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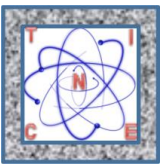
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A new way for the study of the shaking of reinforced concrete structures

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Introduction

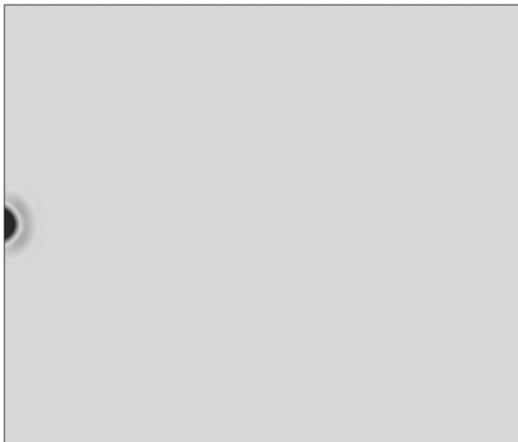
The purpose of our study consists in the research of new ways of calculating the induced vibrations in reinforced concrete structures submitted to commercial aircraft impact. The cutoff frequency for this type of loading is typically within the 40 to 100 Hz range, which would be referred to as the medium frequency range [HER05].

The determination of the shaking induced by an aircraft impact on an industrial structure requires dynamic studies. The response, especially during the transient stage, by using classical finite element method associated with explicit numerical schemes requires calculations with an important calculation time. Also this kind of calculation requires several load cases to be analyzed to consider a wide range of scenarios. By the way, the medium frequency has to be appropriately considered therefore the mesh has to be very refined and consequently, a refined time discretization.

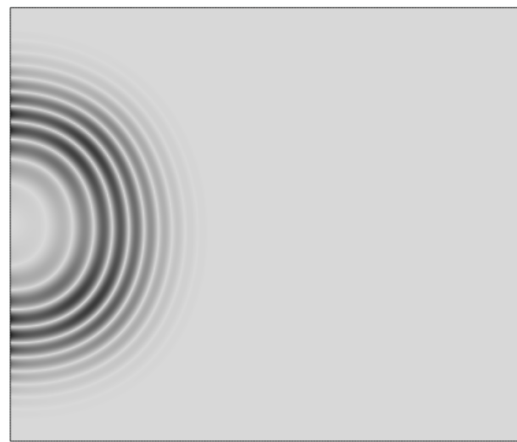
1. Problem description

1.1. Forced vibration problem

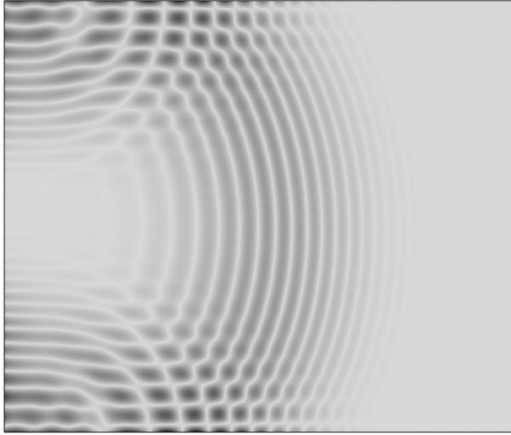
When a structure is subjected to a very brief mechanical shock, as is the case when a projectile impacts a structure, vibrational regimes can be separated in terms of the appearance of the displacement field observed. To illustrate this, we consider the example of a plate timely requested by a shock $F(t)$ (next figure). The four vibrational regimes that can be separated are:



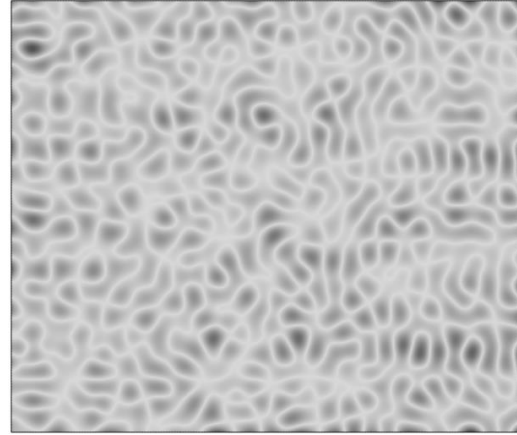
1) Very short time after the shock (0 to 0.5 milliseconds)



2) Short time after the shock (0.5 to 1 milliseconds)



3) Mean time after the shock (1 to 3 milliseconds)



4) Long time after the shock (3 to 10 milliseconds)

Figure 1. Vibrations of a plate subjected to mechanical shock

1) In very short time after the shock, there is a first vibratory regime characterized by the propagation of a wave in a semi-infinite space (wave suffered no reflection at the walls of the structure).

