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Observer and model predictive control for on-line parameter identification in nonlinear systems

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10th International Symposium on Dynamics and Control of Process Systems
18-20 December, Mumbai, India

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Context and motivations

- All model parameters need to be numerically known for simulation, control or optimization of dynamic processes.
- The optimal experiment design (OED) is a classic technique for parameter estimation. (Goodwin and Payne [1979], Ljung [1999])
- The OED usually separates parameter identification from the optimal input design (offline identification).
- Most applications of OED are reliable on linear or approximated linearized models.
- Recently, the coupled online OED techniques and parameter estimation has been developed for open loop stable systems without input/output process constraint. (Jayasankar et al. [2010], Zhu and Huang [2011])

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Outline of the proposed approach

- Our proposed approach of closed-loop OED for online parameter identification has been initially proposed in Flila et al. [2008] for the mono-variable case (SISOSP).
- Synthesize the online OED and online closed-loop parameter identification.
- For linear and nonlinear dynamic model based systems.
- Online optimal input design which optimizes the sensitivities of the measurements with respect to the unknown constant model parameters.
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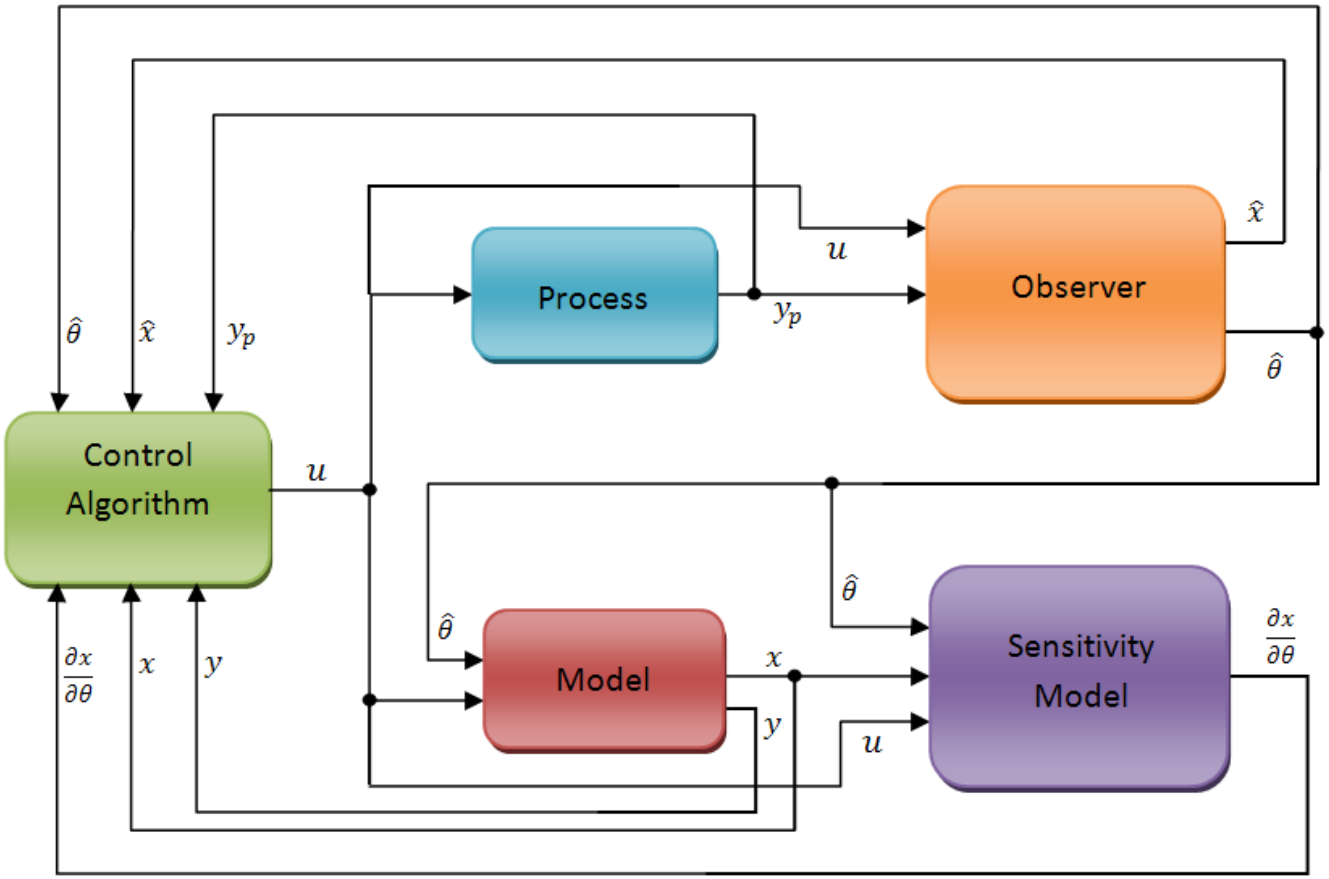
Closed loop control structure



The components

Model (linear or nonlinear)

$$(M)\begin{cases} \dot{x}(t) &= f(x(t),\theta,u(t)) \\ y(t) &= h(x(t),\theta,u(t)) \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the output vector, $u \in \mathcal{U} \subset \mathbb{R}^m$ is the input vector, $\theta \in \mathbb{R}^q$ is the unknown constant parameters vector. f and h are nonlinear functions of suitable dimensions.

Assumptions

1. In this study, the unknown model parameters are all constant.
2. In the system (1), f and h are C^∞ w.r.t. their arguments.

