is used here in order to compare the results by WFE2DM with his results. The periodic grillage structure is constructed from an orthogonal array of aluminium strips as shown in 2. The strips were of rectangular cross-section with  $19.06 \text{ mm} \times 1.59 \text{ mm}$ . At the crossing points, the joints are composed of a set of washers, nut and bolt, which is of  $6 \times 10^{-3} \text{ kg}$ . The joints are taken account as the mass point.



**Figure 2.** A schematic of the experimental structure by [4]. The excitation is at position (A). All dimensions are in mm.

Unit cell used in WFE2DM is defined in the precedent subsection. In the unit cell, the mass point is modelled by Mass21 and the beam is modelled by Beam188 in ANSYS. The latter is a three-dimensional two-node Timoshenko element, with 6 DOFs per node. Since the cross-section of the element is rectangular, the movements of the 6 DOFs are decoupled. The present analysis is restricted to the out-of-plane bending of the beam components so the DOFs related to in-plane movement are fixed to zero, subsequently only  $rot_x$ ,  $rot_y$  and  $uz$  are conserved in each node. To assure the convergence of the problem, the two beams are both meshed with 100 elements. So the  $\mathbf{q}_1$ ,  $\mathbf{q}_2$  as well as  $\mathbf{q}_3$  is of 3 DOFs, the internal DOFs is of 594 DOFs. In the reduced model, only 8 internal modes which represent the natural frequency under 4500 Hz are taken account in. The size



(a) 100Hz,600Hz, 800Hz

(b) 1000Hz, 1200Hz, 1400Hz

**Figure 3.** Fixed frequency contours of dispersion relation, – Langley, \* WFE2DM on completed model, o WFE2DM on reduced model

of the reduced model is about 1% of the full model. The computation time decreases a lot to apply the dynamic condensation on the reduced model.

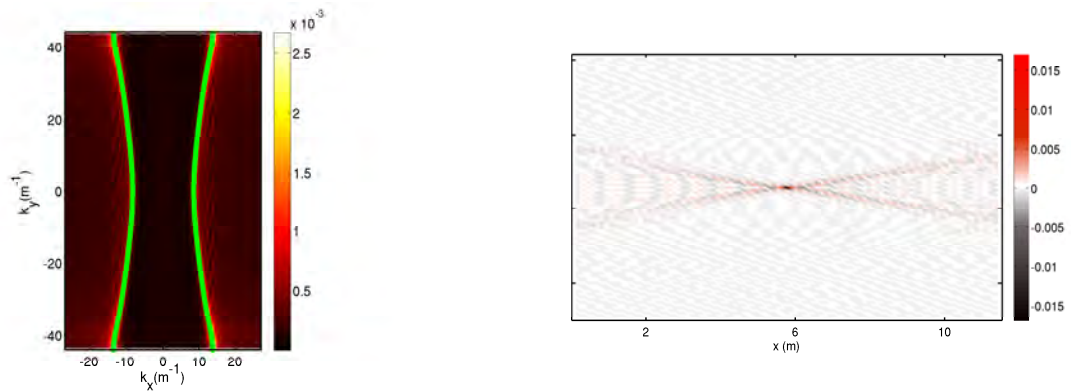
The dispersion relations at 100Hz, 600Hz, 800Hz, 1000Hz, 1200Hz, 1400Hz are given in the Fig. 3. The dispersion relation obtained by the proposed WFE2DM conforms with the theoretic result in the reference [4]. The method applied on the reduced model can predict the dispersion relation with less computation time.

### 3.3 The wave beaming effects in beam grillage

The dispersion relation can also be calculated by the two-dimensional Discrete spatial Fourier Transformation (DFT2D). Here the Finite Element model based on ANSYS which consists of 100 unit cells in x direction and 100 unit cells in y direction. An harmonic excitation is applied in the center of the grillage, then the out of plane displacement is calculated and then DFT2D is applied to obtain the dispersion relation at the frequency of the excitation. At 100Hz, the normal directions of the fixed frequency contour contain all the directions. The conclusion is confirmed by the displacement field. However at higher frequency, such as 600Hz, 800Hz and 1000Hz, as shown in the Fig.4, Fig.5 and Fig.6 respectively, the directivity of the propagation can be observed. It exists some “dead zones” for directions. The normal directions of the fixed frequency contours are restrained to certain angle. So is the displacement field under harmonic excitation at these frequencies.