2) In short time after the shock, there is a vibration system which is again characterized by a phenomenon of wave propagation, but the waves are now guided by the thickness of the plate. This guide phenomenon is usually seen as a large majority of structures such as beams, plates or shells which have one dimension lower than the other. In this case, the structure is often referred to by the term "waveguide".

3) In the mean time after the shock, guided waves are propagated to the edges of the plate, then lead to wave reflection phenomena. The reflected waves interfere with the direct wave which gives rise to localized vibrational modes in some parts of the plate.

4) Finally, long time after the shock, guided waves are reflected many times at the edges of the plate. Now these waves interfere with each other on the entire plate so the structure vibrates anywhere.

The determination of the shaking induced by an aircraft impact on an industrial structure requires dynamic studies on long time after the shock. The response cannot be completely described using classical method. Finite element method associated with explicit numerical schemes could lead to prohibitive computation times. Indeed, the medium frequency range is often ignored unless the calculation is carried out with a very refined mesh (ten linear elements per wavelength for good accuracy) and consequently, a refined time discretization.

1.2. Strategy description

To solve our problem of shock induced vibration in a reinforced concrete structure the strategy implementation is as follows (the figure below). Load is applied to the target structure and the answer is calculated by finite element method in non-linear and on a sufficiently short time. A sensitivity analysis associated with an experimental design (Taguchi methods) should allow us to determine the radius of the damaged area and attenuation of nonlinear area on the

input signal. Then we can apply the temporal attenuated signal across the damaged area to obtain the response of the structure by a calculation with the VTCR (Variational Theory of Complex Rays) method. This calculation requires a shift in frequency domain by FFT (Fast Fourier Transform) and a time recomposition by IFFT (Inverse Fast Fourier Transform).

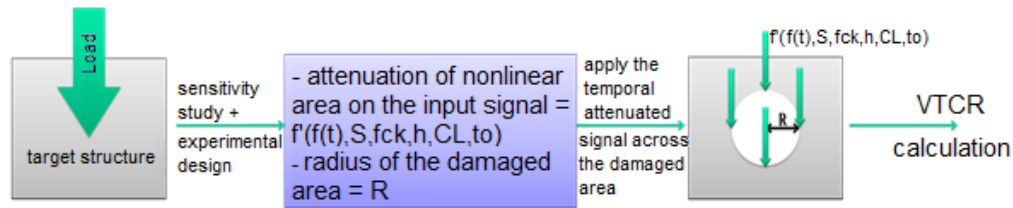


Figure 2. Global calculation strategy

2. Variational Theory of complex rays (VTCR) description

This work, which uses new computational strategies in dynamics, provides an answer to this challenge for the steady state of the solution. The problem is solved in the frequency domain. One needs to solve a forced vibration problem over a frequency range which includes the low- and medium-frequency ranges. The low-frequency and medium-frequency ranges are handled using the Variational Theory of Complex Rays (VTCR).

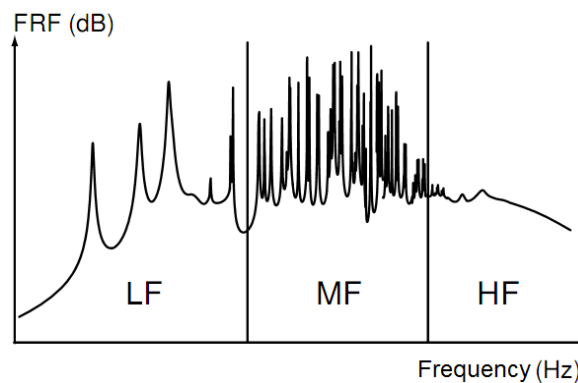


Figure 3. Frequency response function of complex [OHA98]

2.1. The reference problem for an assembly of two substructures

In order to simplify the presentation, we present the problem for an assembly of two substructures, but this formulation can be easily generalized to an arrangement of n substructures.

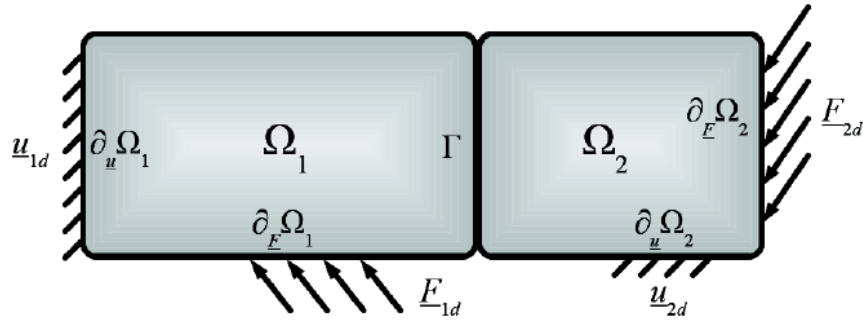


Figure 4. Reference problem for an assembly of two substructures

2.2. The variational formulation associated with the VTCR

The VTCR is a global formulation of the boundary conditions in terms of both displacements and forces. It is based on a priori independent approximations within the substructures. The constitutive relation and dynamic equilibrium equation are exactly verified for each substructure Ω_i to form the corresponding subspace $\mathcal{S}_{ad,i}$:

