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Identification of NNM using sliding window POD analysis

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Summary. In this paper, a method combining the Proper Orthogonal Decomposition (POD) and a time-frequency analysis (TFA) is presented. The POD method is a tool adapted to the study of multi-degrees of freedom systems, which can give some insight of the behaviour of complex nonlinear systems, whereas TFA is suitable for scalar signal with multi-frequency components. The method presented here is of principal interest in the case of swept-sine excitation of complex multi-DOF systems. The method can be used to extract the Non-linear Normal Modes (NNM). It is illustrated on a 3-degrees-of-freedom system.

Introduction

Nonlinearities are nowadays of growing importance in industrial processes. The well-known theory of linear modal analysis is limited in understanding the practical behaviour of real systems, especially when nonlinearities have dominant effects on the behaviour. A large variety of methods have been proposed to characterize and/or identify nonlinear systems. Time-frequency analysis is a powerful tool to study signals with time-dependent frequency content. Moreover, it can be used to characterize and identify linear and nonlinear systems with multiple frequency components. The wavelet transform was for instance used to identify the structural behaviour of complex systems (natural frequencies, mode shapes and damping)[1] [2].

The proper orthogonal decomposition (POD) has now emerged as a structural dynamics analysing tool especially suited for the study of complex multi-degrees-of-freedom systems. The physical interpretation of the proper orthogonal modes (POMs) was investigated by several authors with the links between the POMs and the linear and nonlinear normal modes highlighted [3].

In order to analyze multivariate data with time-dependent frequency content, such as systems following a swept sine excitation, it is thus relevant to combine the POD and a time-frequency analysis (the Gabor Transform). An analysis method combining Gabor Transform and POD will be presented. The properties of the method will be discussed. The method can be used to extract the Non-linear Normal Modes (NNM). The method will be applied to a 3-degrees-of-freedom system containing a cubic non-linearity. The NNM extracted from the simulated tests are compared with the NNM obtained from the harmonic balance.

Sliding window POD analysis

The Gabor Transform can be applied to a scalar signal $s(t)$ to obtain the evolution of the frequency content of the signal on a time-frequency plane :

$$G_s(\tau, f) = \int_{-\infty}^{+\infty} s(t) e^{-\frac{(t-\tau)^2}{2\sigma^2}} e^{-j2\pi f(t-\tau)} dt. \quad (1)$$

The application of the POD to a vector signal $S(t)$ with N components gives a decomposition of the signal on an orthogonal basis separating the time and structure information as follows:

$$S(t) = \sum_{i=1}^N \Phi_i q_i(t) = \sum_{i=1}^N \Phi_i \sqrt{\lambda_i} \eta_i(t) \quad (2)$$

where the vectors Φ_i are the Proper Orthogonal Modes (POMs), the $\eta_i(t)$ are their normalized time evolutions called Proper Orthogonal Components (POCs) and λ_i the energy captured called the Proper Orthogonal Values (POVs).

In order to study a vector signal $S(t)$ and follow its spatial evolution as well as its frequency content, one can combine the two previous methods by applying the POD to the windowed signals

$$S^\tau(t) = S(t) e^{-\frac{(t-\tau)^2}{2\sigma^2}} = \sum_{i=1}^N \Phi_i^\tau \sqrt{\lambda_i^\tau} \eta_i^\tau(t) \quad (3)$$

before applying the Fourier Transform :

$$G_s(\tau, f) = \int_{-\infty}^{+\infty} \sum_{i=1}^N \Phi_i^\tau \sqrt{\lambda_i^\tau} \eta_i^\tau(t) e^{-j2\pi f(t-\tau)} dt = \sum_{i=1}^N \Phi_i^\tau \sqrt{\lambda_i^\tau} \int_{-\infty}^{+\infty} \eta_i^\tau(t) e^{-j2\pi f(t-\tau)} dt = \sum_{i=1}^N \Phi_i^\tau \sqrt{\lambda_i^\tau} \tilde{G}_i(\tau, f). \quad (4)$$

The inverse formula to (4) (not given here) is also valid. One can thus obtain, for each time τ :

- the POMs Φ_i^τ giving the coherent structures characterizing the signal localized around the time τ (they are classified by decreasing energies, and it is possible to reduce the number of POMs from N to M^τ , with a criterion of the type $\sum_{i=1}^{M^\tau} \lambda_i^\tau \geq 0.99 \sum_{i=1}^N \lambda_i^\tau$);

- the energies λ_i^τ giving the repartition of energy between the POMs;
- the time-frequency analysis of the POCs $\tilde{G}_i(\tau, f) = \int_{-\infty}^{+\infty} \eta_i^\tau(t) e^{-j2\pi f(t-\tau)} dt$.