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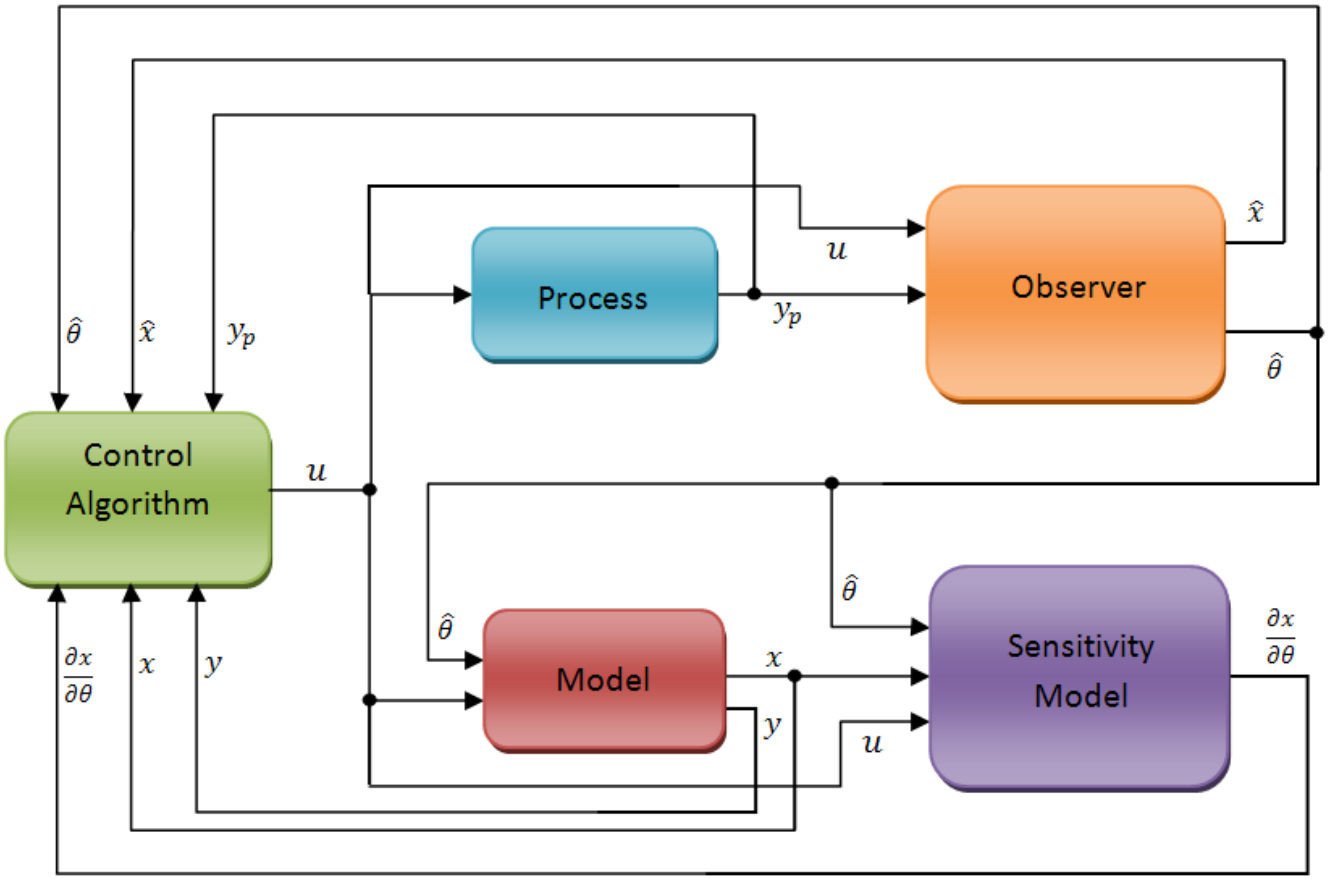

The components

Observer design for augmented system

System augmented with the unknown constant model parameters.

$$(M_a)\begin{cases} \dot{x}(t) &= f(x(t),\theta(t),u(t)) \\ \dot{\theta} &= 0 \\ y(t) &= h(x(t),\theta(t),u(t)) \end{cases} \tag{2}$$

The augmented state vector $x_a = [x\ \theta]^T$ and the vector function $f_a = [f\ 0]^T$

Observer definition

A general form for an observer for system (2) can be given by

$$(O)\begin{cases} \dot{\hat{x}}_a(t) = f_a(\hat{x}_a(t),u(t)) + g_a(t, h(\hat{x}_a(t),u(t)) - y_p(t)) \\ \text{with: } g_a(t,0) = 0, \end{cases} \tag{3}$$