$$\mathcal{S}_{ad,i} = \left\{ (\underline{U}_i; \underline{\sigma}_i) \in \mathcal{U}_i \times \mathcal{S}_i, \begin{cases} \text{div } \underline{\sigma}_i = -\rho_i \omega^2 \underline{U}_i & \text{in } \Omega_i \\ \underline{\sigma}_i = \underline{K}_i : \underline{\varepsilon}(\underline{U}_i) & \text{in } \Omega_i \end{cases} \right\} \quad (1)$$

The variational formulation can be expressed as:

$$\mathcal{A} \left(\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \begin{pmatrix} \delta s_1 \\ \delta s_2 \end{pmatrix} \right) = \mathcal{L} \left(\begin{pmatrix} \delta s_1 \\ \delta s_2 \end{pmatrix} \right), \quad \forall (\delta s_1, \delta s_2) \in (\mathcal{S}_{ad,1}^0, \mathcal{S}_{ad,2}^0) \quad (2)$$

with:

$$\begin{aligned} \mathcal{A} \left(\begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \begin{pmatrix} \delta s_1 \\ \delta s_2 \end{pmatrix} \right) &= \text{Re} \left\{ -i\omega \left[\int_{\partial_{\underline{u}}\Omega_1} \underline{\delta\sigma}_1 \underline{n}_1 \cdot \underline{U}_1^* dS + \int_{\partial_{\underline{F}}\Omega_1} \underline{\sigma}_1 \underline{n}_1 \cdot \delta \underline{U}_1^* dS + \int_{\partial_{\underline{u}}\Omega_2} \underline{\delta\sigma}_2 \underline{n}_2 \cdot \underline{U}_2^* dS \right. \right. \\ &\quad \left. \left. + \int_{\partial_{\underline{F}}\Omega_2} \underline{\sigma}_2 \underline{n}_2 \cdot \delta \underline{U}_2^* dS + \frac{1}{2} \int_{\Gamma} \left\{ (\underline{\delta\sigma}_1 \underline{n}_1 - \underline{\delta\sigma}_2 \underline{n}_2) \cdot (\underline{U}_1 - \underline{U}_2)^* \right. \right. \right. \\ &\quad \left. \left. \left. + (\underline{\sigma}_1 \underline{n}_1 - \underline{\sigma}_2 \underline{n}_2) \cdot (\delta \underline{U}_1 - \delta \underline{U}_2)^* \right\} dS \right\} \right\} \\ \mathcal{L} \left(\begin{pmatrix} \delta s_1 \\ \delta s_2 \end{pmatrix} \right) &= \text{Re} \left\{ -i\omega \left[\int_{\partial_{\underline{u}}\Omega_1} \underline{\delta\sigma}_1 \underline{n}_1 \cdot \underline{U}_{1d}^* dS + \int_{\partial_{\underline{F}}\Omega_1} \underline{F}_{1d} \cdot \delta \underline{U}_1^* dS + \int_{\partial_{\underline{u}}\Omega_2} \underline{\delta\sigma}_2 \underline{n}_2 \cdot \underline{U}_{2d}^* dS \right. \right. \\ &\quad \left. \left. + \int_{\partial_{\underline{F}}\Omega_2} \underline{F}_{2d} \cdot \delta \underline{U}_2^* dS \right] \right\} \end{aligned}$$

where Re designates the real part of a quantity and $*$ the conjugate.

- Could verify the fixtures on Ω_1
- Could verify the loads on Ω_1
- Could verify the fixtures on Ω_2
- Could verify the loads on Ω_2
- Could verify the boundary conditions on Γ

It is easy to prove that the variational form is equivalent to the reference problem, provided that:

- the reference problem has a solution,
- the Hooke's operator \underline{K}_i is positive definite,
- the damping coefficients are such that $\eta_i > 0$.

