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# Analysis of Stability of Structural Modes in Milling Processes

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## Abstract

Several methods exist for the analysis of the dynamic stability of milling processes. The methods based on the time domain analysis have been developed for systems with natural modes parallel to the coordinate system of the machine, that is, axes defined by the spindle (Z), the feed direction (X), and perpendicular to both (Y). For finishing operations, at which the main flexibility comes from the spindle / tool side, the approach is acceptable, but for roughing operations, or for any process in which the structural modes of the machine are involved, the method can not be applied for the analysis of the stability against chatter vibration. The paper shows a modification of the method of Semi-discretization based on a modal content approach. The new method allows to include in the simulation modes in any spatial direction, and further is not limited to a maximum number of three modes, as the previous developments. A comparison of the results with the methods of Semi-discretization and Multi-frequency Analysis is performed, showing excellent agreement.

## Keywords:

Milling, Chatter, Simulation

## 1 INTRODUCTION

In spite of the continuous advance in machine tool technology and milling tools development, cutting process parameters such as spindle speed, feed rate, depth and width of cut tend to be selected conservatively, to avoid the risk of damaging costly workpieces and machine tools during manufacturing. Nevertheless, there is an increasing demand for optimisation of cutting conditions in order to reduce production time and augment material removal rate. Therefore, the capability of simulating the cutting process behaviour, such as machine/tool vibration, cutting forces, demanded spindle torque and power, are of huge importance.

Self-excited or chatter vibrations are well known among milling machine tool users. Nowadays, chatter is one of the most important restrictions of the milling process. Machine tool chatter vibrations prevent obtaining the required accuracy and roughness in the workpiece, and reduce the lifetime of the cutter and of the mechanical components of the machine.

Chatter vibrations appear in many different ways in the milling process, and there are different techniques for its prediction. For example, in high speed rough milling of aluminium, the natural modes limiting the stability are associated to the tool and the toolholder, or to the spindle (chatter frequency is roughly between 300 and 3000 Hz). However in face milling of steel, the critical modes tend to be related to the whole machine tool structure (chatter frequency is roughly between 15 and 100 Hz).

Generally speaking, most of the currently existing methods for estimating process stability are oriented towards one set of process types, and depending on the cutting operation, it might be possible that some or all of these methods cannot be applied. Several methods based on the time domain analysis have been developed for systems with natural modes parallel to the coordinate system of the machine, that is, axes defined by the spindle direction (Z), the feed direction (X), and perpendicular to both (Y). For high speed finishing operations, at which the main flexibility tends to come

from the spindle / tool side, this approach is acceptable, but for steel roughing operations, or for any process in which the structural modes of the machine are involved, these methods cannot be applied for the analysis of the stability against chatter vibration.

## 2 STATE OF THE ART

Chatter first basic research works were carried out in mid 40's [1]. Initially it was thought that chatter was originated by a negative damping caused by the cutting process. Later on, Tobias and Fishwick [2], and Tlustý and Poláček [3], stated that the main reason for self-excited vibrations is the regeneration of the chip thickness and the mode coupling. Merritt [4] presented the vibration regeneration as a closed-loop scheme, which made chatter easier to be understood and predicted. Nevertheless, almost all theoretical and experimental researches were only focused on continuous cutting processes, like turning.

The simulation of milling processes is more complex than for other machining processes due to its discontinuous nature: the direction of the chip thickness, and therefore the chip load, change continuously, and furthermore the number of teeth cutting at any moment can vary. In addition, the cutting forces constantly vary in magnitude and direction, producing forced vibrations and affecting to the actual chip thickness.

The first milling process modelling attempts were carried out by Koegnisberger and Tlustý [5], Sridhar et al. [6], and Opitz and Bernardi [7]. Opitz [7] applied turning stability theory to milling process stability. They replaced the time dependent cutting coefficient by a constant term, the average term over the real cutting period of each cutter. This solution was shown afterwards to be particularly correct for the case of a single dominant mode, in which case coincides with the single frequency solution of Altintas and Budak [8].

In fact, the basic theory did not advance significantly until this research by Altintas and Budak [8] was presented, where the general single frequency multimodal solution

was developed. The next important progress was due to Budak and Altintas [9] who presented the general solution for milling by using Multi-frequency.

Later, Davies et al. [10] presented a solution for a very discontinuous case of milling (very low radial immersion), and found a new unstable situation: the period doubling.

