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Numerical study of the nonlinear dynamics of horizontal drillings

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Summary. This paper presents a model to describe the nonlinear dynamics of a drillstring in horizontal configuration, which is intended to correctly predict the three-dimensional dynamics of this complex structure. This model uses a sophisticated beam theory, with effects of rotatory inertia and shear deformation, which is capable of reproducing the large displacements that the beam undergoes. Also, it considers the effects of torsional friction and normal shock due to the transversal impacts between the rotating beam and the borehole wall, as well as, the force and the torque induced by the bit-rock interaction. This is done as a first effort to solve a robust optimization problem, which seeks to maximize the rate of penetration of the drillstring into the soil, to reduce the drilling process costs. Numerical simulations show that the dynamic behavior of the drillstring is strongly correlated with the number of shocks, which causes the column to assume complex spatial configurations, and force a reduction in the drilling process efficiency.

Introduction

Oil prospecting uses, for drilling of exploratory wells, an equipment called a drillstring, which is a long column under rotation, composed by a sequence of connected drill-pipes and auxiliary equipment. The dynamics of this column is very complex, because under normal operational conditions it is subjected to longitudinal, lateral and torsional vibrations, which presents a nonlinear coupling, and the structure is subjected to shocks due to the drill-bit/soil and drill-pipes/borehole impacts [1].

The study of the nonlinear dynamics of a drillstring in the vertical configuration was consistently treated in a series of recent papers [2, 3, 4, 5, 6]. On the other hand, for a drillstring in the horizontal configuration, until the best of the author's knowledge, there is only one work in the open literature [7], which examines the horizontal drillstring dynamics, but it considers only the longitudinal dynamics. Therefore, further studies are necessary for a better understand of the horizontal drillstring three-dimensional dynamics.

Due to the relevance of the problem in the context of engineering, and to the lack of works in the scientific literature, the nonlinear dynamics of a horizontal drillstring is a research topic of great interest. In this sense, this work aims to develop a deterministic modeling of the nonlinear dynamics of a drillstring in horizontal configuration. Also, it intends to conduct a numerical study to describe accurately the behavior of this dynamical system during its operation. This is a first step to the development of stochastic modeling of the same problem, which is necessary to solve a robust optimization problem that seeks to maximize the rate of penetration of the column into the soil (to reduce the drilling process costs) [3].

Modeling of the nonlinear dynamics

Mechanical system of interest

The mechanical system of interest in this work is sketched in Figure 1. It consists of a horizontal rigid pipe (illustrated as the pair of stationary rigid walls), perpendicular to gravity acceleration g , which contains in its interior a deformable tube under rotation (rotating beam), subjected to three-dimensional displacements. This deformable tube has a length L , cross section area A , and is made of a material with mass density ρ , elastic modulus E , and shear modulus G . It loses energy through a mechanism of viscous dissipation, proportional to the mass operator, with damping coefficient c . Concerning the boundary conditions, the rotating beam is blocked for transversal displacements in both extremes; blocked to transversal rotations on the left extreme; and, on the left extreme, has a constant angular velocity around x equal to Ω , and an imposed longitudinal velocity V_0 .

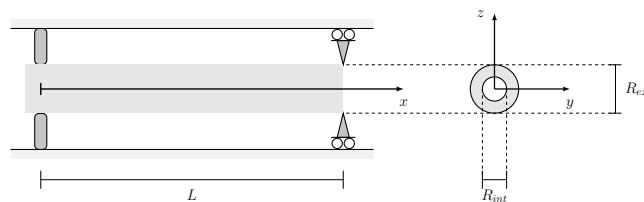


Figure 1: Schematic representation of the rotating beam which models the horizontal drillstring.

The beam theory adopted takes into account the rotatory inertia and shear deformation of the beam cross section. Also, it is assumed that the beam is undergoing small rotations in the transverse directions, large rotation in x , small strains, and large displacements the three spatial directions, which couples the longitudinal, transverse and torsional vibrations.

