

Robust sequence storage in bistable oscillators

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Motivation

- Interesting computational tasks, like image segmentation or memory storage, can be implemented with a network of coupled oscillators. In practical applications, it is hardly used due to the computational cost of integrating nonlinear equations.
- At nanoscale, fluctuations cannot be neglected so that the robustness of proposed implementations should be assessed.

Nanoscale devices, like MTJ [1], have rich dynamics, including oscillatory ones. Such components may relieve us of the burden of integrating nonlinear equations. We propose a phenomenological model of bistable oscillators and we show that in a network, it achieves robust storage of sequences.

A bistable oscillator

Each unit i is described by 2 state variables: an activity variable $s_i \in \mathbb{R}$ and an oscillatory variable $\phi_t \in [0, 2\pi[$. The coupled dynamics for the network are

$$\begin{aligned} \frac{ds}{dt} &= -s + w_0 f(s) + \sigma(\cos\phi - I_0) + I_{ext} + d\xi_t \\ \frac{d\phi}{dt} &= \omega + (\beta - \rho s)\sin\phi \end{aligned}$$

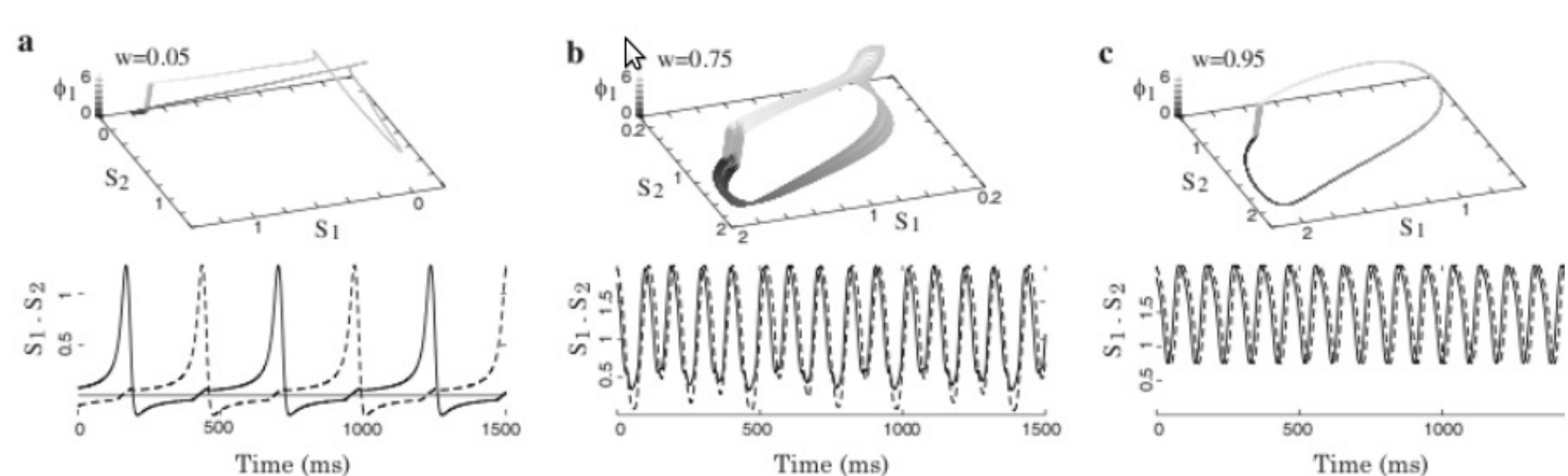
with

$$I_i^{ext} = p_i^{(k)}(t) + \sum_j w_{ji} f(s_j).$$

(see Table for the definitions and values of parameters)

Deterministic dynamics (d=0) [2] The value of $I_0 = \sin^{-1}(-\omega/\beta)$ is chosen so that the resting state is a fixed point.