Stepan and Insperger [11] and Insperger and Stepan [12] [13] [14] proposed an analytical solution for stability prediction of general milling operations with single-degree-of-freedom. For that, they applied the Semi-discretization technique, in which the delay differential equation representing dynamics of milling is approximated by series of piece-wise ordinary differential equations. This model proved to be an efficient time domain based analytical solution.

Another successful strategy was presented by Baily et al. [15], called Temporal Finite Element Analysis. The method is similar to the Semi-discretization, but it uses a sophisticated interpolation function to obtain a more accurate approach of the vibration inside the segment between any two discretised points. In this way, the computing time required for preparing the transition matrix is longer, but the size of it, and consequently the time for obtaining its eigenvalues is shorter.

Merdol and Altintas [16] proved that the existence of the period doubling instability can also be explained by the method of Multi-frequency.

All these developments were based on modal models of the system with modes parallel to the Cartesian axes of the process, and limited to three modes at most. Only MAL Inc. [17] proposes the use of experimental FRF functions, in which case any modal directions can be considered.

Insperger programmed the Semidiscretization method for modes in any direction, although this approach is not published yet. In this method a geometrical transformation between the mode directions and the Cartesian displacements is done, but the state of the vibration is defined, as in previous approaches, by the three coordinate displacements and their derivatives (velocities). Still this important advance has limitations: it is not possible to consider more than three modes, and also it is not possible to consider two parallel modes (in X direction for example). Furthermore, when two modes are very close to parallel, numerical problems arise.

### 3 CONTENT OF THE PAPER

As shown in the previous section, Semi-discretization method has been well documented for systems limited to have modes parallel to the X and Y directions. Also, the approaches in the bibliography are not valid for systems with more than one mode in any direction (X or Y).

This paper will present the way to enhance the Semidiscretization method to n-mode systems in any spatial directions, what is necessary to simulate most of the actual roughing operations for hard materials.

The arrangement of this paper is as follows. In section 4 the basic characteristics of the process of roughing dies and moulds is presented. In section 5, the theoretical development of the extended Semi-discretization method is included. In section 6, the experimental mold machining process is detailed. Afterwards in section 6, a comparison between different models is performed.

Section 8 compares the results obtained by using skew modes and by using modes projected to the Cartesian axes. Finally, Section 9 compares the behaviour of the machine for different feed directions / senses, and Section 10 summarises the conclusions.

## 4 ROUGHING OF DIES AND MOULDS

Dies and moulds are characterised by having deep pockets, obliging to use long, relatively slender tools to avoid collisions of the headstock and the part.

The use of long tools gives rise to low radial stiffness, and as a consequence, to a very limited material removal rate due to early appearance of chatter instability.

The solution for this problem consists in using very special tools with low lead angle, as shown in Figure 1. These tools are used with low axial immersion, in the range of 1 mm, giving rise to considerable chip width (in the order of 5 mm). High productivity is obtained by means of very large feed per tooth, in the order of 1.5 mm, producing a chip thickness of roughly 0.3 mm.

With these conditions, the tool is able to produce chip flows in the same order or bigger of more conventional tools. The advantage of these tools comes from a double fact:

- the radial component of the cutting force is much lower than for conventional tools
- a radial displacement (vibration) of the tool has a very low effect on the chip thickness

Both these effects tend to produce a very low directional factor for modes in the XY plane, and therefore, the weakness of the tool in this plane is compensated for by the low directional factor.



Figure 1: Hitachi dye and mould rough milling tool.

As the tool gets a reasonable cutting capacity, the modes of the machine structure, and specially those having large displacement component in the direction Z, come into the game and can give rise to the chip flow limitation by the appearance of chatter instability.

Machine structural modes have, in general, displacement components in the three Cartesian coordinates, and therefore their behaviour cannot be simulated by the methods working with modes in X, Y and Z pure directions only.

Also, the number of modes to consider can be high, as it might be necessary to include the modes of the spindle (typically 2 required), of the tool (again, 2 required), and of the machine (at least 2 in the typical case).

## 5 THEORETICAL DEVELOPMENT USING SEMIDISCRETIZATION METHOD FOR N-MODES

The Semi-discretization method is a well-known technique in the analysis of rigid solids, or in the fluid mechanics. As was mentioned in the state of the art, the application of this theory to stability modelling of the cutting process was carried out by Insperger and Stépán [13], and fully detailed for milling and turning processes with modes in X and Y directions by Insperger [14]. In this section the Semidiscretization method will be enhanced to n-mode systems in any spatial directions.