In this way, the following kinematic hypothesis is adopted

$$\begin{aligned} u_x(x, y, z, t) &= u - y\theta_z + z\theta_y, \\ u_y(x, y, z, t) &= v + y(\cos\theta_x - 1) - z\sin\theta_x, \\ u_z(x, y, z, t) &= w + z(\cos\theta_x - 1) + y\sin\theta_x, \end{aligned} \quad (1)$$

where u_x , u_y , and u_z respectively denote the displacement of a beam point in x , y , and z directions, at the instant t . Also, u , v , and w are the displacements of a beam neutral fiber point in x , y , and z directions, respectively, while θ_x , θ_y , and θ_z represent rotations of the beam around the x , y , and z axes respectively. Note that, to analyze the dynamics of the rotating beam, the physical quantities of interest are the fields u , v , w , θ_x , θ_y , and θ_z , which depend on the position x and the time t .

Friction and shock effects

This rotating beam is also able to generate normal shocks and torsional friction in random areas of the rigid tube, which are respectively described by the Hunt and Crossley shock model [8], and the standard Coulomb friction model. Therefore, the force of normal shock is given by

$$F_{FS} = -k_{FS_1} \delta_{FS} - k_{FS_2} \delta_{FS}^3 - c_{FS} |\dot{\delta}|^3 \delta_{FS}, \quad (2)$$

and the Coulomb frictional torque by

$$T_{FS} = -\mu_{FS} F_{FS} R_{bh} \operatorname{sgn}(\dot{\theta}_x). \quad (3)$$

In the above equations, k_{FS_1} , k_{FS_2} and c_{FS} are constants of the shock model, while μ_{FS} is a friction coefficient, R_{bh} is the borehole radius, and $\operatorname{sgn}(\cdot)$ the sign function. The $\dot{\cdot}$ is an abbreviation for time derivative, and the parameter $\delta_{FS} = r - \text{gap}$, where $r = \sqrt{v^2 + w^2}$, is dubbed *indentation*, and is a measure of penetration in the wall of a beam cross section, such as illustrated in Figure 2.

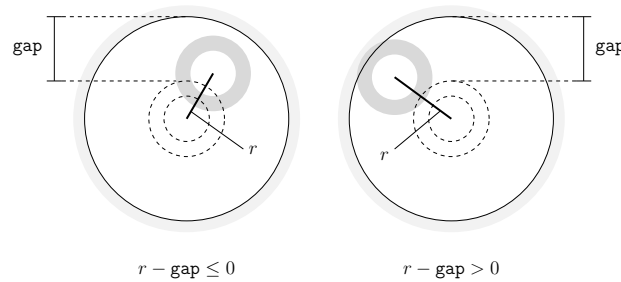


Figure 2: Illustration of the indentation parameter in a situation without impact (left) or with impact (right).

Bit-rock interaction effects

At the right extreme of the rotating beam act a force and a torque, which emulate the effects of interaction between the drill-bit and the soil. They are respectively given by

$$F_{BR} = \begin{cases} \Gamma_{BR} \left(\exp(-\alpha_{BR} \dot{u}(L, \cdot)) - 1 \right), & \text{for } \dot{u}(L, \cdot) > 0 \\ 0, & \text{for } \dot{u}(L, \cdot) \leq 0 \end{cases} \quad (4)$$

and

$$T_{BR} = -\mu_{BR} F_{BR} \xi_{BR}(\dot{\theta}_x), \quad (5)$$

where Γ_{BR} is the bit-rock limit force; α_{BR} is the rate of change of bit-rock force; μ_{BR} bit-rock friction coefficient; and ξ_{BR} is a regularization function, which takes into account the dimension of length, to the Eq.(5) gives a torque. The expression for the bit-rock interaction models above were, respectively, proposed by [7] and [10].