2.3. Principle of approximate formulations

The VTCR uses two scales of approximation $(\underline{U}^h, \underline{\sigma}^h)$, each with a strong mechanical meaning, defined by identifying three zones: the interior zone, the edge zone and the corner zone. For example, in the neighborhood of a point \underline{X} of the interior zone, the solution is assumed to be properly described locally as the superposition of an infinite number of local vibration modes which can be written in the following manner:

$$\underline{U}^h(\underline{X}, \underline{Y}, \underline{P}) = \underline{W}^h(\underline{X}, \underline{P}) e^{i\omega \underline{P} \cdot \underline{Y}} \quad (3)$$

$$\underline{\sigma}^h(\underline{X}, \underline{Y}, \underline{P}) = \underline{C}^h(\underline{X}, \underline{P}) e^{i\omega \underline{P} \cdot \underline{Y}} \quad (4)$$

where both \underline{X} and \underline{Y} represent the position vector, \underline{X} being associated with slow variations and \underline{Y} with rapid variations. More precisely, the terms related to the position vector \underline{X} vary slowly when \underline{X} moves along the structure, whereas the terms related to the position vector \underline{Y} vary rapidly when \underline{Y} moves along the structure. \underline{P} is a vector characterizing the local vibration mode. In order for these local modes $(\underline{U}^h, \underline{\sigma}^h)$ to be admissible, they must be in \mathcal{S}_{ad}^h and verify the constitutive and dynamic equilibrium equation. Thus, we get some properties of \underline{P} . For instance, let us consider the out-of-plane bending motions of thin, flat, homogeneous and isotropic plates. According to Kirchhoff's thin plate theory, the steady-state displacement w of the plate's mid-surface in the direction perpendicular to the plate is governed by the dynamic equation:

$$\frac{(1+i\eta)Eh^3}{12(1-\nu^2)} \Delta \Delta w = \rho h \omega^2 w \quad \text{on } \Omega \quad (5)$$

where Δ is the Laplacian operator, E the Young's modulus, h the plate's thickness, ν the Poisson's ratio, ρ the mass density, ω the frequency, and η the damping factor.

The complex rays for the interior, edge and corner modes are:

$$\begin{cases} w_{int}^h(\underline{x}, \underline{p}) = W_{int}^h(\underline{x}, \underline{p}) \cdot e^{\frac{\eta}{4}\sqrt{\omega} \underline{p} \cdot \underline{x}} \cdot e^{i\sqrt{\omega} \underline{p} \cdot \underline{x}} \\ w_{bord}^h(\underline{x}, \underline{p}) = W_{bord}^h(\underline{x}, \underline{p}) \cdot e^{-\frac{\eta}{4}\sqrt{\omega} \frac{r_0^2}{p n} \underline{n} \cdot \underline{x}} \cdot e^{\sqrt{\omega} p n \underline{n} \cdot \underline{x} + i\sqrt{\omega} p t \underline{t} \cdot \underline{x}} \\ w_{coin}^h(\underline{x}, \underline{p}) = W_{coin}^h(\underline{x}, \underline{p}) \cdot e^{-i\frac{\eta}{4}\sqrt{\omega} \underline{p} \cdot \underline{x}} \cdot e^{\sqrt{\omega} \underline{p} \cdot \underline{x}} \end{cases} \quad (6)$$

This complex interior ray corresponds to a plane bending wave which propagates through the plate in the \underline{P} direction.

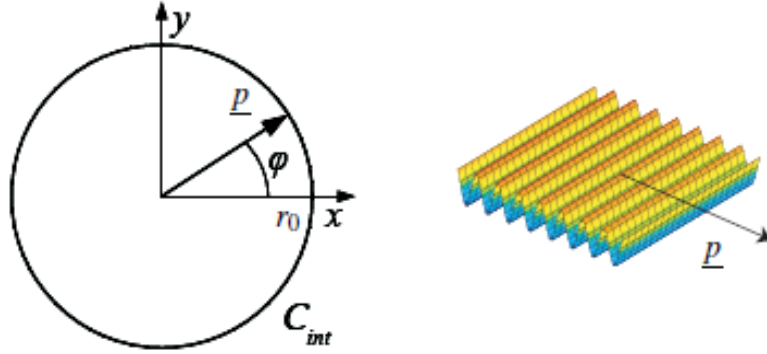


Figure 5. Description of interior modes
Examples of such modes are shown in the next figure.

