Analysis of the nonlinear dynamics of a horizontal drillstring

A. Cunha Jr, Christian Soize, R. Sampaio

Analysis of the nonlinear dynamics of a horizontal drillstring

Americo Cunha Jr[1,2], Christian Soize[2], and Rubens Sampaio[1]

[1]PUC–Rio, Department of Mechanical Engineering
Rua Marquês de São Vicente, 225, Gávea, Rio de Janeiro - RJ, Brazil — 22453-900
email: americo.cunhajr@gmail.com, rsampaio@puc-rio.br

[2]Université Paris-Est, Laboratory of Multi-Scale Modeling and Simulation
5, Boulevard Descartes 77454, Marne-la-Vallée Cedex 2, France
email: christian.soize@univ-paris-est.fr

ABSTRACT: A drillstring is a long column under rotation, composed by a sequence of connected drill-pipes and auxiliary equipment, which is used to drill the soil in oil prospecting. During its operation, this column presents a three-dimensional dynamics, subjected to longitudinal, lateral, and torsional vibrations, besides the effects of friction and shock. Due to the relevance of this equipment in some engineering applications, this work aims to analyze the nonlinear dynamics of a drillstring in horizontal configuration. A computational model, which uses a nonlinear beam theory with rotatory inertia and shear deformation of the beam cross section, as well as the coupling between longitudinal, transverse, and torsional vibrations is proposed. This model also takes into account the effects of friction and shock, induced by the lateral impacts between the drillstring and the borehole wall. The model equations are discretized using the Galerkin/finite element method, and then projected in vector space of low dimension to reduced the computational cost of the simulation. The initial value problem that results from the discretization is integrated using the Newmark method. Numerical simulations are conducted to investigate the effects of the lateral shock in nonlinear dynamic of the column. The simulation results show that the lateral shock induces geometrical configurations "close" to flexural modes of higher order. It is also noted that the geometric nonlinearity induces a coupling between the mechanisms of longitudinal and transverse vibration, so that the system responds longitudinally despite not being excited in this direction.

KEY WORDS: nonlinear dynamics, drillstring dynamics, nonlinear beam theory, geometric nonlinearity

1 Introduction

Oil prospecting uses an equipment called drillstring, which is a long column under rotation, composed by a sequence of connected drill-pipes and auxiliary equipment. The dynamics of this column is very complex because, under normal operational conditions, it is subjected to longitudinal, lateral and torsional vibrations, which present a nonlinear coupling, and the structure is also subjected to shocks due to the drill-bit/soil and drill-pipes/borehole impacts [1].

By being a matter of practical interest, with many applications in engineering, the dynamics of a drillstring has been analyzed in many scientific papers [2], [3], [4], [5], [6], [7], [8]. These works investigate the couplings between the different mechanisms of vibration to which the structure is subject, and most of them use probabilistic approaches to quantify the uncertainties in the system response that are induced by the variability of the model parameters and the errors made in the conception of the model. A common feature to all these works, is that the drillstring configuration considered is vertical. To the best of the authors' knowledge, there is only one study in the open literature that models a horizontal drillstring [9], which takes into account only the longitudinal dynamics of the structure. Therefore, the scientific literature lacks models to describe the nonlinear dynamics of a horizontal drillstring.

In this sense this work intends to analyze the three-dimensional nonlinear dynamics of a drillstring in horizontal configuration. For this purpose, it is presented a deterministic computational model which uses a nonlinear beam theory that takes into account the rotatory inertia and shear deformation of the beam cross section, as well as the coupling between longitudinal, transverse and torsional vibrations. This model also includes the effects of torsional friction and lateral shocks induced by the impacts between the drillstring and the borehole wall. Numerical simulations are conducted in order to investigate the influence of the lateral shocks in the nonlinear dynamic behavior the mechanical system under analysis.

The organization of the paper is as follows. The section 2 presents the mathematical modeling of the physical system of interest. In the section 3 are presented the numerical results of simulations which were performed to analyze the nonlinear dynamics of the horizontal drillstring. Finally, in the section 4, the conclusions of the work are reemphasized, and some paths for future works are pointed out.

2 Mathematical modeling

2.1 Mechanical system of interest

The bottom part of the horizontal drillstring is modeled as an annular beam, blocked to move transversely in both extremes, and blocked to rotate transversely in the left extreme. This beam is free to rotate around the x axis, and to move longitudinally. It has a length L, cross section area A, and is made of a material with mass density ρ, elastic modulus E, and shear modulus G. Due to the horizontal configuration, it is subject to a gravitational field, which induces an acceleration g. It loses energy through a mechanism of viscous dissipation, proportional to the mass operator, with damping coefficient c. The advance of the beam is controlled by the imposed constant velocity V_0 at the left end. A sketch of this physical system can be seen in the Figure 1.