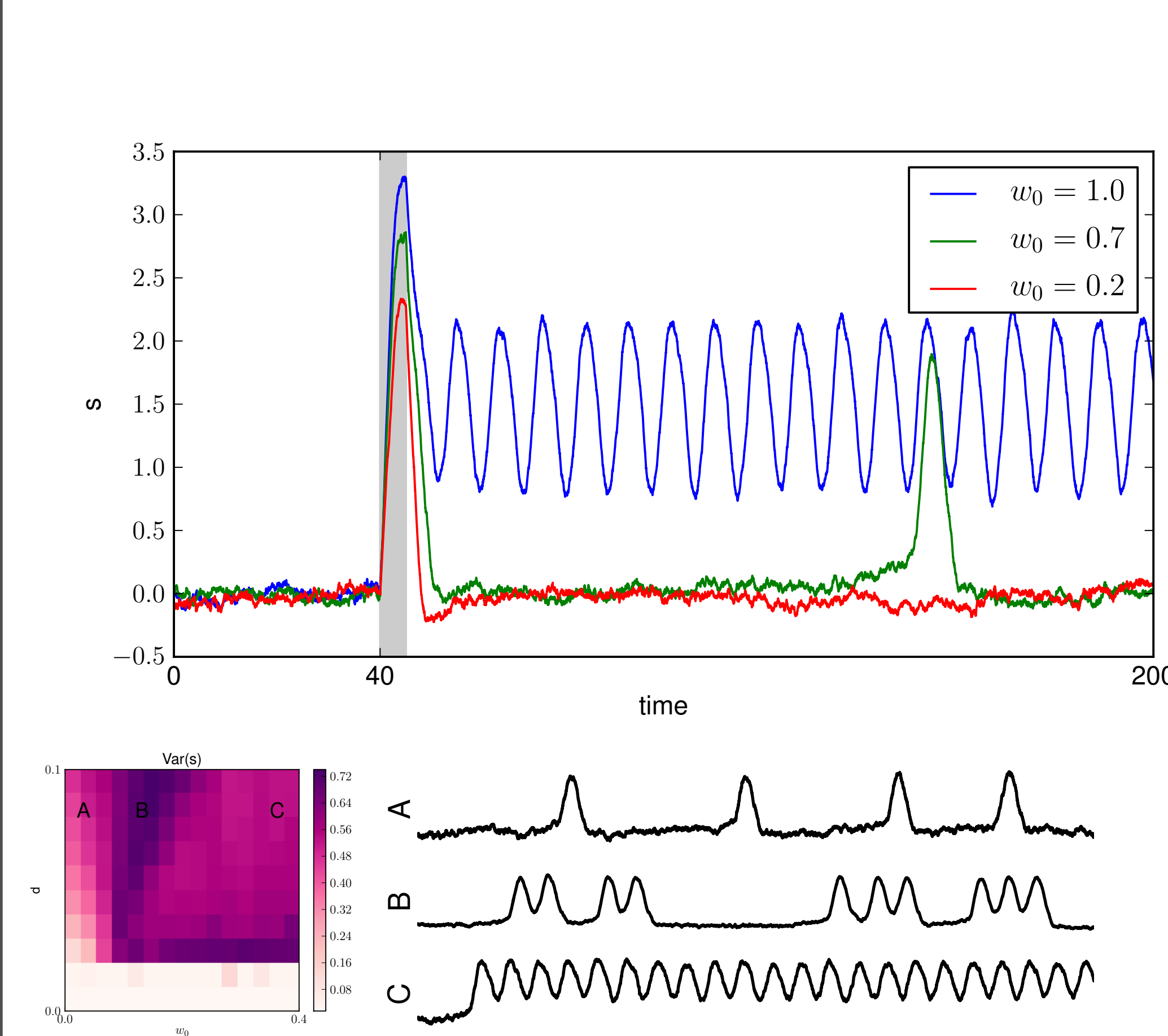
- the stability of this resting state depends on the parameter $\mu = \rho\sigma$ ($\mu_c = 0.86$ in our case).
- for strong coupling (w_0 for a single unit), there is bistability between quiescent resting state and oscillatory state.
- for two coupled units, there are 3 kinds of dynamics: antiphase at low connection strength, inphase at high connection strength and complex rhythms at intermediate connection strength.