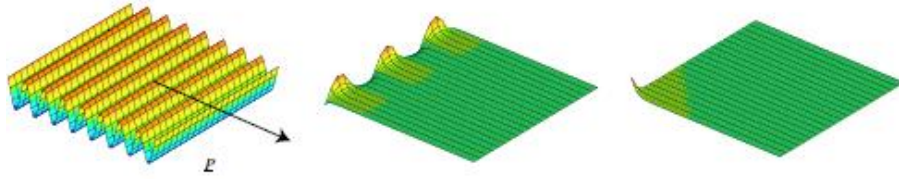


Figure 6. Interior, edge and corner modes for a homogeneous plate

2.4. The discretized problem

The displacement of any point of the substructure is generated by a basis of admissible complex rays. The unknown is the generalized amplitude $\underline{w}^h(\underline{X}, \underline{P})$ of the basis (an n th-order polynomial in \underline{X} and a large-wavelength quantity). Accounting for all the directions φ in C leads to an integral over C . This integral takes the form:

$$\begin{aligned} w(\underline{x}) = & \int_{\varphi \in C_{int}} W_{int}^h(\underline{x}, \varphi) e^{\underline{k}_{int}(\varphi) \cdot \underline{x}} d\varphi + \int_{\varphi \in C_{bord}} W_{bord}^h(\underline{x}, \varphi) e^{\underline{k}_{bord}(\varphi) \cdot \underline{x}} d\varphi \\ & + \int_{\varphi \in C_{coin}} W_{coin}^h(\underline{x}, \varphi) e^{\underline{k}_{coin}(\varphi) \cdot \underline{x}} d\varphi \end{aligned} \quad (7)$$

with

$$\begin{cases} \underline{k}_{int}(\varphi) = \frac{\eta}{4} \sqrt{\omega \underline{p}(\varphi)} + i \sqrt{\omega \underline{p}(\varphi)} \\ \underline{k}_{bord}(\varphi) = -i \frac{\eta}{4} \sqrt{\omega} \frac{r_0^2}{p_n(\varphi)} \underline{n} + \sqrt{\omega} p_n(\varphi) \underline{n} + i \sqrt{\omega} p_t(\varphi) \underline{t} \\ \underline{k}_{coin}(\varphi) = -i \frac{\eta}{4} \sqrt{\omega \underline{p}(\varphi)} + \sqrt{\omega \underline{p}(\varphi)} \end{cases} \quad (8)$$

In order to obtain a finite-dimension problem, this integral can be discretized and one can consider the amplitude $\underline{w}^h(\underline{X}, \underline{P}(\varphi))$ to be constant in each angular sector. The angular distributions of the plane waves for all points in the substructure are assumed to be well-described by this discontinuous angular distribution.

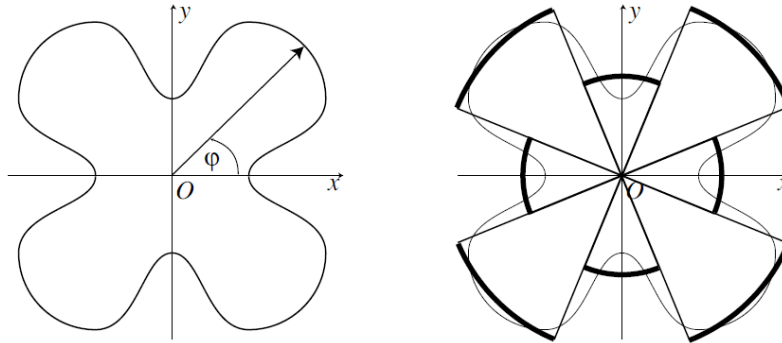


Figure 7. The discretized amplitudes

3. Numerical Example

3.1. First numerical example : one simply supported plate

In order to study the convergence of the VTCR for plate problems, let us consider the example of next figure. A clamped steel plate with the following mechanical properties is subjected to shear loading on one of its free edges.

- Young's modulus = 210 GPa
- Poisson's ratio = 0.3
- Mass density = 7800 kg/m³
- Damping coefficient = 0.01
- Thickness of the plate = 0.003 m

A reference solution using the CASTEM code was obtained with ten linear elements per wavelength for good accuracy.