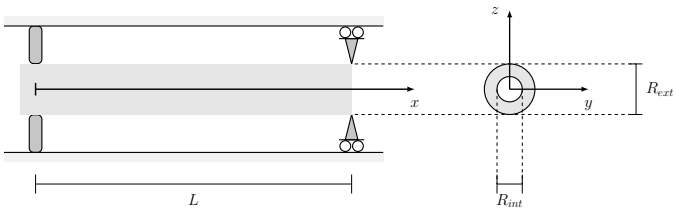


Figure 1 Sketch of the annular rotating beam used to model the bottom part of the horizontal drillstring.

In this beam model, it is supposed small rotations in the transversal directions and large displacements for the beam neutral fiber displacements, so that the following kinematic hypothesis is adopted

$$\begin{aligned} u_x(x,y,z,t) &= u - y\theta_z + z\theta_y, \\ u_y(x,y,z,t) &= v + y\,(\cos\theta_x - 1) - z\sin\theta_x, \\ u_z(x,y,z,t) &= w + z\,(\cos\theta_x - 1) + y\sin\theta_x, \end{aligned} \tag{1}$$

where u_x, u_y, and u_z respectively denote the displacement of a beam point in x, y, and z directions, at the instant t. Also, u, v, and w are the displacements of a beam neutral fiber point in x, y, and z directions, respectively, while θ_x, θ_y, and θ_z represent rotations of the beam around the x, y, and z axes respectively.

Note that, to analyze the dynamics of this beam, the physical quantities of interest are the fields u, v, w, θ_x, θ_y, and θ_z, which depend on the position x and the time t.

2.2 Friction and shock effects

The effects of normal shock and torsional friction between the beam and the borehole wall are modeled in terms of a measure of penetration in the wall of a beam cross section [10], dubbed *indentation*, which is defined as

$$\delta_{FS} = r - \mathtt{gap}, \tag{2}$$

where the lateral displacement of the neutral fiber is

$$r = \sqrt{v^2 + w^2}, \tag{3}$$

and $\mathtt{gap}$ denotes the spacing between the undeformed beam and the borehole wall. One has that $\delta_{FS} > 0$ in case of an impact, or $\delta_{FS} \leq 0$ otherwise, as can be seen in Figure 2.

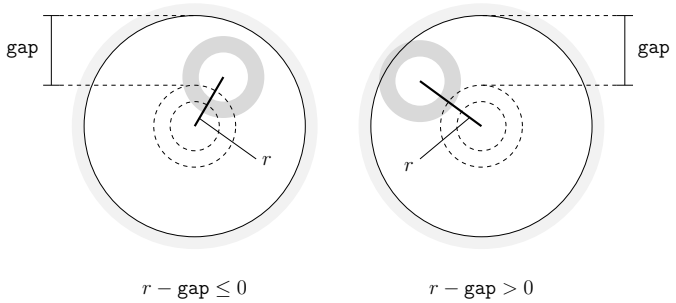


Figure 2 Illustration of the indentation parameter in a situation without impact (left) or with impact (right).

When an impact occurs, it begins to act on the beam cross section a normal force of the form

$$F_{FS} = -k_{FS_1}\,\delta_{FS} - k_{FS_2}\,\delta_{FS}^3 - c_{FS}\,|\delta|^3\dot{\delta}_{FS}, \tag{4}$$

and a Coulomb frictional torque of the form

$$T_{FS} = -\mu_{FS}\,F_{FS}\,R_{bh}\,\mathtt{sgn}\left(\dot{\theta}_x\right). \tag{5}$$

In the above equations, k_{FS_1}, k_{FS_2} and c_{FS} are constants of the Hunt and Crossley shock model [11], while μ_{FS} is a friction coefficient, R_{bh} is the borehole radius, and $\mathtt{sgn}\,(\cdot)$ the sign function.

It is adopted the hypothesis that the mechanical contact between the beam and the borehole wall occurs exactly in the nodes of the finite element mesh, so that the forces and torques of shock are modeled as concentrated forces and torques in these nodes.