such that
i) if $\hat{x}_a(0) = x_a(0)$, then $\hat{x}_a(t) = x_a(t), \quad \forall t \geq 0$;
ii) if $\forall x_a(0),\ \forall \hat{x}_a(0)$, then $\lim_{t\to+\infty} \|\hat{x}_a(t) - x_a(t)\| = 0$,
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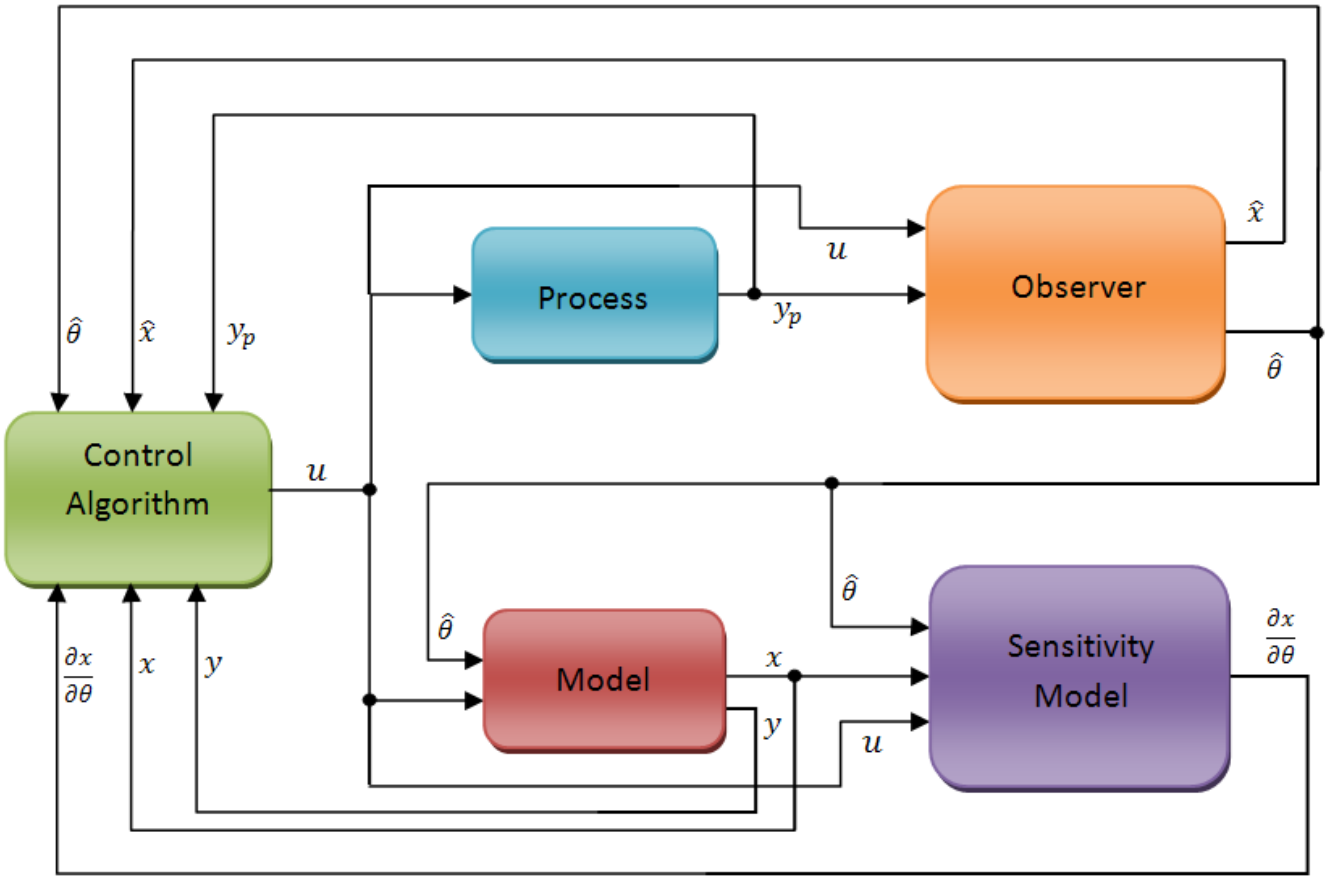

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Sensitivity model

Using the definition of the sensitivity function $(\cdot)_\theta = \frac{\partial(\cdot)}{\partial\theta}$ of a variable $(\cdot)$ with respect to the parameters θ, and the dynamical model (M), we give the sensitivity model as follows

$$(M_\theta)\begin{cases} \dot{x}_\theta(t) &= \dfrac{\partial f(x(t),\theta,u(t))}{\partial x}x_\theta + \dfrac{\partial f(x(t),\theta,u(t))}{\partial\theta} \\ y_\theta(t) &= x_\theta(t), \end{cases} \tag{4}$$

where $x_\theta \in R^{n\times q}$ and $y_\theta \in R^{n\times q}$ are the matrices of sensitivities of the states (the outputs) with respect to the parameters.

The relative-sensitivity function

$$\begin{cases} \bar{x}_\theta(i,j) &= \dfrac{\theta_j}{x_i}x_\theta(i,j); \quad i=1,\cdots,n; j=1,\cdots,q \\ \bar{y}_\theta(i,j) &= \dfrac{\theta_j}{y_i}y_\theta(i,j); \quad i=1,\cdots,r; j=1,\cdots,q. \end{cases} \tag{5}$$

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Model Predictive Control (MPC): general framework