It is well known that cutting forces can excite vibration modes in anyone of the three directions X, Y and Z, and consequently a superficial undulation in the workpiece is

produced. Therefore, the chip thickness will not be constant and will vary depending on the frequency of vibration, the spindle speed and the mode direction with respect to the chip thickness direction.

The mechanical model of the milling process for a system with a single mode can be seen in the Figure 1, where the dynamical parameters are defined as  $m$  equivalent mass of the mode,  $c$  equivalent damping coefficient, and  $k$  equivalent stiffness. The angular spindle speed is expressed as  $\Omega$  in rad/s, and the projection of the cutting force in the modal direction as  $F_m$ .

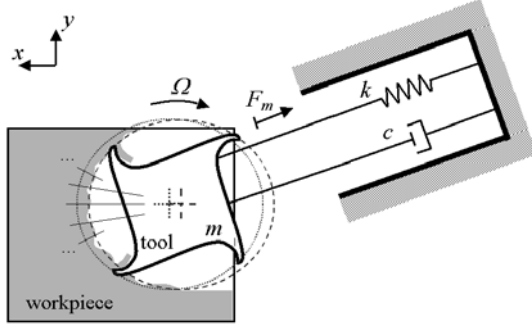


Figure 2: Mechanical model of milling process.

In the modal approach, the relative displacement between tool and part will be calculated as the addition of the modal components. If  $\phi_i$  represents the displacement of the  $i^{\text{th}}$  mode, and  $q_i$  is the modal content of this  $i^{\text{th}}$  mode, the Cartesian relative displacement will be:

$$\begin{Bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{Bmatrix} = \begin{bmatrix} \phi_{x1} & \dots & \phi_{xm} \\ \phi_{y1} & \dots & \phi_{ym} \\ \phi_{z1} & \dots & \phi_{zm} \end{bmatrix} \begin{Bmatrix} q_1 \\ \vdots \\ q_m \end{Bmatrix} \quad (1)$$

It is well known that the modes can have any size, depending on the modal mass that was selected. In the following, a unit modal mass will be assumed, in which case the equivalent mass between part and tool for any mode can be calculated as:

$$m = (\phi_x^2 + \phi_y^2 + \phi_z^2)^{-1} \quad (2)$$

The dynamic equilibrium equation in modal coordinates is:

$$\begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \ddots \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \vdots \\ \ddot{q}_m \end{Bmatrix} + \begin{bmatrix} 2\zeta_1\omega_1 & & & \\ & \ddots & & \\ & & 2\zeta_m\omega_m & \\ & & & \ddots \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_m \end{Bmatrix} + \begin{bmatrix} \omega_1 & & & \\ & \ddots & & \\ & & \omega_m & \\ & & & \ddots \end{bmatrix} \begin{Bmatrix} q_1 \\ \vdots \\ q_m \end{Bmatrix} = \begin{bmatrix} \phi_{x1} & \dots & \phi_{xm} \\ \phi_{y1} & \dots & \phi_{ym} \\ \phi_{z1} & \dots & \phi_{zm} \end{bmatrix}^T \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix} \quad (3)$$

where  $\omega_i$  and  $\zeta_i$  are respectively the natural frequency and damping ratio of the  $i^{\text{th}}$  mode, and  $f_x$ ,  $f_y$ , and  $f_z$  are the Cartesian components of the force applied to the system.

The first step to develop Semi-discretization method consists in the definition of the repetition time, which will be the tooth passing period, and the time interval  $[t_i, t_{i+1})$  of  $\Delta t$  increments for  $i=0,1,\dots,n$ . So that the time increment at step can be expressed as  $\Delta t = \tau / n$  where  $\tau$  is the time delay (tooth passing period) and  $n$  is the approximation parameter of the Semi-discretization method.

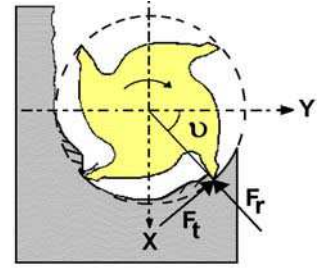


Figure 3: Milling cutting forces.