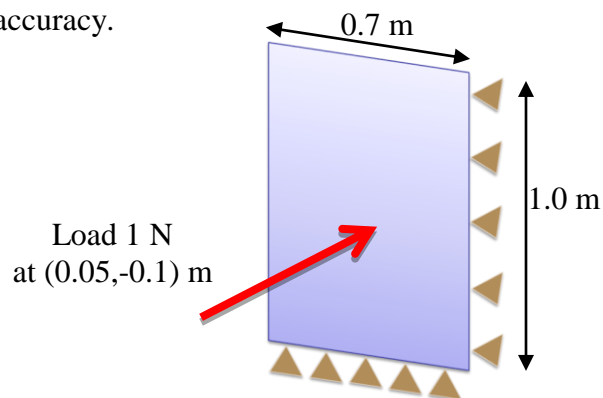
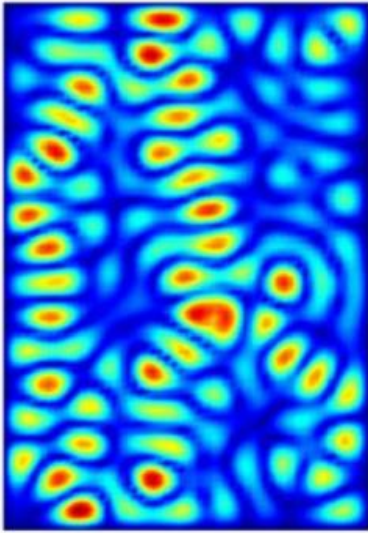
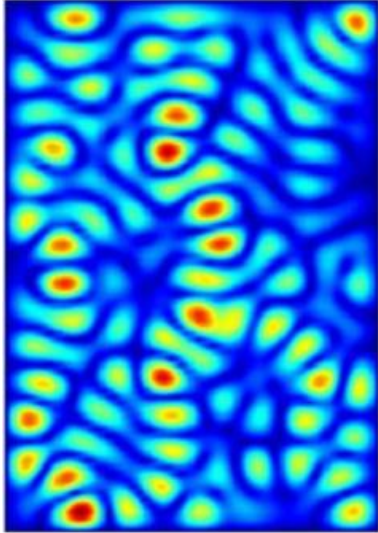
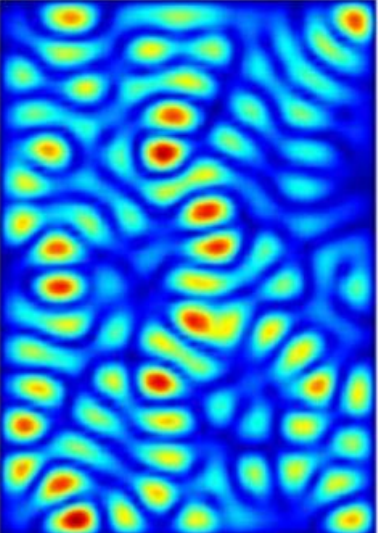


Figure 8. First example: The reference problem and the action of the environment

Next figure shows the solutions obtained with CASTEM and with the VTCR. One can see that the two solutions are very similar, even though the VTCR was obtained with only 180 DOFs.

One can easily note the computational efficiency of the VTCR in such a structural vibration problem.

Castem	Analytical results	VTCR
39046 DOF (10 elements per wavelength)		100 interior modes 4*20 edge modes
		

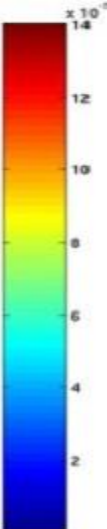


Table 1. The FE (with Castem) solution (left), the analytical solution and the VTCR solution with 180 dofs (right)

3.2. Second numerical example : shell structures

In order to study the convergence of the VTCR method for shell problems, we consider now a concrete structure where the mechanical properties of concrete are the following:

- Young's modulus = 34 GPa
- Poisson's ratio = 0.2
- Mass density = 2500 kg/m³
- Damping coefficient = 0.04

We simplify the geometry of the structure with a shell assembly of 0.15 m thick. This structure is subjected to shear loading (blue line in the next figure) on one of its free edges. A reference solution using the CASTEM code was obtained with ten linear elements per wavelength for good accuracy. This example is shown in the following figure.

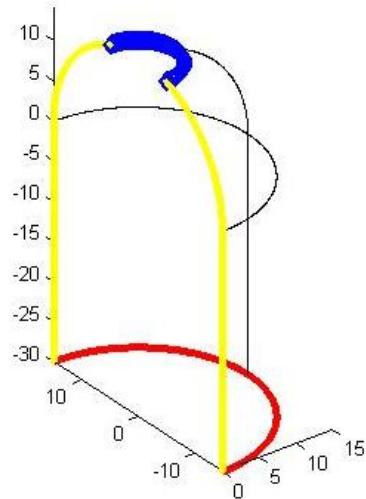


Figure 9. Second example: The reference problem and the action of the environment

Next figure shows the solutions obtained with CASTEM and with the VTCR. One can see that the two solutions are very similar, even though the VTCR was obtained, as in the first example, with less DOFs than the finite element method. One can easily note the computational efficiency of the VTCR in such a structural vibration problem.