Variational formulation of the nonlinear dynamics

Using a modified version of the extended Hamilton's principle, to include the effects of dissipation, one can write the weak form of the nonlinear equation of motion of the mechanical system as

$$\mathcal{M}(\boldsymbol{\psi}, \ddot{\boldsymbol{U}}) + \mathcal{C}(\boldsymbol{\psi}, \dot{\boldsymbol{U}}) + \mathcal{K}(\boldsymbol{\psi}, \boldsymbol{U}) = \mathcal{F}_{NL}(\boldsymbol{\psi}, \boldsymbol{U}, \dot{\boldsymbol{U}}, \ddot{\boldsymbol{U}}), \quad (6)$$

where \mathcal{M} represents the mass operator, \mathcal{C} is the damping operator, \mathcal{K} is the stiffness operator, and \mathcal{F}_{NL} is the nonlinear force operator. Also, the field variables and their weight functions are lumped in the vectors fields

$$\boldsymbol{U} = \begin{pmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\psi} = \begin{pmatrix} \psi_u \\ \psi_v \\ \psi_w \\ \psi_{\theta_x} \\ \psi_{\theta_y} \\ \psi_{\theta_z} \end{pmatrix}. \quad (7)$$

The weak form of the initial conditions reads

$$\mathcal{M}(\boldsymbol{\psi}, \boldsymbol{U}(0)) = \mathcal{M}(\boldsymbol{\psi}, \boldsymbol{U}_0), \quad (8)$$

and

$$\mathcal{M}(\boldsymbol{\psi}, \dot{\boldsymbol{U}}(0)) = \mathcal{M}(\boldsymbol{\psi}, \dot{\boldsymbol{U}}_0), \quad (9)$$

where \boldsymbol{U}_0 and $\dot{\boldsymbol{U}}_0$, respectively, denote the initial displacement, and the initial velocity fields.

Discretization of the model equations

After the discretization of the Eqs.(6), (8) and (9), by means of the Galerkin/finite element method [9], one arrives the following initial value problem

$$[M] \ddot{\boldsymbol{Q}}(t) + [C] \dot{\boldsymbol{Q}}(t) + [K] \boldsymbol{Q}(t) = \boldsymbol{f}(\boldsymbol{Q}(t), \dot{\boldsymbol{Q}}(t), \ddot{\boldsymbol{Q}}(t)), \quad (10)$$

and

$$\boldsymbol{Q}(0) = \boldsymbol{Q}_0, \quad \text{and} \quad \dot{\boldsymbol{Q}}(0) = \dot{\boldsymbol{Q}}_0, \quad (11)$$

where $\boldsymbol{Q}(t)$ is the generalized displacement vector, $\dot{\boldsymbol{Q}}(t)$ is the generalized velocity vector, $\ddot{\boldsymbol{Q}}(t)$ is the generalized acceleration vector, $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, and \boldsymbol{f} is a nonlinear force vector, which contains contributions of an inertial force and a force of geometric stiffness.

The discretization of the structure uses a finite element mesh with 500 elements. As each element has 6 degrees of freedom per node, this results in a semi-discrete model with 3006 degrees of freedom. To reduce the computational cost of the simulations, the initial value problem of Eqs.(10) and (11) is projected in a vector space of dimension 56 to generate a reduced order model, which is integrated using the Newmark method [9], and the nonlinear system of algebraic equations, resulting from the time discretization, is solved by a fixed point iteration.

The construction of the reduced order model took into account all the rigid body modes of the system; all the flexural modes of vibration located at frequency band $[0, 10]$ Hz, and all the longitudinal and torsional modes that are in the band $[0, 200]$ Hz.

Numerical simulation of the nonlinear dynamics

In order to simulate the nonlinear dynamics of the rotating beam, the physical parameters presented in the Table 1 are adopted. The dynamics is investigated for a "temporal window" of 20s, with a nominal time step $\Delta t = 2.5$ ms, which is refined whenever necessary to capture the effects of shock. For the initial conditions, are adopted: a zero displacement; a constant angular velocity Ω around the x axis; and a constant longitudinal velocity V_0 .