- Advantages:
  - many theoretical papers published
  - +4000 applications in the world (Qin and Badgwell [2003])
- Idea:
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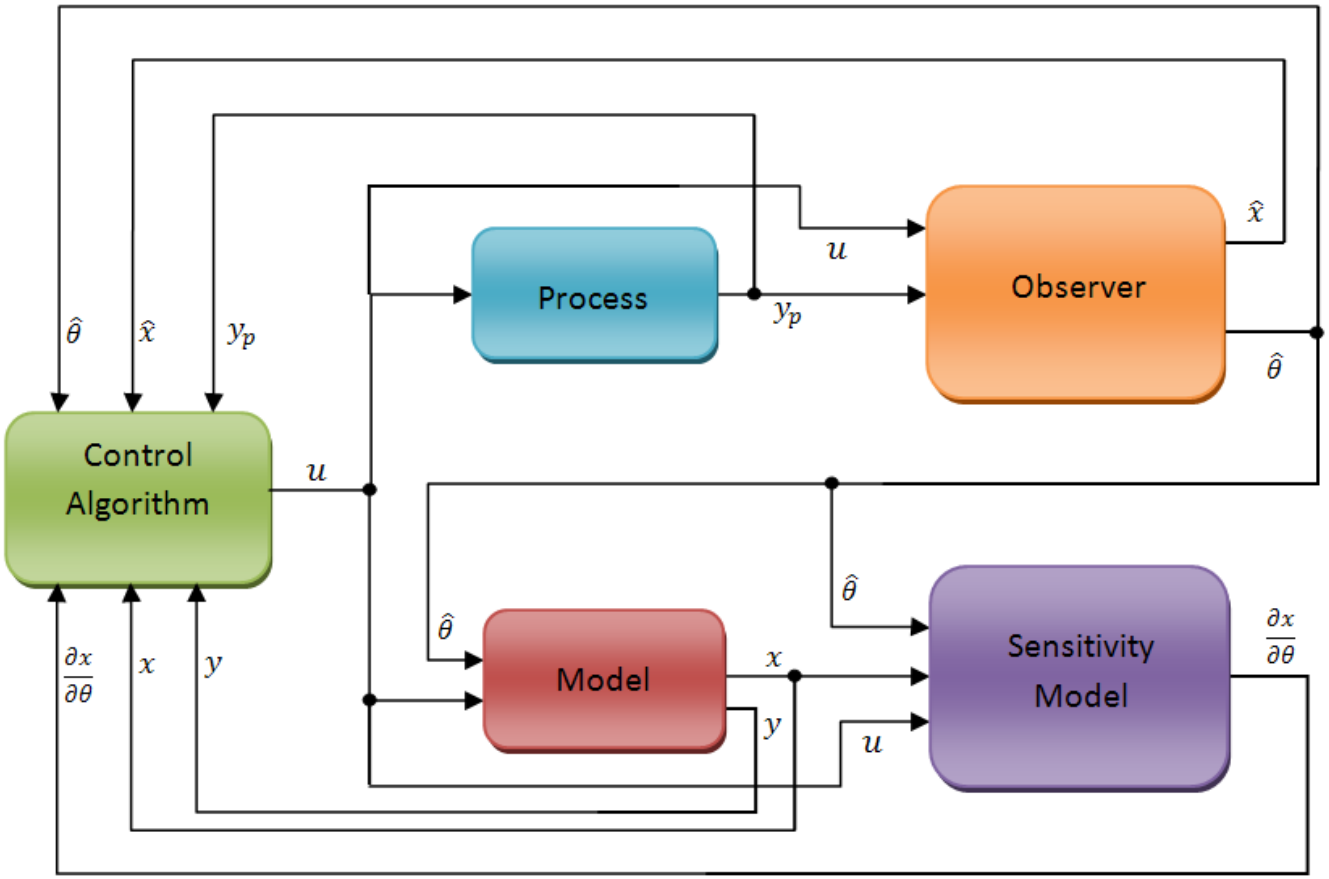

Optimal control law design

- Time discretization: $t = k \times T_s$ (k =current time index)
- At each k, over the prediction horizon N_p: maximize the norm of the sensitivity $\frac{\partial y_p}{\partial \theta}$ to get "rich" data for parameter identification.

Sensitivity matrix

$$\bar{y}_{\theta l|k} = \begin{bmatrix} \bar{y}_\theta(1,1)|_{l|k} & \bar{y}_\theta(1,2)|_{l|k} & \dots & \bar{y}_\theta(1,q)|_{l|k} \\ \bar{y}_\theta(2,1)|_{l|k} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \bar{y}_\theta(r,1)|_{l|k} & \dots & \dots & \bar{y}_\theta(r,q)_{l|k} \end{bmatrix}. \quad (6)$$

which gives at the current instant k the prediction at a future time $l \leq k$ ($l \in [k \; k+N_p]$) of the normalized outputs sensitivity $\bar{y}_\theta$.

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Optimal control law design

- Cost function

$$\begin{cases} F(\bar{y}_{\theta l|k}, u_{l|k}, y_p(k), \hat{x}_a(k)) = \frac{1}{N_p} \sum_{l=k+1}^{k+N_p} M_{l|k} \\ u_{l|k} = \{u(k) \dots u(l) \dots u(k+N_p)\}, l \in [k \ k+N_p]. \end{cases} \quad (8)$$

- E-optimality

$$\begin{cases} u^*_{l|k} = & arg\max_{u_{l|k}} \left(J(u_{l|k}) = \frac{\lambda_{min}(F)}{\lambda_{max}(F)} \right) \\ u_{l|k} = & \{u(k) \dots u(l) \dots u(k+N_p)\}, l \in [k \ k+N_p]. \end{cases} \quad (9)$$

- Handling specified constraints:
  - Constraints on the inputs (physical limitations of the actuator):

$$u_{min} \leq u(k) \leq u_{max}, \ \forall k \quad (10)$$

    Velocity constraints may also be added.
  - Constraints on the estimated states and/or the measured outputs (dealing with safety, operating zone, production, ...):

$$g_{min} \leq g(\hat{x}(k), y_p(k), u(k)) \leq g_{max}, \ \forall k \quad (11)$$

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The unstable nonlinear model of delta wing

The unstable nonlinear model of the system is:

$$\begin{cases} \dot{x}_1(t) & = x_2(t) \\ \dot{x}_2(t) & = \alpha_1\theta_1 x_1(t) + (\alpha_1\theta_2 - \alpha_2)x_2(t) + \alpha_1\theta_3 x_1^3(t)... \\ & \quad ... + \alpha_1\theta_4 x_1^2 x_2(t) + \alpha_1\theta_5 x_1 x_2^2(t) + \alpha_3 u(t) \\ y(t) & = [x_1(t)\ x_2(t)], \end{cases} \tag{12}$$

where:

- u is the control input to the model
- α is the known constant parameter vector
- θ is the vector of five unknown constant parameters
- input constraints: $-0.01 \leqslant u \leqslant 0.01$ (unstable zone in open loop)
- output constraints: $-0.5 \leqslant y_{p1} \leqslant 0.5$ (defined by trial and error)

Objective: based on (12) online identify the 5 unknowns parameters.
(more development details on appendix)

Review of Jain et al. [2005]

- Method used in Jain et al. [2005][1]:
  - A feedback linearizing control is used to cancel the nonlinear terms in the plant model and to alter the linear terms such that the closed-loop behavior matches with a specified linear reference model.
  - The reference model is taken to be a second-order linear system.
  - An external forcing signal is chosen to be a damped sinusoidal input in the adaptive control law.
  - Both designs of the reference model and the forcing input are not really discussed.
- Simulation results:
  - Two linear parameters (θ_1 and θ_2) converge to their targets at $t > 1000s$.
  - The convergence of the tree nonlinear parameter estimations(θ_3, θ_4 and θ_5) to their targets is not achieved.

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  - Both designs of the reference model and the forcing input are not really discussed.
- Simulation results:
  - Two linear parameters (θ_1 and θ_2) converge to their targets at $t > 1000s$.
  - The convergence of the tree nonlinear parameter estimations(θ_3, θ_4 and θ_5) to their targets is not achieved.

[1] Jain, H., Kaul, V., and Ananthkrishnan, N. (2005). Parameter estimation of unstable, limit cycling systems using adaptive feedback linearization: example of delta wing roll dynamics. *Journal of Sound and Vibration*, 939-960.

Simulation results

- Input applied: $u(t)$
- Process outputs: $y_{p1}(t)$ and $y_{p2}(t)$
- Parameter estimation: θ_1, θ_2, and θ_3
- Parameter estimation: θ_4 and θ_5
- Initial and final estimation errors:

Simulation results

- Input applied: $u(t)$
- Process outputs: $y_{p1}(t)$ and $y_{p2}(t)$
- Parameter estimation: θ_1, θ_2, and θ_3
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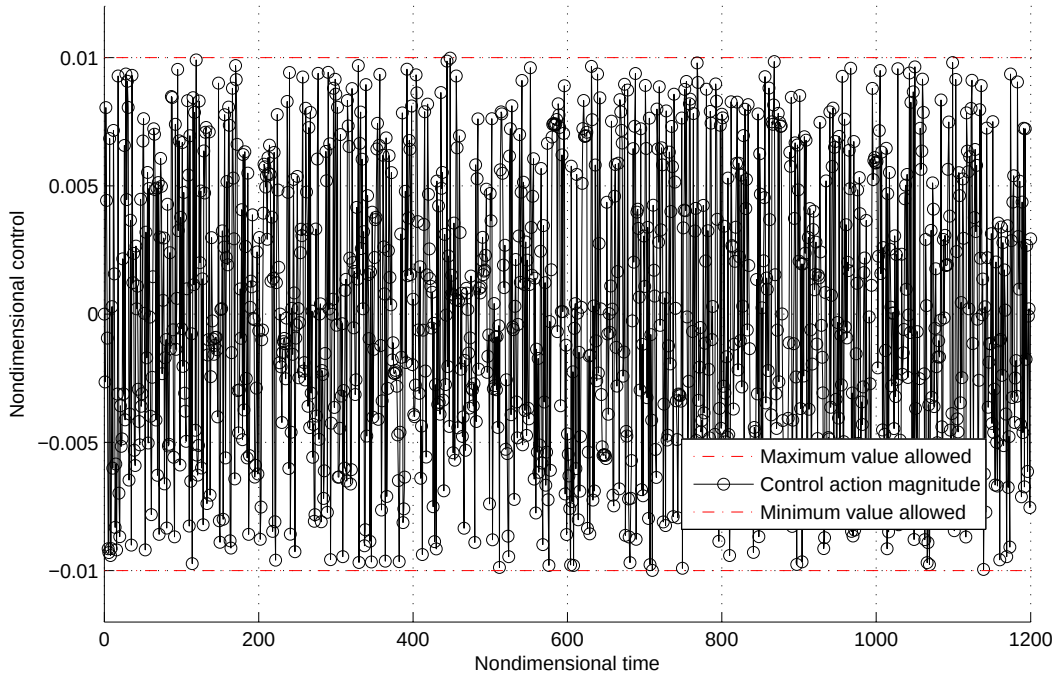
- Initial and final estimation errors:



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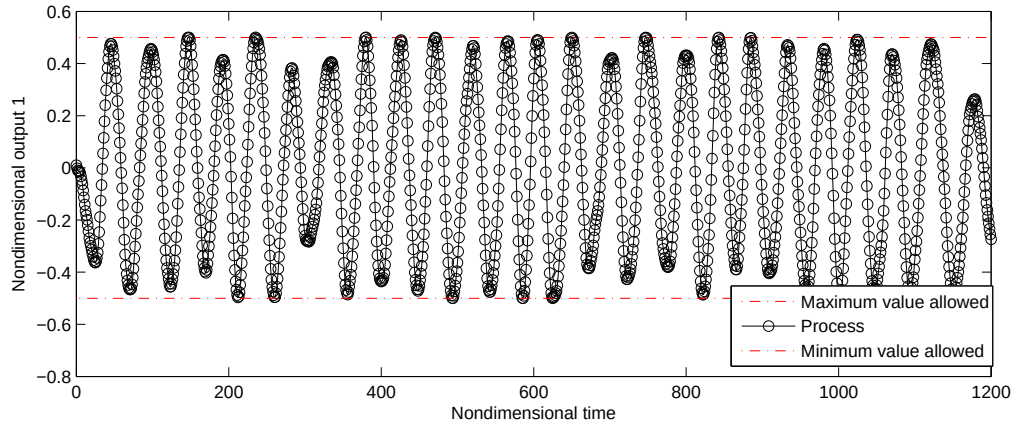