Parameters

β : self-coupling of ϕ	1.2
ω : intrinsic frequency	1.
ρ : s to ϕ coupling	.9
σ : ϕ to s coupling	.9
d : noise intensity	.1 (varied)
w_0 : self-coupling of s	1 (varied)
$f(s)$: soft thresholding function	$\frac{\tanh(10(s-0.5)) - 1}{2}$
I_0 : pulse strength	1
N : Number of units	225
K : number of active units in a pattern	30

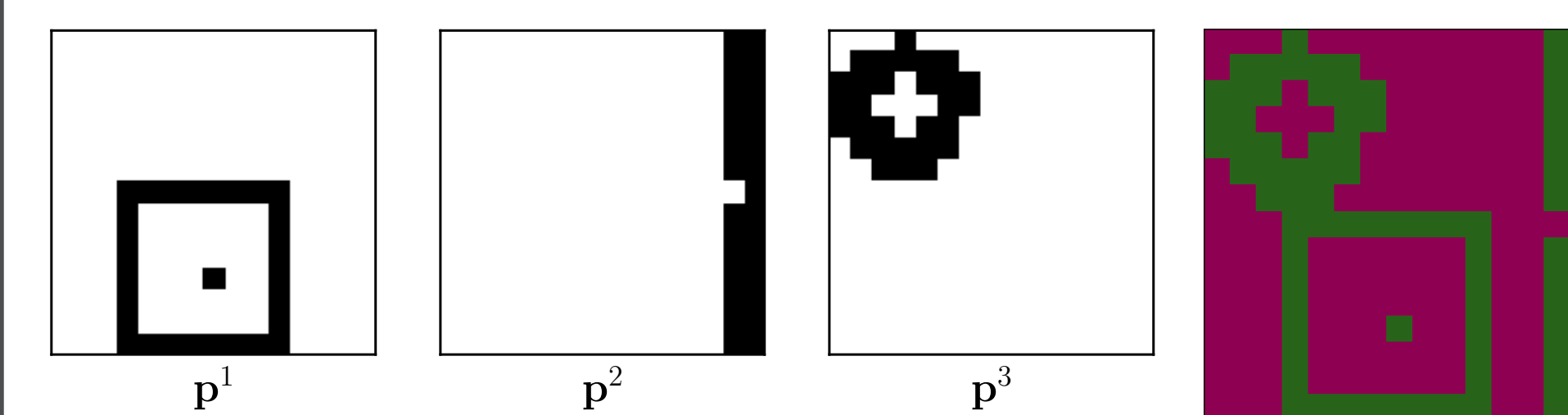
Stochastic dynamics of a single oscillator



Simulations of the stochastic dynamics (using strong order 1 scheme) show:

- Bistable behavior is preserved at large coupling and small stochastic perturbation (see Left).
- 3 ways to destabilize the down state depending on coupling w_0 and d (see below).
- The unit is robust to noise in a wide parameter range.