The longitudinal displacement and velocity of the drill-bit (located at the beam right extreme), can be seen in Figure 3. Note that the velocity exhibits a mean value of 1.05 mm/s, close to the velocity $V_0 = 1.39$ mm/s, which is imposed on the left end of the beam. Also, throughout the "temporal window" analyzed, one can observe packages where the the velocity of the drill-bit presents large fluctuations, which can reach up to 200 times the mean value. These fluctuations in the velocity are reflected in the longitudinal displacement as retroactive movements. A serious consequence of these retroactive movements is the reduction of the drilling process efficiency.

Table 1: . Physical parameters used in the simulation.

parameter	value	unit
ρ	7900	kg/m^3
E	203	GPa
G	78	GPa
R_{int}	50	mm
R_{ext}	80	mm
R_{bh}	95	mm
A	5500π	mm^2
L	100	m
c	0.01	—
g	9.81	m/s^2
Ω	50	rpm
V_0	5	m/h
k_{FS_1}	10^{10}	N/m
k_{FS_2}	10^{16}	N/m^3
c_{FS}	0.001	$(N/m^3)/(m/s)$
μ_{FS}	0.0005	—
Γ_{BR}	250	kN
α_{BR}	180	$1/(m/s)$
μ_{BR}	0.04	—

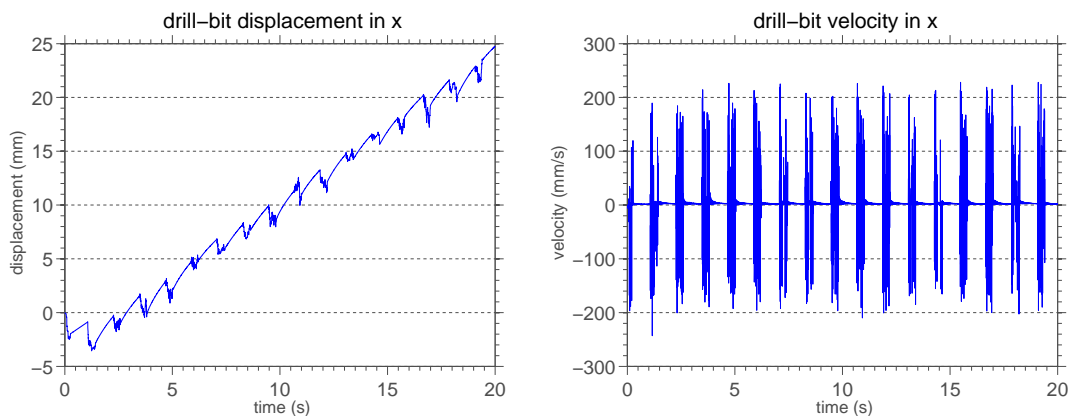


Figure 3: Illustration of the drill-bit displacement (left) and of the drill-bit velocity (right).

The packages of fluctuation observed correspond to transitory periods of the dynamical system, and are highly correlated with the process of collision between beam and the borehole wall, as can be seen, if the reader compares the graphs in the Figure 3 with the evolution of the number of shocks, which is shown in the left part of Figure 4. Whenever there is a shock, the system “loses its memory” about the previous dynamic behavior, and undergoes a new transient period until it reaches a steady state again. This behavior is repeated 17 times in the “temporal window” analyzed.

With regard to the distribution of impacts along the beam, which is shown on the right part of the Figure 4, it is clear that they do not occur near the beam ends. This is natural due to the restrictions of movement imposed by the boundary conditions. A greater number of impacts tends to be observed on the right part of the structure because the right extreme is not blocked to the transverse rotations.

Observing the cross section of the beam at $x = L/2$, for which the transversal displacement (left) and velocity (right) are shown in Figure 5, one can see that the transient packages are also present in the inner parts of the mechanical system. And it is clearly visible from Figure 5 that these transient packages also present a strong correlation with the number of shocks shown in Figure 4.