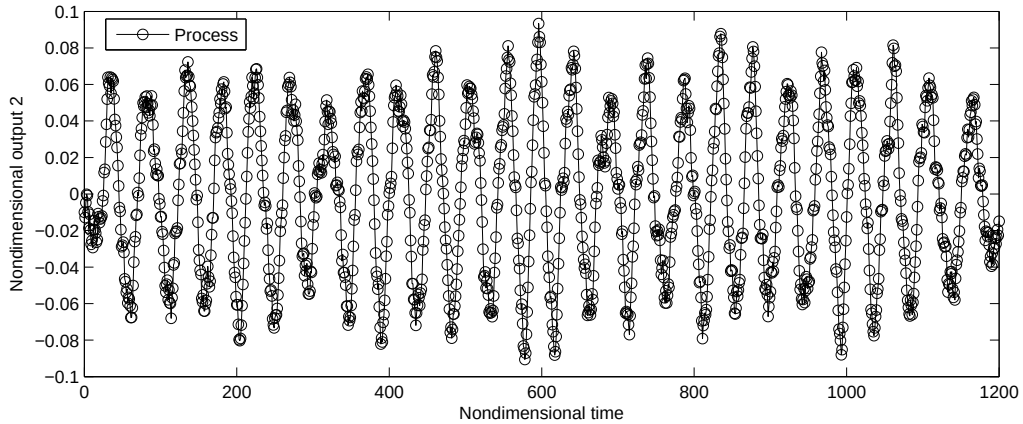

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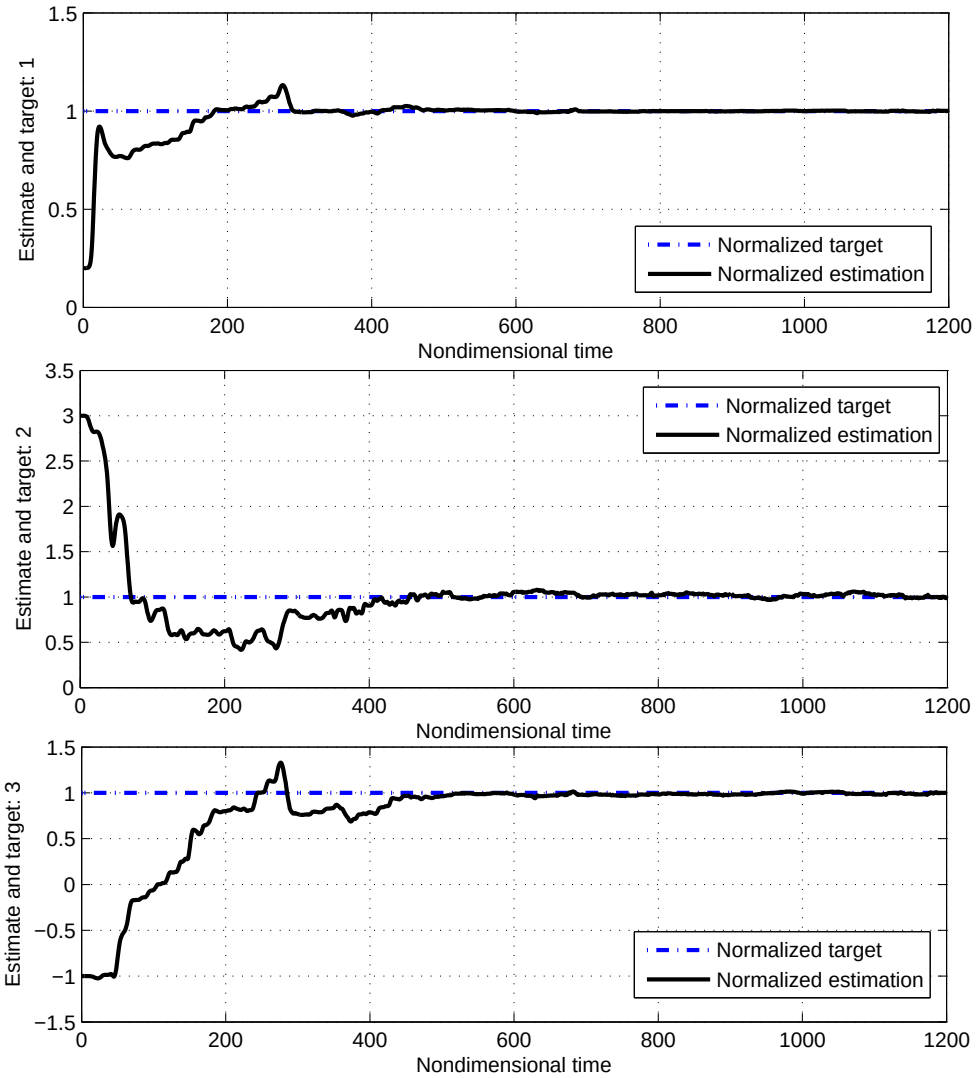