The total cutting angle ( $360^\circ/\text{number of teeth}$ ) is divided into sufficiently small arcs, according to the defined time increment. So that, the cutting angle achieves from  $v_0$  to  $v_n$ , then the arcs are  $v_0, v_1, v_2, v_3, \dots, v_n$ . At each of these positions the displacement vectors of the tool at the previous pass are stored (4). These would represent the trajectory of tool tip in the previous pass.

$$\{\delta_0 \quad \delta_1 \quad \delta_2 \quad \delta_3 \quad \dots \quad \mathbf{q}_n \quad \dot{\mathbf{q}}_n\}^T \quad (4)$$

where  $\delta_i$  represents a vector containing the three Cartesian displacements at the previous tooth pass at position  $i$ .

The representation of the system will be obtained by the 'state', which is a minimum set of variables univocally representing the instantaneous vibration. The state in this approach will consist of the modal participations ( $\mathbf{q}$ ) and their derivatives respect time ( $\dot{\mathbf{q}}$ ).

Starting with the tool tip at the last angle, it is needed to know the state at that position, that is, the modal displacement and velocity for all the modes. For that position, these variables will be stored instead of the displacements.

With position and speed defined by the parameters with sub index 'n', the state when a new tooth reaches the angle '0' coincides with the state at 'n', because the arc length was selected as being exactly the tooth pass. Therefore,  $\mathbf{q}_0 = \mathbf{q}_n$

Now, at angle '0' chip width is calculated as,

$$h = \begin{bmatrix} \sin(\kappa)\sin(\varphi) \\ \sin(\kappa)\cos(\varphi) \\ -\cos(\kappa) \end{bmatrix}^T \left( \begin{bmatrix} \phi_{x1} & \dots & \phi_{xm} \\ \phi_{y1} & \dots & \phi_{ym} \\ \phi_{z1} & \dots & \phi_{zm} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} - \delta_0 \right) \quad (5)$$

Cutting force at state  $\mathbf{q}_0$  is,

$$\begin{Bmatrix} f_t \\ f_r \\ f_a \end{Bmatrix} = -b \begin{bmatrix} k_t \\ k_r \\ k_a \end{bmatrix} \begin{bmatrix} \sin(\kappa)\sin(\nu) \\ \sin(\kappa)\cos(\nu) \\ -\cos(\kappa) \end{bmatrix}^T \left( \mathbf{\Phi} \mathbf{q}_0 - \delta_0 \right) \quad (6)$$

Projection of force into the modes is,

$$f_\phi = -b \mathbf{\Phi}^T \begin{bmatrix} \cos(\nu) & \sin(\nu) & 0 \\ -\sin(\nu) & \cos(\nu) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} f_t \\ f_r \\ f_a \end{Bmatrix} \quad (7)$$

As a first approach, the integration of the trajectory can be done based on a constant force with this value along the Semi-discretization segment. A more accurate solution can be obtained by calculating the displacement between 0 and 1 with inclusion of the velocity at position 0. Similarly, excess material from previous pass could be obtained by linear interpolation between 0 and 1.

Now the new state at position 1 can be calculated by:

$$\mathbf{q}_1 = \mathbf{A}_{01} \mathbf{q}_0 + \mathbf{R}_{01} \mathbf{f}_\phi \equiv \mathbf{A}_{01} \begin{Bmatrix} q_0 \\ \dot{q}_0 \end{Bmatrix} - \mathbf{B}_{01} \mathbf{q}_0 + \mathbf{C}_{01} \delta_0 \quad (8)$$

with

$$\mathbf{A}_{01} = \begin{bmatrix} \mathbf{A11} & \mathbf{A12} \\ \mathbf{A21} & \mathbf{A22} \end{bmatrix}$$

$$\mathbf{A11} = \text{diag} \begin{bmatrix} \left( \cos(v_1 \cdot \tau) - \frac{\alpha_1}{v_1} \cdot \sin(v_1 \cdot \tau) \right) \cdot e^{\alpha_1 \cdot \tau} \\ \vdots \\ \left( \cos(v_m \cdot \tau) - \frac{\alpha_m}{v_m} \cdot \sin(v_m \cdot \tau) \right) \cdot e^{\alpha_m \cdot \tau} \end{bmatrix} \quad (9)$$

$$\mathbf{A12} = \text{diag} \begin{bmatrix} \frac{1}{v_1} \sin(v_1 \cdot \tau) e^{\alpha_1 \cdot \tau} \\ \vdots \\ \frac{1}{v_m} \sin(v_m \cdot \tau) e^{\alpha_m \cdot \tau} \end{bmatrix}$$