2.3 Weak formulation of the nonlinear dynamics

Starting from a modified version of the extended Hamilton's principle, to include the effects of dissipation, one can write the weak form of the nonlinear equation of motion of the physical system as

$$\mathcal{M}\left(\boldsymbol{\psi},\ddot{\boldsymbol{U}}\right)+\mathcal{C}\left(\boldsymbol{\psi},\dot{\boldsymbol{U}}\right)+\mathcal{K}\left(\boldsymbol{\psi},\boldsymbol{U}\right)=\mathcal{F}_{NL}\left(\boldsymbol{\psi},\boldsymbol{U},\dot{\boldsymbol{U}},\ddot{\boldsymbol{U}}\right), \tag{6}$$

where $\mathcal{M}$ represents the mass operator, $\mathcal{C}$ is the damping operator, $\mathcal{K}$ is the stiffness operator, and $\mathcal{F}_{NL}$ is the nonlinear force operator. Also, the field variables and their weight functions are lumped in the vectors fields

$$\boldsymbol{U}=\begin{pmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{pmatrix}, \qquad \text{and} \qquad \boldsymbol{\psi}=\begin{pmatrix} \psi_u \\ \psi_v \\ \psi_w \\ \psi_{\theta_x} \\ \psi_{\theta_y} \\ \psi_{\theta_z} \end{pmatrix}. \tag{7}$$

The weak form of the initial conditions reads

$$\mathcal{M}\left(\boldsymbol{\psi},\boldsymbol{U}(0)\right)=\mathcal{M}\left(\boldsymbol{\psi},\boldsymbol{U}_0\right), \tag{8}$$

and

$$\mathcal{M}\left(\boldsymbol{\psi},\dot{\boldsymbol{U}}(0)\right)=\mathcal{M}\left(\boldsymbol{\psi},\dot{\boldsymbol{U}}_0\right), \tag{9}$$

where $\boldsymbol{U}_0$ and $\dot{\boldsymbol{U}}_0$, respectively, denote the initial displacement, and the initial velocity fields.

2.4 Discretization of the model equations

The Galerkin/finite element method [12] is employed to construct an approximation to the solution of the boundary/initial value problem corresponding to the weak formulation given by Eqs.(6), (8) and (9).

To interpolate the longitudinal displacement and the axial rotation fields, linear shape functions are used, while the other fields are interpolated by cubic splines.

In this way, one gets an initial value problem of the form

$$[\mathcal{M}]\,\ddot{\boldsymbol{Q}}(t)+[\mathcal{C}]\,\dot{\boldsymbol{Q}}(t)+[\mathcal{K}]\,\boldsymbol{Q}(t)=\boldsymbol{\mathcal{F}}_{NL}\left(\boldsymbol{Q}(t),\dot{\boldsymbol{Q}}(t),\ddot{\boldsymbol{Q}}(t)\right), \tag{10}$$

$$\boldsymbol{Q}(0)=\boldsymbol{Q}_0, \qquad \text{and} \qquad \dot{\boldsymbol{Q}}(0)=\dot{\boldsymbol{Q}}_0, \tag{11}$$

where $\boldsymbol{Q}(t)$ is the generalized displacement vector, $\dot{\boldsymbol{Q}}(t)$ is the generalized velocity vector, $\ddot{\boldsymbol{Q}}(t)$ is the generalized acceleration vector, $[\mathcal{M}]$ is the mass matrix, $[\mathcal{C}]$ is the damping matrix, $[\mathcal{K}]$ is the stiffness matrix. Also, $\boldsymbol{\mathcal{F}}_{NL}$, $\boldsymbol{Q}_0$, and $\dot{\boldsymbol{Q}}_0$ are vectors which, respectively, represent the nonlinear force, the initial generalized position, and the initial generalized velocity.

The initial value problem defined by Eqs.(10) and (11) is integrated using the Newmark method [13], [12], which is an integration scheme proper for differential equations that comes from of structural dynamics. The nonlinear system of algebraic equations, resulting from the time discretization, is solved by a fixed point iteration [14].

3 Numerical experimentation

In order to simulate the nonlinear dynamics of the mechanical system defined in the section 2.1, the physical parameters presented in the Table 1 are adopted. The dynamics is investigated during the interval of time $[t_0,\, t_f] = [0,\, 1.5]\ s$, being adopted as initial conditions: a zero displacement; a constant angular velocity Ω around the x axis; and a constant longitudinal velocity V_0.