The impacts between the drillstring and the borehole wall generate nonlinear elastic deformations in the beam, but without residual deformation effects. In this contact also occurs energy dissipation, due to the normal shock, and the torsional friction, induced by the rotation of the beam. These mechanical contacts also activate flexural modes of vibration associated to high natural frequencies, so that the mechanical system assumes complex spatial configurations, as can be seen, for several instants, in Figure 6.

It is also very clear from the Figure 6 that, the mechanical contacts between the beam and the borehole wall, do not occur all the time among discrete points, they can also be seen along continuous line segments.

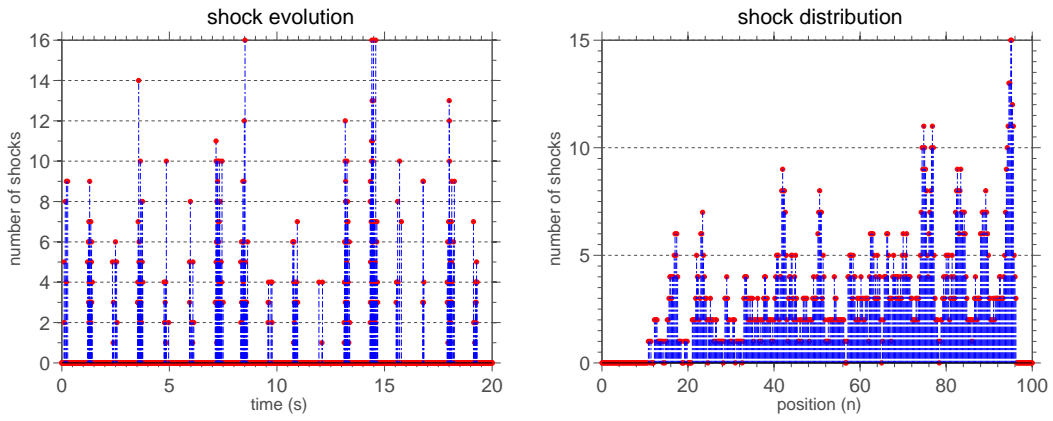


Figure 4: Illustration of the number of shocks per unit of time (left) and per position (right).

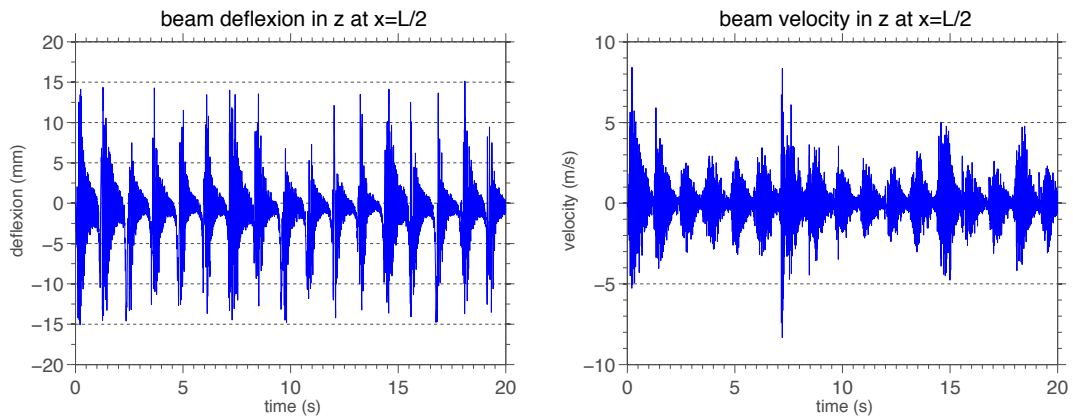


Figure 5: Illustration of the displacement in z (left) and of the velocity in z (right) when $x = L/2$.