$$\mathbf{A21} = \text{diag} \begin{bmatrix} -\frac{v_1^2 + \alpha_1^2}{v_1} \cdot \sin(v_1 \cdot \tau) e^{\alpha_1 \cdot \tau} \\ \vdots \\ -\frac{v_m^2 + \alpha_m^2}{v_m} \cdot \sin(v_m \cdot \tau) e^{\alpha_m \cdot \tau} \end{bmatrix} \quad (10)$$

$$\mathbf{A22} = \text{diag} \begin{bmatrix} \left( \cos(v_1 \cdot \tau) + \frac{\alpha_1}{v_1} \cdot \sin(v_1 \cdot \tau) \right) \cdot e^{\alpha_1 \cdot \tau} \\ \vdots \\ \left( \cos(v_m \cdot \tau) + \frac{\alpha_m}{v_m} \cdot \sin(v_m \cdot \tau) \right) \cdot e^{\alpha_m \cdot \tau} \end{bmatrix}$$

The term  $\mathbf{B}_{01}$  of equation (6) is calculated as,

$$\mathbf{B}_{01} = \begin{bmatrix} \tau^2 & \tau^3 \\ 2 & 6 \\ \tau & \tau^2 \\ & 2 \end{bmatrix} \begin{bmatrix} \mathbf{Bf} & 0 \\ 0 & \mathbf{Bf} \end{bmatrix} \quad (11)$$

$$\mathbf{Bf} = b [\Phi]^T \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & 0 \\ -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} k_r \\ k_r \\ k_a \end{bmatrix} \begin{bmatrix} \sin(\kappa) \sin(\varphi) \\ \sin(\kappa) \cos(\varphi) \\ -\cos(\kappa) \end{bmatrix}^T [\Phi]$$

In the same way, the term  $\mathbf{C}_{01}$  of equation (11) is expressed as,

$$\mathbf{C}_{01} = \begin{bmatrix} \tau^2 \\ 2 \\ \tau \end{bmatrix} \mathbf{Cf} \quad (12)$$

$$\mathbf{Cf} = b [\Phi]^T \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & 0 \\ -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} k_t \\ k_r \\ k_a \end{bmatrix} \begin{bmatrix} \sin(\kappa) \sin(\varphi) \\ \sin(\kappa) \cos(\varphi) \\ -\cos(\kappa) \end{bmatrix}^T$$

Now, the new displacements of the tool/part will be,

$$\begin{aligned} \hat{\delta}_0 &= [\Phi_0] \mathbf{q}_0 = \mathbf{T} \mathbf{q}_0 \\ \hat{\delta}_1 &= [\Phi_1] \mathbf{q}_1 = \mathbf{T} \mathbf{q}_1 \end{aligned} \quad (13)$$

Writing down the complete vector for position 0,

$$\hat{\delta}_0 = \mathbf{T} \mathbf{q}_0 = \mathbf{T} \mathbf{q}_n = \begin{bmatrix} 0 & \dots & 0 & \mathbf{T} \end{bmatrix} \begin{Bmatrix} \delta_0 \\ \vdots \\ \delta_{n-1} \\ \mathbf{q}_n \end{Bmatrix} \quad (14)$$

$$\mathbf{q}_0 = \mathbf{q}_n$$

And for position 1,

$$\mathbf{q}_1 = \mathbf{A}_{01} \begin{Bmatrix} \mathbf{q}_0 \\ \dot{\mathbf{q}}_0 \end{Bmatrix} - \mathbf{B}_{01} \cdot \mathbf{q}_0 + \mathbf{C}_{01} \cdot \delta_0 = \mathbf{D}_{01} \cdot \mathbf{A}_{n0} \cdot \mathbf{q}_n + \mathbf{C}_{01} \cdot \delta_0 \quad (15)$$

$$\begin{Bmatrix} \hat{\delta}_0 \\ \hat{\delta}_1 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 & \mathbf{T} \\ \mathbf{T} \cdot \mathbf{C}_{01} & 0 & \dots & 0 & \mathbf{T} \cdot \mathbf{D}_{01} \end{bmatrix} \begin{Bmatrix} \delta_0 \\ \vdots \\ \delta_{n-1} \\ \mathbf{q}_n \end{Bmatrix}$$

In the same way, it is possible to advance to the other time steps and obtain their step matrices. When the calculation arrives to the step  $n$ , the total transition matrix will have been obtained.