Table 1. Physical parameters used in the simulation.

parameter	value	unit
ρ	7900	kg/m^3
E	203	GPa
G	78	GPa
R_{int}	50	mm
R_{ext}	80	mm
R_{bh}	95	mm
A	5500π	mm^2
L	100	m
c	1×10^{-2}	—
g	9.81	m/s^2
Ω	50	rpm
V_0	5	m/h
k_{FS_1}	1×10^{10}	N/m
k_{FS_2}	1×10^{16}	N/m^3
c_{FS}	1×10^{-3}	N/m^4s
μ_{FS}	5×10^{-4}	—

The discretization of the structure uses a finite element mesh with 500 elements. As each element has 6 degrees of freedom per node, this results in a semi-discrete model with 3006 degrees of freedom, which is projected in a vector space of dimension 56 to generate a reduced order model to efficient computation.

For temporal integration, the numerical scheme uses as nominal time step $\Delta t = 2.5\ ms$, which is refined when necessary to capture the effects of shock on the dynamics.

3.1 Transverse dynamics of the drillstring and the lateral impacts

In this section it is analyzed the transverse dynamics of the beam, whose neutral fiber displacement in the z direction is presented, for some instants, in the Figure 3. Illustrations of the mechanical system, sectioned by the plane $y = 0$, for the same instants, can be seen in the Figure 4.

One can note that, for none of the instants analyzed the neutral fiber assumes a parabolic shape, as occurs when a beam is subjected to its own weight. For $t = 18\ ms$, this is a pure consequence of the nonlinear inertial effects, induced by the rotation of the beam, combined with the geometric nonlinearity due to the large displacements. At

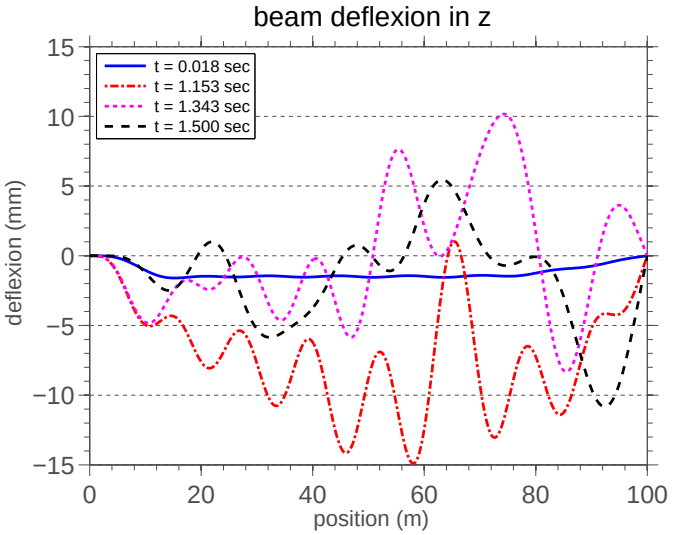


Figure 3 This figure presents the beam neutral fiber displacement, in the z direction, for some instants.

this moment, the beam configuration is "close" to the first flexural mode, as can be seen in the Figure 4(a).

Then, the beam begins to impact the wall of the borehole, which generates configurations "close" to high-order flexural modes, as shown in the Figure 4(b), for $t = 1.153\ s$. Note that, at this instant, the beam/wall contact is not punctual, it occurs along three line segments.

This mechanical interaction between the two bodies generates a nonlinear elastic deformation in both bodies, but without residual deformation effects. In this contact also occurs energy dissipation, due to the normal shock, and the torsional friction, induced by the rotation of the beam.

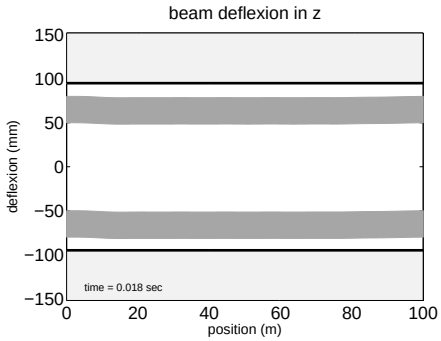
In the instants of analysis that follow, additional impacts do not occur, and the beam continues to assume configurations that are "close" to flexural modes of order higher than one, as can be see Figures 4(c) and 4(d).