## 4. Conclusion

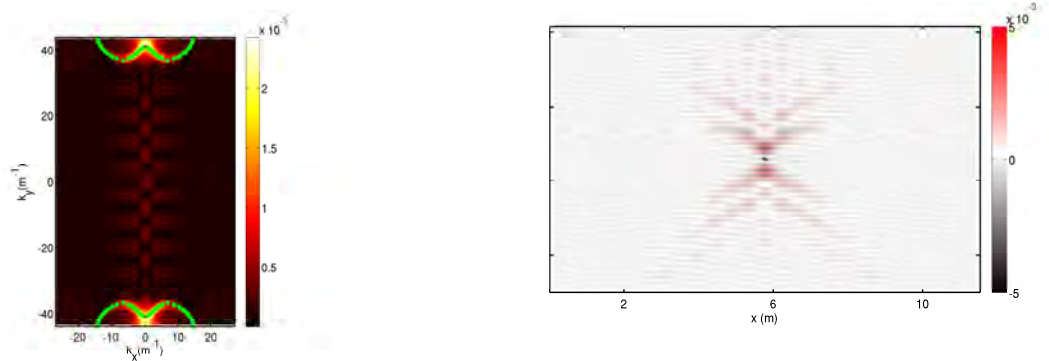
In this paper we propose the WFE2DM to study a two-dimensional periodic structures such as beam grillage. The model reduction based on Craig-Bampton is applied to decrease computation time and proved to give as accurate result as the full model. The fixed frequency contours of dispersion relation are obtained by the WFE2DM and are validated by the theoretical result existing in the reference. The wave beam effects in the beam grillage are studied, by using the fixed frequency contour of dispersion relation and by numerical simulation of the finite beam grillage. There is a good correlation between the results obtained by the two methods, it exists some dead directions of the propagation while other directions are privileged. Some experimental studies on the two-dimensional periodic structure such as beam grillage will be considered in the future work.



(a) Fixed frequency contour of dispersion relation

(b) Displacement field of the beam grillage

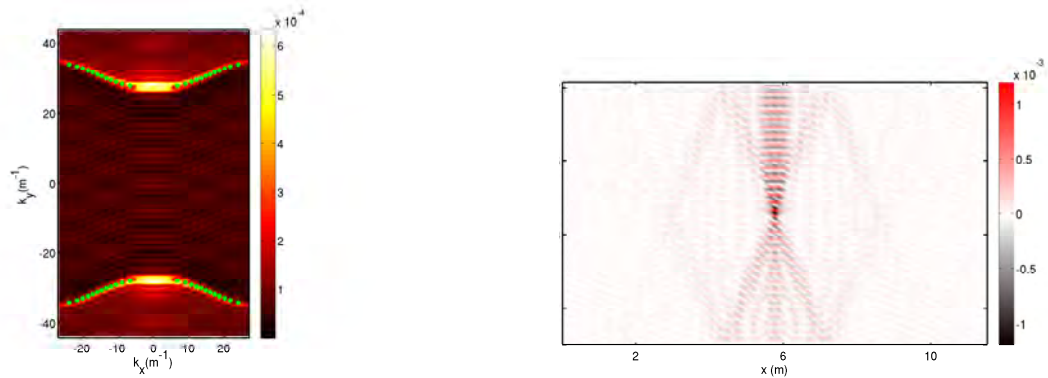
**Figure 4.** At 600Hz (a) Phase constant surface by the DFT2D, the line with ‘o’ indicates the result by WFE2DM on reduced model. (b) The displacement field of the beam grillage with 100 periods in each direction



(a) Fixed frequency contour of dispersion relation

(b) Displacement field of the beam grillage

**Figure 5.** At 800Hz (a) Phase constant surface by the DFT2D, the line with ‘o’ indicates the result by WFE2DM on reduced model. (b) The displacement field of the beam grillage with 100 periods in each direction



(a) Fixed frequency contour of dispersion relation

(b) Displacement field of the beam grillage

**Figure 6.** At 1000Hz (a) Phase constant surface by the DFT2D, the line with ‘o’ indicates the result by WFE2DM on reduced model. (b) The displacement field of the beam grillage with 100 periods in each direction

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