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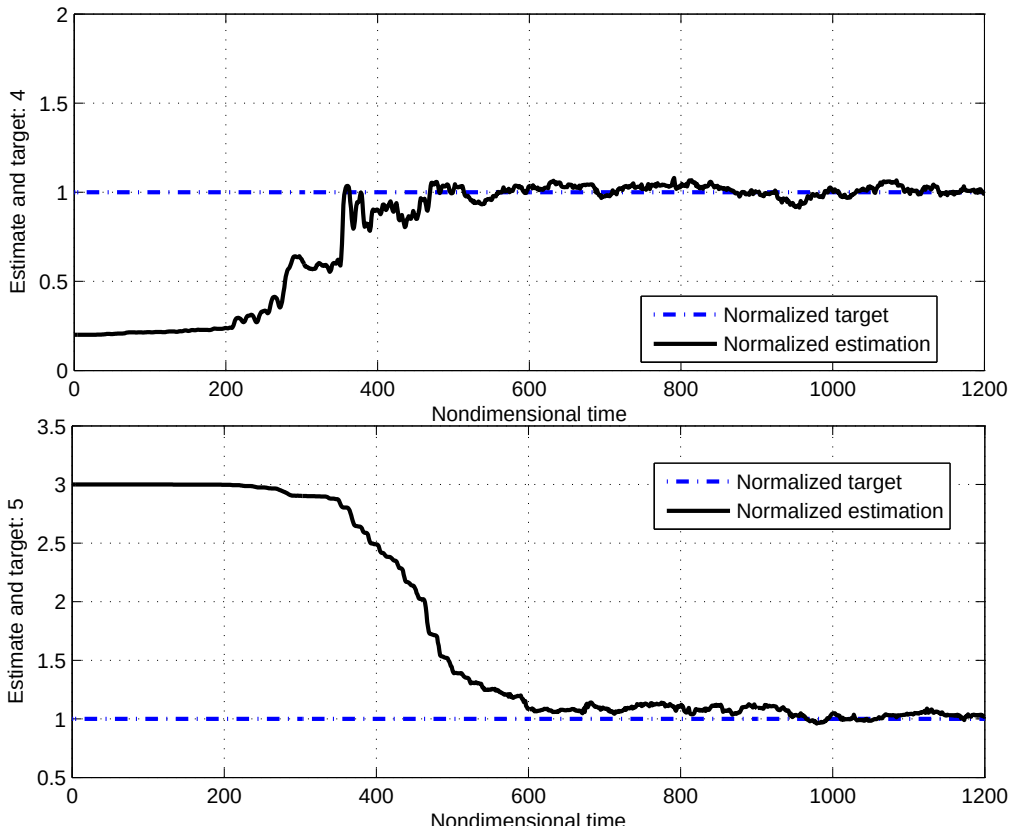

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- Initial and final estimation errors:

Simulation results

- Input applied: $u(t)$
- Process outputs: $y_{p1}(t)$ and $y_{p2}(t)$
- Parameter estimation: θ_1, θ_2, and θ_3
- Parameter estimation: θ_4 and θ_5
- Initial and final estimation errors:

θ	1	2	3	4	5
Initial error (%)	80	-200	200	80	-200
Final error (%)	0.1	0.4	0.8	-0.26	-2.88

Conclusion

- The proposed approach is able to
  - + design online the optimal experiment under constraints;
  - + identify online model parameters at the same time.
- The combination of an observer and a predictive control in closed loop improves the speed of the parameter estimation.
- The sensitivity criteria improve the accuracy of parameter estimation and leads to an optimal control at the same time.
- The input and output constraints specify the physical limitations imposed by the system and ensure the efficiency of the OED.
- The proposed approach may be adapted and tuned for any user defined dynamic model.

Contacts, acknowledgements and time for discussion

Software

- ODOE4OPE (Optimal Design Of Experiments for Online Parameter Estimation)
  Email: odoe4ope@univ-lyon1.fr; Website: http://odoe4ope.univ-lyon1.fr

People

- Acsystème company: Expertise in automation, process control, signal processing, optimization, software development, ...
  Website: http://www.acsysteme.com/en
- LAGEP: Laboratory of Process Control and Chemical Engineering, UMR 5007 CNRS-UCBL1
  Website: http://www-lagep.univ-lyon1.fr

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- The french company Acsystème
- The french ministry of higher education and research, for the financial support of this CIFRE PhD thesis 2011/0876

Annex A: Delta Wing

- Simulation condition

$$\begin{cases} \alpha = [0.354\ 0.001\ 1] \\ \theta_{Target} = [-0.05686\ 0.03254\ 0.07334\ -0.3597\ 1.46681] \\ \text{initial values of model states: } [x_{m1}(0)\ x_{m2}(0)] = [0\ 0] \\ \text{initial estimate of state 2: } \widehat{x}_2(0) = -0.01; \\ \text{prediction horizon: } N_p = 5 \\ \text{time of the simulation: } T_{fin} = 1200s \\ \text{sampling time: } T_s = 1s. \end{cases} \tag{13}$$

Annex A: Delta Wing

- Observer design for system delta wing
  - System augmented

$$\begin{cases} \dot{x}_a(t) &= A_a(y(t))x_a(t) + B_a(u(t)) \\ y(t) &= C_a x_a(t), \end{cases} \tag{14}$$

which is a state affine system up to output nonlinear injection with:

$$x_a(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}; B_a(u(t))=\begin{bmatrix} 0 \\ -\alpha_3 u(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; C_a^T=\begin{bmatrix} I_{2\times 2} \\ 0_{5\times 2} \end{bmatrix};$$

$$A_a(y(t))=\begin{bmatrix} 0_{2\times 1} & A(y(t)) \\ 0_{5\times 1} & 0_{5\times 6} \end{bmatrix}; A^T(y(t))=\begin{bmatrix} 1 & -\alpha_2 \\ 0 & \alpha_1 y_1(t) \\ 0 & \alpha_1 y_2(t) \\ 0 & \alpha_1 y_1^3(t) \\ 0 & \alpha_1 y_1^2(t) y_2(t) \\ 0 & \alpha_1 y_1(t) y_2^2(t) \end{bmatrix},$$

where $I_{2\times 2}$ is the 2×2 identity matrix, $0_{a\times b}$ is the $a \times b$ matrix of zeros.