3.2 Coupling between the longitudinal and the transverse dynamics of the drillstring

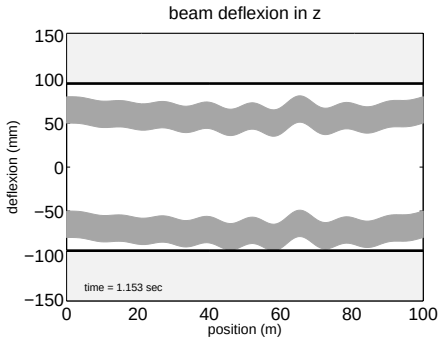
In what follows, it is analyzed the longitudinal dynamic behavior of the beam. The reader can see the beam neutral fiber displacement, in the x direction, for several instants, in the Figure 5.

Note that, before impact, for time $t = 18\ ms$, the beam has virtually no longitudinal displacement. However, after the beginning of the impacts the entire structure presents longitudinal displacement, alternating between traction (on the left part) and compression (on the right part) around the center of the beam.

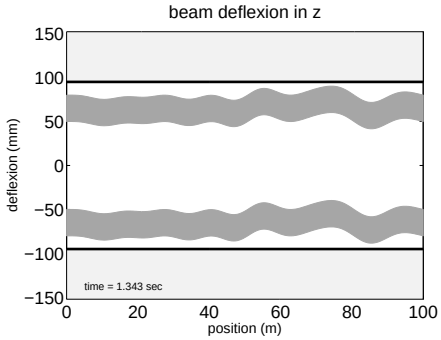
Finally, it is worth mentioning that it is surprising that the beam presents a longitudinal dynamics, since



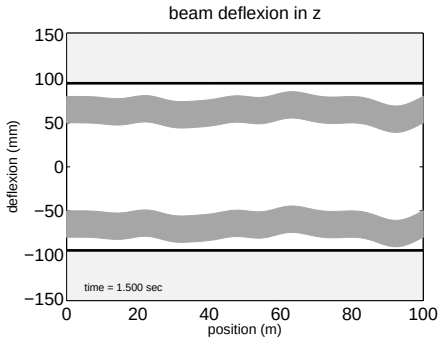
(a) $t = 0.018\ s$



(b) $t = 1.153\ s$



(c) $t = 1.343\ s$



(d) $t = 1.5\ s$

Figure 4 This figure illustrates the mechanical system under analysis, sectioned by the plane $y = 0$, for the several instants.

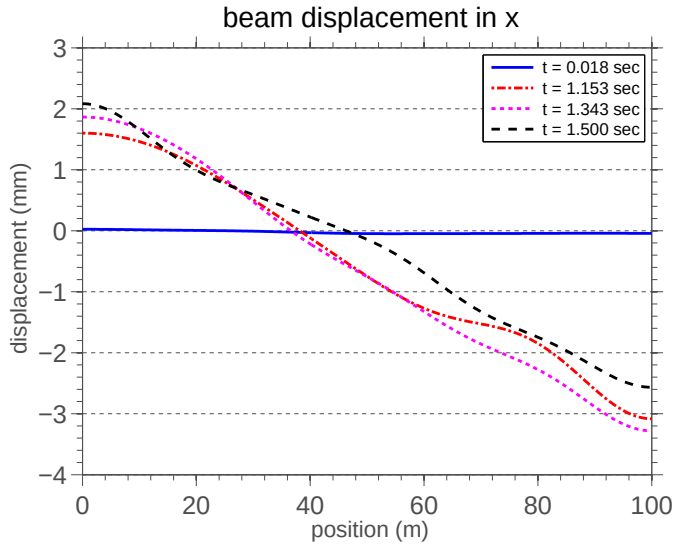


Figure 5 This figure presents the beam neutral fiber displacement, in the x direction, for several instants.

the mechanical system is not excited longitudinally by no charge. This behavior is a result of the nonlinear coupling between the transverse and longitudinal dynamics of the dynamic structure, that is carried by the forces coming from the geometric nonlinearity of the problem.

4 Final remarks and future works

In this work it was discussed the the effects induced by lateral shock in nonlinear dynamic behavior of a annular beam that emulates a drillstring in horizontal configuration. The analysis of the system was performed, through the spatial configurations adopted by the beam at different instants, and the respective power spectral density functions.

The results show that the shock significantly modifies the dynamics of the mechanical system, inducing configurations "close" to high-order flexural modes. It is also observed a nonlinear coupling between the mechanisms of longitudinal and transverse vibrations, caused by the geometric nonlinearity of the problem. This coupling generates a longitudinal dynamics in the mechanical system, even in the absence of axial excitation.