Robust associative memory for sequences



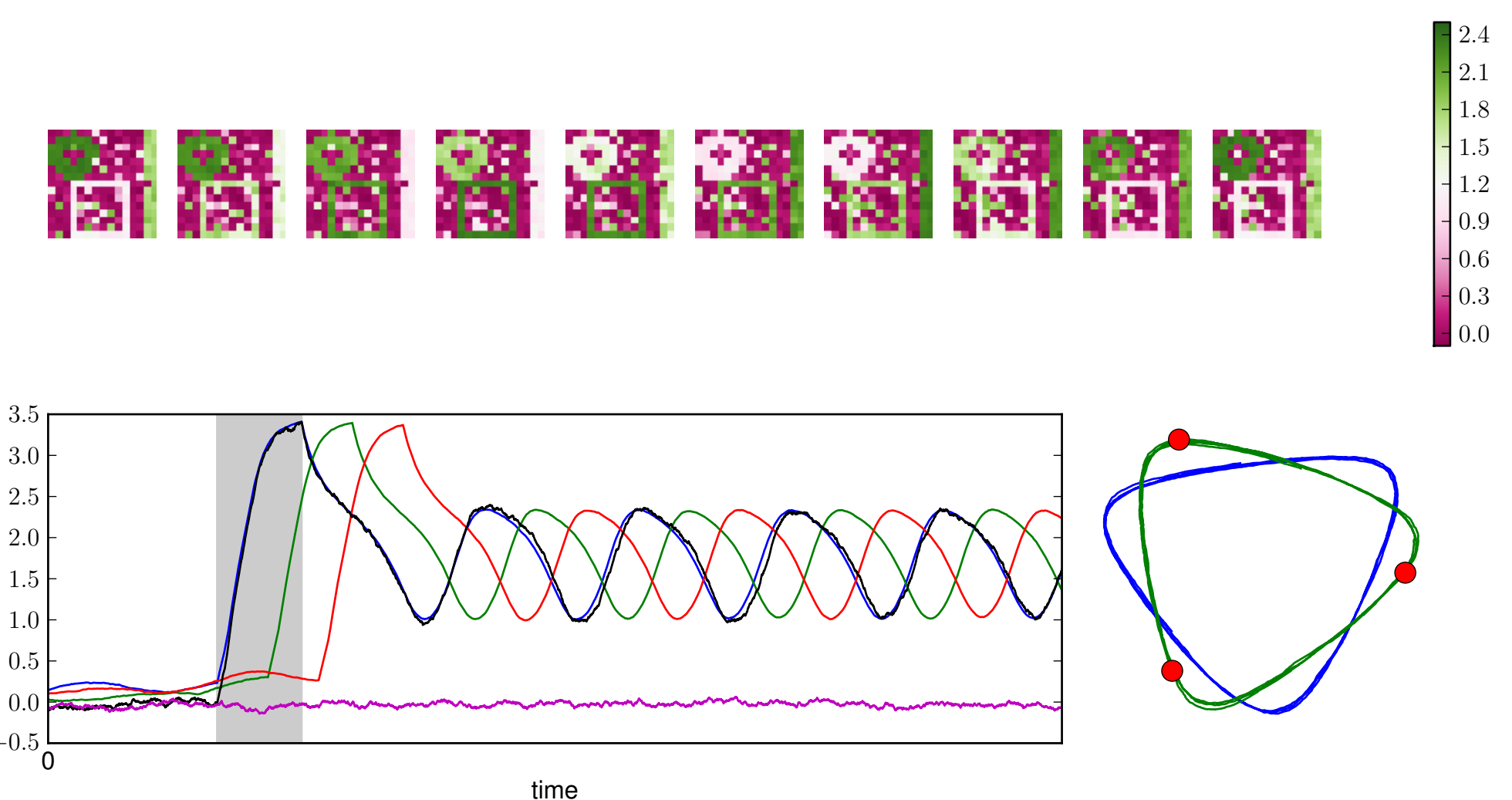
Aim: Storing patterns $\xi^p = (\xi_1^p, \dots, \xi_N^p)$ with $p \in [1, P]$. Fixed point attractors [3] in Hopfield networks are commonly used with connections