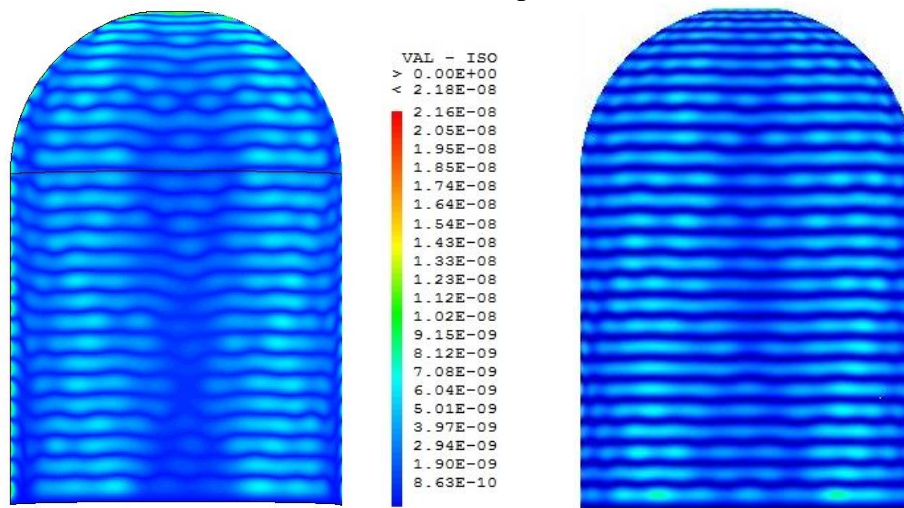


Figure 10. The FE (with Castem) solution with 36,400 dofs (left) and the VTCR solution with 1200 dofs (right)

3.3. Third numerical example : Civil engineering structure

In this section we use the VTCR to calculate the medium frequency response of a structure subjected to a sinusoidal loading. We calculate at first the discrete Fourier transform of the load. The VTCR then gives us the frequency response at a chosen point of the structure. The response time is obtained by the inverse Fourier transform. We therefore consider a concrete structure where the mechanical properties of concrete are calculated according to the rules of BAEL:

- Concrete B30 : ≈ 30 MPa
- Young's modulus = 34 GPa

- Poisson's ratio = 0.2
- Mass density = 2500 kg/m³
- Damping coefficient = 0.04

In this study, we use an hysteretic damping. We simplify the geometry of the structure with a plate assembly of 0.15 m thick. Our structure is then subjected to a sinusoidal concentrated load applied at the center of a side plate. The load is modeled by moving uniformly distributed on the boundary of a square hole in the point of application of the load. This loading is modeled by a red arrow in the following figure.

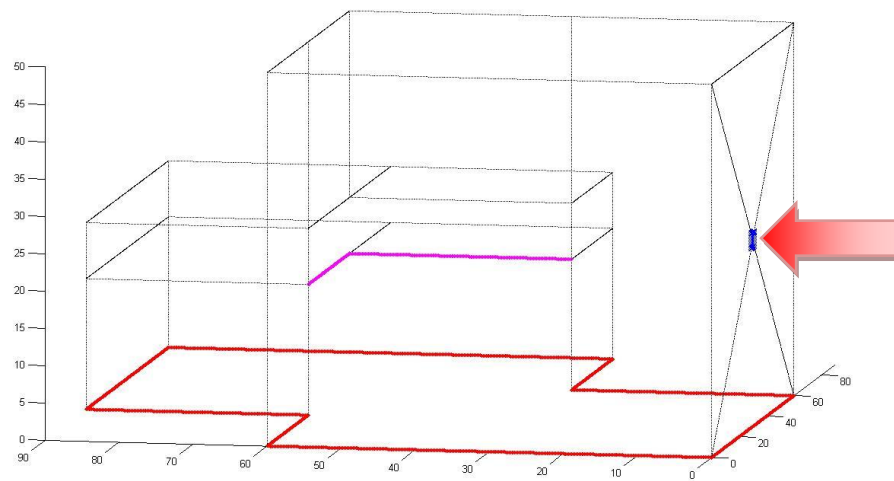


Figure 11. Geometry of second numerical example

The computational strategy is as follows. We calculate the discrete Fourier transform by the FFT load and use to calculate, using the VTCR, the frequency response corresponding to each frequency on a selected point of the structure. The program selects the frequencies having a significant amplitude to describe the good time loading. The time response is then obtained by applying the IFFT to the frequency response.

We consider a one-time loading P_1 of the form:

$$F_{P_1}(t) = 100 \sin(2\pi 10t + 10) - 200 \sin(2\pi 20t + 20) + 300 \sin(2\pi 30t + 30) - 400 \sin(2\pi 40t + 40) \quad (9)$$