In a future work, in order to have a drillstring deterministic model more realistic, it is expected includes in the modeling the effects of fluid-structure interaction, that results from the flow of drilling fluid inside and outside of the drillstring. The authors also intend to develop a stochastic modeling of the problem, in order to quantify the uncertainties of the model [15], [16]. Uncertainties that are due to the variability of its parameters, and/or epistemic in nature, i.e, result of the ignorance about the physics of the problem [17].

Acknowledgments

The authors are indebted to the Brazilian agencies CNPq, CAPES, and FAPERJ for the financial support given to this research.

References

[1] P. D. Spanos, A. M. Chevallier, N. P. Politis, and M. L. Payne, "Oil and gas well drilling: a vibrations perspective," *The Shock and Vibration Digest*, vol. 35, pp. 85–103, 2003.

[2] P. D. Spanos, A. M. Chevallier, and N. P. Politis, "Nonlinear stochastic drill-string vibrations," *Journal of Vibration and Acoustics*, vol. 124, pp. 512–518, 2002, doi:10.1115/1.1502669.

[3] M. A. Trindade, C. Wolter, and R. Sampaio, "Karhunen–Loève decomposition of coupled axial/bending vibrations of beams subject to impacts," *Journal of Sound and Vibration*, vol. 279, pp. 1015–1036, 2005, doi:10.1016/j.jsv.2003.11.057.

[4] R. Sampaio, M.T. Piovan, and G. Venero Lozano, "Coupled axial/torsional vibrations of drill-strings by means of non-linear model," *Mechanics Research Communications*, vol. 34, pp. 497–502, 2007, doi:10.1016/j.mechrescom.2007.03.005.

[5] T. G. Ritto, C. Soize, and R. Sampaio, "Non-linear dynamics of a drill-string with uncertain model of the bit–rock interaction," *International Journal of Non-Linear Mechanics*, vol. 44, pp. 865–876, 2009, doi:10.1016/j.ijnonlinmec.2009.06.003.

[6] T. G. Ritto, C. Soize, and R. Sampaio, "Stochastic dynamics of a drill-string with uncertain weight-on-hook," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 32, pp. 250–258, 2010, doi:10.1590/S1678-58782010000300008.

[7] T. G. Ritto, C. Soize, and R. Sampaio, "Robust optimization of the rate of penetration of a drill-string using a stochastic nonlinear dynamical model," *Computational Mechanics*, vol. 45, pp. 415–427, 2010, doi:10.1007/s00466-009-0462-8.

[8] T. G. Ritto and R. Sampaio, "Stochastic drill-string dynamics with uncertainty on the imposed speed and on the bit-rock parameters," *International Journal for Uncertainty Quantification*, vol. 2, pp. 111–124, 2012, doi:10.1615/Int.J.UncertaintyQuantification.v2.i2.

[9] T. G. Ritto, M. R. Escalante, R. Sampaio, and M. B. Rosales, "Drill-string horizontal dynamics with uncertainty on the frictional force," *Journal of Sound and Vibration*, vol. 332, pp. 145–153, 2013, doi:10.1016/j.jsv.2012.08.007.

[10] G. Gilardi and I. Sharf, "Literature survey of contact dynamics modelling," *Mechanism and Machine Theory*, vol. 37, pp. 1213–1239, 2002, doi:10.1016/S0094-114X(02)00045-9.

[11] K. H. Hunt and F. E. Crossley, "Coefficient of restitution interpreted as damping in vibroimpact," *Journal of Applied Mechanics*, vol. 42, pp. 440–445, 1975, doi:10.1115/1.3423596.

[12] T. J. R. Hughes, *The Finite Element Method*, Dover Publications, New York, 2000.

[13] N. M. Newmark, "A method of computation for structural dynamics," *Journal of the Engineering Mechanics Division*, vol. 85, pp. 67–94, 1959.

[14] R. W. Hamming, *Numerical Methods for Scientists and Engineers*, Dover Publications, New York, 2nd edition, 1987.

[15] G. I. Schuëller, "A state-of-the-art report on computational stochastic mechanics," *Probabilistic Engineering Mechanics*, vol. 12, pp. 197–321, 1997.

[16] G. I. Schuëller, "On the treatment of uncertainties in structural mechanics and analysis," *Computers & Structures*, vol. 85, pp. 235–243, 2007.

[17] C. Soize, "Stochastic modeling of uncertainties in computational structural dynamics — Recent theoretical advances," *Journal of Sound and Vibration*, vol. 332, pp. 2379—2395, 2013, doi:10.1016/j.jsv.2011.10.010.