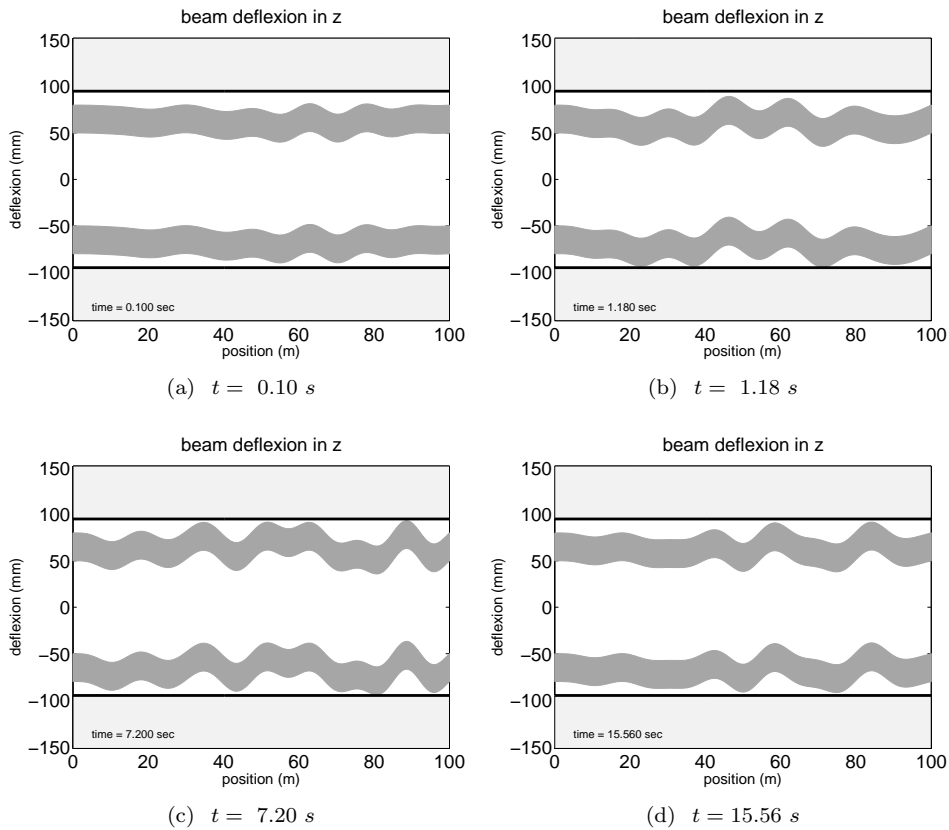


Figure 6: Illustration of the mechanical system under analysis, sectioned by the plane $y = 0$, for the several instants.

Conclusions and perspectives

A model was developed in this work to describe the nonlinear dynamics of horizontal drillstrings. The model uses a sophisticated beam theory, with effects of rotatory inertia and shear deformation, which is capable of reproducing the large displacements that the beam undergoes. This model also considers the effects of friction and shock due to the transversal impacts between the beam and the borehole wall, as well as, the force and the torque induced by the bit-rock interaction.

Numerical simulations show that the dynamics of the drill-bit is severely affected by the impacts between the beam and the borehole wall, with the appearance of transients packages that leads to a reduction in the drilling process efficiency. The numerical study also shows that the dynamics in the middle of the structure is also correlated with the number of shocks per unit time, which causes great fluctuations in the transversal displacement and velocity at this point. The mechanical impacts also cause the beam to assume complex spatial configurations, which are formed by flexural modes associated to high natural frequencies.

The physical system studied in this work is subject to a series of uncertainties [11], that are due to the variability of its parameters [12], and/or epistemic in nature, i.e, result of the ignorance about the physics of the problem [13]. Therefore, in a next step, the authors intend to develop a stochastic modeling of the nonlinear dynamics of the horizontal drillstring, in order to quantify these uncertainties and to solve an robust optimization problem, which seeks to maximize the rate of penetration of the column into the soil [3].

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