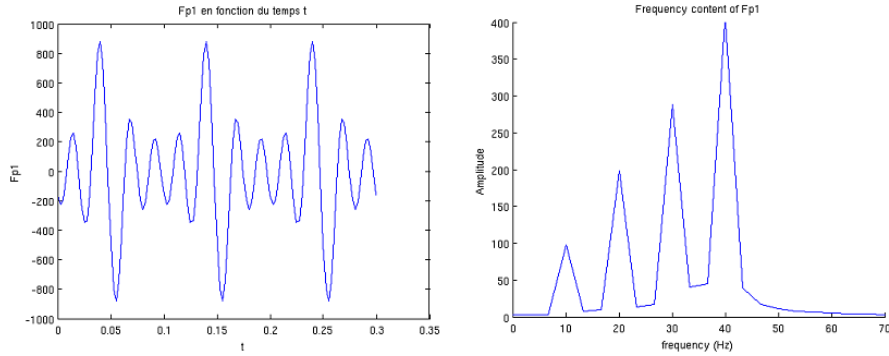


Figure 12. Loading applied and associated Fourier transform

Two hundred rays are sufficient to properly represent the frequency response. The following table shows the solution obtained in each of four frequencies studied.

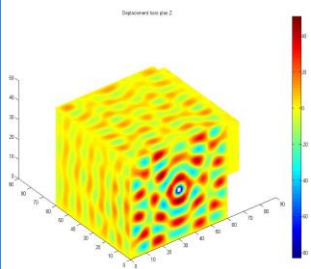
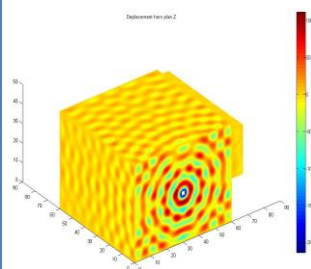
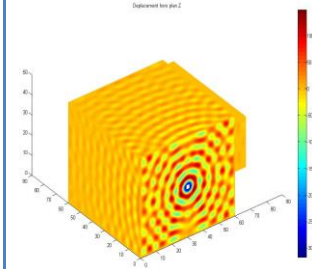
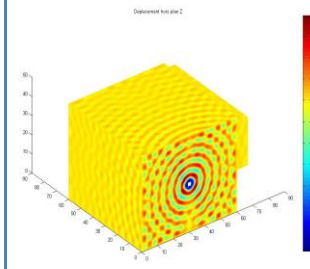
10 Hz	20 Hz	30 Hz	40 Hz
102 interior modes 4*25 edge modes			
			

Table 2. Solution of second numerical example

This study allows us to realize low cost in terms of degrees of freedom used by the VTCR for solving such a problem. The following figure shows the difference in time resolution between resolution VTCR and CASTEM for this problem.

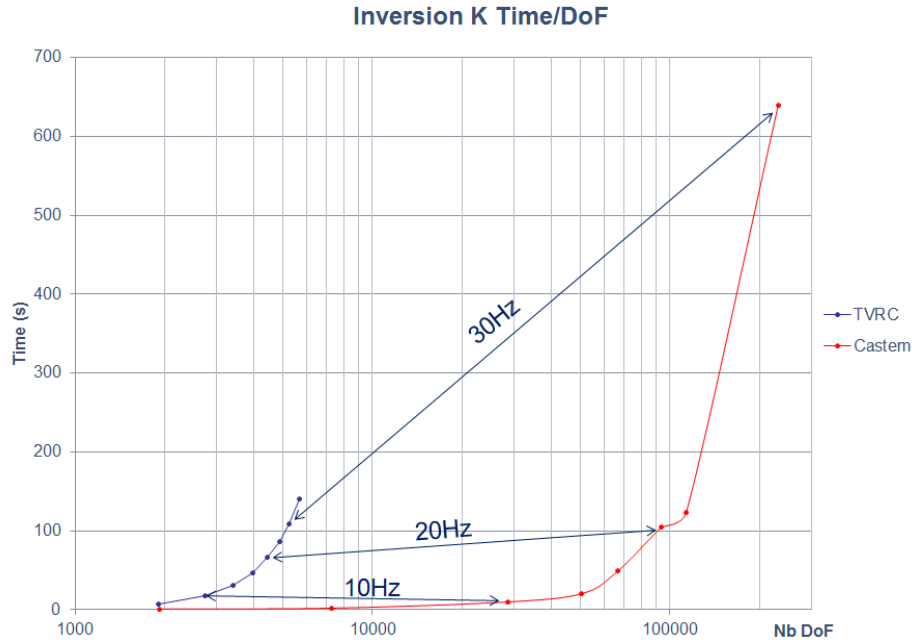


Figure 12. Comparison VTCR and CASTEM

Conclusions

This methodology provides three interests for the calculation of the induced vibration:

- Modeling a large FE Model is not necessary,
- The total frequency content is well considered without the need of a rich discretization,
- The calculation cost is less important as for FE calculation.

A set of examples will be provided in comparison with FE calculations regarding the behavior of a representative structure submitted to a time history loading. An attention is paid to the important gain of calculation time provided.

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