$$w_{ij} = \frac{1}{K} \sum_{k \in (1,2,3)} p_i^{(k)} p_j^{(k)}.$$

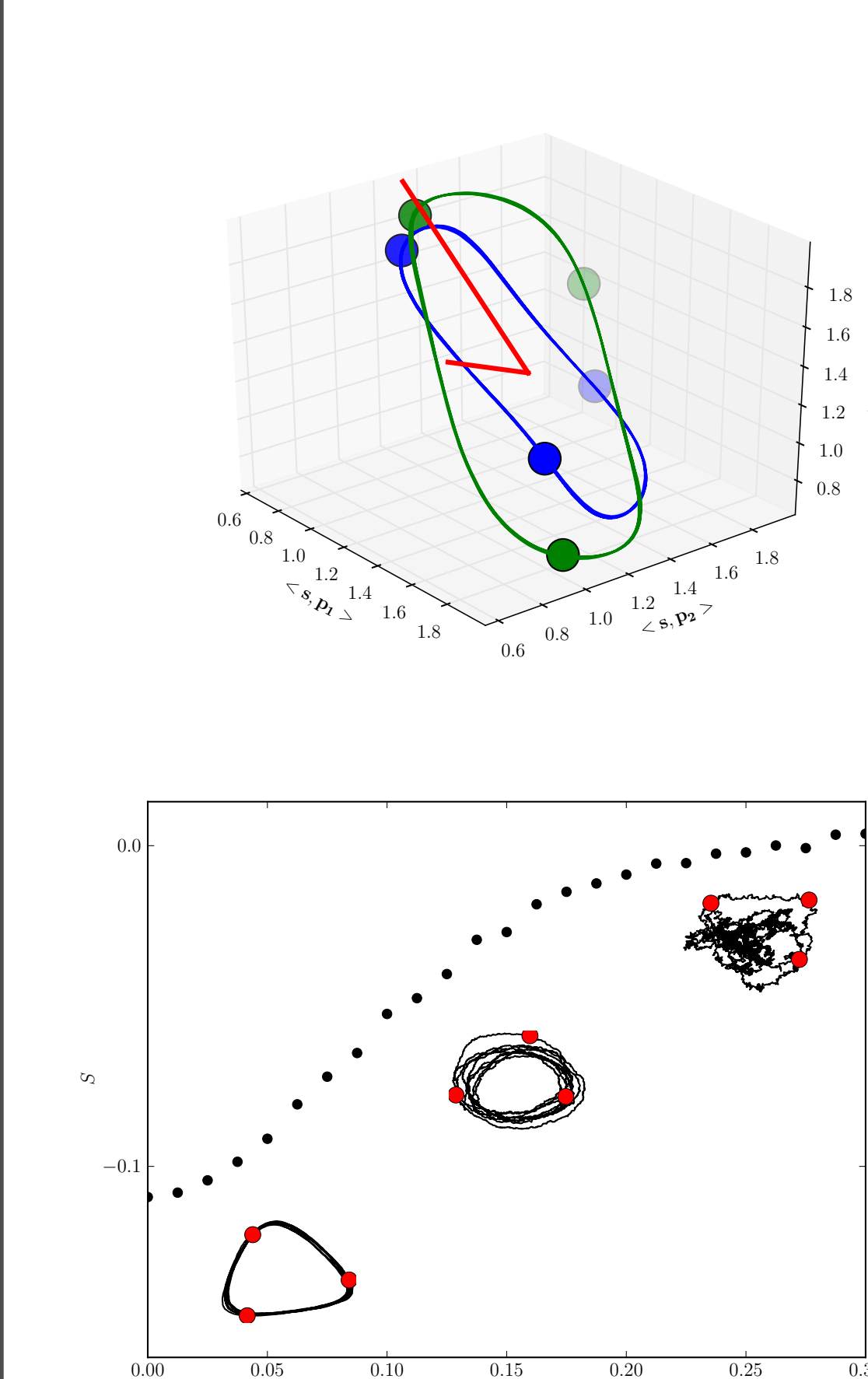
The recall of multiple patterns are merged into a single fixed point attractor.

The superposition catastrophe is solved in oscillatory networks when the identity of patterns is coded in the phase of the oscillation. This also allow to read out the sequence order of the presented patterns [4]. In reduced space (see Below), the trajectories for the sequences (1, 2, 3) in blue and (1, 3, 2) in green are different. The recall of a pattern \mathbf{p}_k is monitored by its overlap with the network activity

$$o_t^{(k)} = \mathbf{s}_t \cdot \mathbf{p}^{(k)}.$$



Reduced dynamics and robustness



Reduced dynamics and separability

When viewed in the overlaps coordinates (o_1, o_2, o_3) , the cyclic dynamics for the recall of a sequence can be reduced to its first and second principal components. The separability index of the two sequences read out can be measured by computing the average of overlap correlations:

$$S \propto \sum_{k \neq l} \sum_t \bar{o}_t^{(k)} \cdot \bar{o}_t^{(l)}.$$

Tunable recall As noise increases, first the sequence order is lost and then the patterns in the sequence are not recognized. By modulation of the noise, it is thus possible to switch from sequence to pattern recognition.

References

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- [2] D.C. *et al* (2009) Working memory dynamics and spontaneous activity in a flip-flop oscillations network model with a Milnor attractor. Cog. Neurod.
- [3] J. Hopfield *et al* (1982) Neural networks and physical systems with emergent collective computational abilities. PNAS
- [4] O. Jensen *et al* (2006) Maintenance of multiple working memory items by temporal segmentation. Neurosc.