Annex A: Delta Wing

- Exponential observer for (14)

Theorem

Assuming that $v(t) := C_a \chi_a(t, x_a(0))$ *is regularly persistent for*

$$\begin{cases} \dot{x}_a(t) &= A_a(v(t))x_a(t) + B_a(u(t)) \\ y(t) &= C_a x_a(t), \end{cases} \tag{15}$$

for any $x_a(0)$*, then the system admits an exponential observer of the form*

$$\begin{cases} \dot{\hat{x}}_a(t) &= A_a(y(t))\hat{x}_a(t) + B_a(u(t)) \\ & \quad \ldots - RS_\mu(t)^{-1} C_a^T (C_a \hat{x}_a(t) - y_p(t)) \\ \dot{S}_\mu(t) &= -\mu S_\mu(t) - A_a(y(t))^T S_\mu(t) - S_\mu(t) A_a(y(t)) \\ & \quad \ldots + C_a^T R C_a, \end{cases} \tag{16}$$

where S_μ *is a symmetric positive definite* 7×7 *matrix, the positive constant* $\mu > 0$ *and* $R > 1$ *are the observer tuning parameters.*

Annex A: Delta Wing

- Sensitivity model of the delta wing

$$
\begin{cases}
\dot{x}_{1\theta_1} = & x_{2\theta_1} \\
\dot{x}_{1\theta_2} = & x_{2\theta_2} \\
\dot{x}_{1\theta_3} = & x_{2\theta_3} \\
\dot{x}_{1\theta_4} = & x_{2\theta_4} \\
\dot{x}_{1\theta_5} = & x_{2\theta_5} \\
\dot{x}_{2\theta_1} = & \alpha_1(x_1 + \theta_1 x_{1\theta_1}) + (\alpha_1\theta_2 - \alpha_2)x_{2\theta_1} + 3\alpha_1\theta_3 x_1^2 x_{1\theta_1} \ldots \\
& \ldots + \alpha_1\theta_4(2x_1 x_{1\theta_1} x_2 + x_1^2 x_{2\theta_1}) + \alpha_1\theta_5(x_{1\theta_1} x_2^2 + 2x_1 x_2 x_{2\theta_1}) \\
\dot{x}_{2\theta_2} = & \alpha_1\theta_1 x_{1\theta_2} + (\alpha_1(x_2 + (\theta_2 x_{2\theta_2}) - \alpha_2 x_{2\theta_2})) + 3\alpha_1\theta_3 x_1^2 x_{1\theta_2} \ldots \\
& \ldots + \alpha_1\theta_4(2x_1 x_{1\theta_2} x_2 + x_1^2 x_{2\theta_2}) + \alpha_1\theta_5(x_{1\theta_2} x_2^2 + 2x_1 x_2 x_{2\theta_2}) \\
\dot{x}_{2\theta_3} = & \alpha_1\theta_1 x_{1\theta_3} + (\alpha_1\theta_2 - \alpha_2)x_{2\theta_3} + \alpha_1(x_1^3 + 3\theta_3 x_1^2 x_{1\theta_3}) \ldots \\
& \ldots + \alpha_1\theta_4(2x_1 x_{1\theta_3} x_2 + x_1^2 x_{2\theta_3}) + \alpha_1\theta_5(x_{1\theta_3} x_2^2 + 2x_1 x_2 x_{2\theta_3}) \\
\dot{x}_{2\theta_4} = & \alpha_1\theta_1 x_{1\theta_4} + (\alpha_1\theta_2 - \alpha_2)x_{2\theta_4} + 3\alpha_1\theta_3 x_1^2 x_{1\theta_4} \ldots \\
& \ldots + \alpha_1(x_1^2 x_2 + \theta_4(2x_{1\theta_4} x_1 x_2 + x_1^2 x_{2\theta_4})) + \alpha_1\theta_5(x_{1\theta_4} x_2^2 + 2x_1 x_2 x_{2\theta_4}) \\
\dot{x}_{2\theta_5} = & \alpha_1\theta_1 x_{1\theta_5} + (\alpha_1\theta_2 - \alpha_2)x_{2\theta_5} + 3\alpha_1\theta_3 x_1^2 x_{1\theta_5} + \ldots \\
& \ldots \alpha_1\theta_4(2x_{1\theta_5} x_1 x_2 + x_1^2 x_{2\theta_5}) + \alpha_1(x_1 x_2^2 + \theta_5(x_{1\theta_5} x_2^2 + 2x_1 x_2 x_{2\theta_5})).
\end{cases}
\tag{17}
$$

Perspective: Economic Model Predictive Control (EMPC)

- EMPC optimizes directly in real time the economic performance of the process, rather than tracking to setpoint.
- Closed-loop stability analysis based on the Lyapunov function.
- Bibliography:

[1] Rawlings, J.B. and Angeli, D. and Bates, C.N. (2012), *Fundamentals of Economic Model Predictive Control*, 51st IEEE Conference on Decision and Control, 3851-3861, Maui, Hawaii, USA.

[2] Amrit, R. and Rawlings, J.B. and Angeli, D. (2011), *Economic optimization using model predictive control with a terminal cost*, Annual Reviews in Control, 35, 178-186.

[3] Christofides, P.D. and Liu, J.F. and Heidarinejad, M. (2013), *Algorithms for improved fixed-time performance of Lyapunov-based economic model predictive control of nonlinear systems*, Journal of Process Control, 23, 404-414.