Extraction of the NNM of a 3 degrees-of-freedom system

The simulated system is shown in Fig. 1. It comports three masses related with linear springs and linear dashpots (light damping). The central mass is related to the ground with a cubic spring. Assuming that $m = 1$ kg and $k = 500$ m/N, the

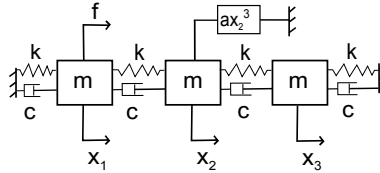


Figure 1: Nonlinear system.

natural frequencies of the underlying linear system are $f_1^0 = 2.75$ Hz, $f_2^0 = 5.01$ Hz and $f_3^0 = 6.51$ Hz. We will choose $a = 1$ and $c = 0.58$ m/sN (corresponding to a damping ratio of 1.% on the first mode of the underlying linear system) for the present simulations. The system is excited by applying a force on the first mass following a linear swept sine from 0.05 to 10. Hz over 400 seconds with constant amplitude F .

Fig. 2 (left) shows the results obtained by analyzing the time evolution of the displacement of the system, $S(t) = (x_1(t), x_2(t), x_3(t))^T$, obtained with a low amplitude force $F = 10$ N. The first POM Φ_1^T is dominant (Fig. 2 (left-top))

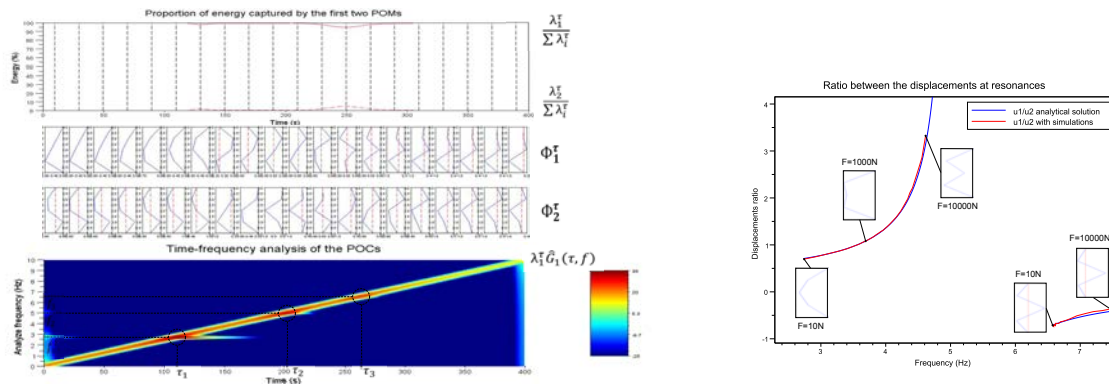


Figure 2: Sliding window POD analysis (left). Extracted NNM in an amplitude ratio-frequency plot (right).

and takes successively the shape of the normal modes of the underlying linear system (Fig. 2 (left-center)). The resonance frequencies can thus be detected with the maxima of the $\lambda_1^\tau \tilde{G}_i(\tau, f)$ (Fig. 2 (left-bottom)), giving the times (τ_1, τ_2, τ_3) where resonance motions occur with their corresponding frequencies (f_1, f_2, f_3) and mode shapes ($\Phi_1^{\tau_1}, \Phi_1^{\tau_2}, \Phi_1^{\tau_3}$).

The previous analysis was repeated for various excitation amplitude levels F from 10. N to 10000. N. The extracted frequencies and deformations versus the excitation amplitude level F can be considered as the nonlinear modes of the system. We observe that the second NNM is not affected by the non-linearity as foreseen by the theory. Fig. 2 (right) shows the evolution of the ratio of the first and second components of $\Phi_1^{\tau_j}$ for $j = 1$ (the first NNM) and $j = 3$ (the third NNM). The results are in good agreement with the NNM obtained using a first order harmonic balance method.

Conclusions

The method described allows following at the same time the evolution of the structural dynamic of a system, as well as its frequency behaviour. It was applied to extract the NNM of a basic nonlinear system, which were compared to the ones obtained from an analytical analysis, showing that the method can in this case identify the NNM of the system.

References

- [1] Yin H.P, Duhamel D., Argoul P. (2004) Natural frequencies and dampings estimation using wavelet transform of a frequency response function. *Journal of Sound and Vibration* **271**:999-1014.
- [2] Bellizzi S., Guillemain P., Kronland-Martinet R. (2001) Identification of coupled non-linear modes from free vibration using time-frequency representations. *Journal of Sound and Vibration* **232(2)**:191-213.
- [3] Kerschen G., Golinvial J.C., Feeny B., Vakakis A.F., Bergman L.A. (2005) The Method of Proper Orthogonal Decomposition for Dynamical Characterization and Order Reduction of Mechanical Systems : An Overview. *Nonlinear Dynamics* **41**:147-169.