The stability analysis of the system reduces to obtaining the eigenvalue of maximum amplitude of the transition matrix. If any eigenvalue has a magnitude larger than 1, then the system is unstable, whereas when all magnitudes are lower than 1 the system is stable.

## 6 TEST CASE

To show the importance of including the actual mode direction instead of the projection of the modal displacement direction to the Cartesian axes, simulations will be run on real values for the machine dynamic behaviour, material / tool specific cutting pressures, and process parameters.

### 6.1 Machine dynamics

The machine selected is a portal milling machine, with ram displacement oriented in the vertical direction. For the application of milling moulds and dyes, the preferred tool orientation is vertical. The dynamic behaviour of the machine can be represented by two modes with displacement in the planes XZ and YZ. The parameters obtained from an experimental modal analysis, for a particular extension of the ram are shown in table 1.

$\omega$ [Hz]	$k$ [N/ $\mu\text{m}$ ]	$\zeta$ [%]	$\phi_x$	$\phi_y$	$\phi_z$
32.5	21.45	1.60	0.951	0.015	-0.307
42.5	48.70	1.90	0.200	0.980	0.014

Table 1. Machine tool modal parameters.

### 6.2 Tool, part material, and specific cutting forces

The tool selected is of a type specially devoted to milling of moulds and dyes. Figure 4 shows a typical application for this tool, in rough milling the mould for a front bumper of a car.

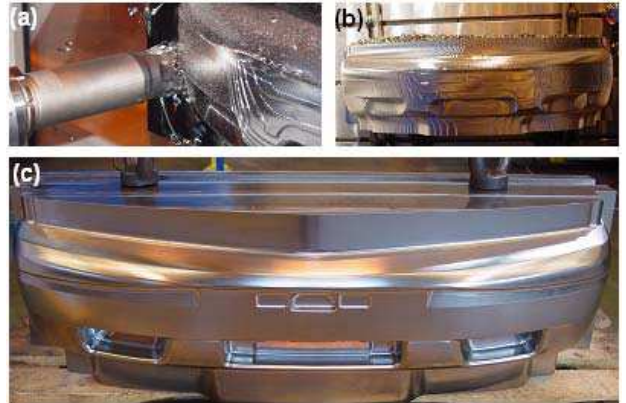
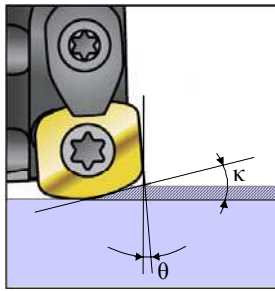


Figure 4: Automotive front bumper mould.

Figure 5 shows the shape of this tools' inserts, and their 'equivalent' lead angle.





Cutting tool: Hitachi  
 Model: ASR5080-5  
 Insert type: EDNW15T4TN CY250  
 Operation: Roughing  
 Diameter:  $\varnothing$  80 mm  
 z: 5  
 $\kappa$ : 15°  
 ae: 55 mm  
 f: 5000 mm/min  
 N: 600 r/min  
 Vc: 150 m/min

Figure 5: Cutting tool geometry.

The recommended peripheral speed of the tool is 150 m/min, although a variation between 100 to 200 m/min can be acceptable. For the tool diameter of 80 mm, this speed range corresponds to a rotational speed range from 400 to 800 r/min.

The cutting force is estimated by a mechanistic model. The cutting coefficients of this kind of tool on a carbon steel were measured by milling tests, following the procedure proposed by Altintas [18]. The tests were done for several peripheral velocities, several axial immersions, and several feeds per tooth. The average results obtained from the experiments were:

$K_t = 1600$  MPa,  $K_r = 161$  MPa,  $K_a = 621$  MPa.

The equivalent lead angle varies with the axial penetration. For the sake of simplicity, a constant lead angle of 15° was assumed for the simulations.

### 6.3 Process parameters

The process considered is a downmilling with radial immersion 55 mm. This radial immersion results in an angular immersion from 68° to 180°.

For the proposed five fluted Hitachi cutting tool, the maximum axial depth of cut ( $a_p$ ) is limited due to insert design, to 2 mm. In the practice, the axial immersion takes values in the order of 1 mm.

## 7 COMPARISON BETWEEN STABILITY MODELS

As a first result, the lobe diagrams obtained with the Semi-discretization approach will be compared with the results obtained by Multi-frequency. Figure 6 shows these two results. Comparing the results obtained by Multi-frequency and by Semi-discretization, it is concluded that both methods give the same stability limits.

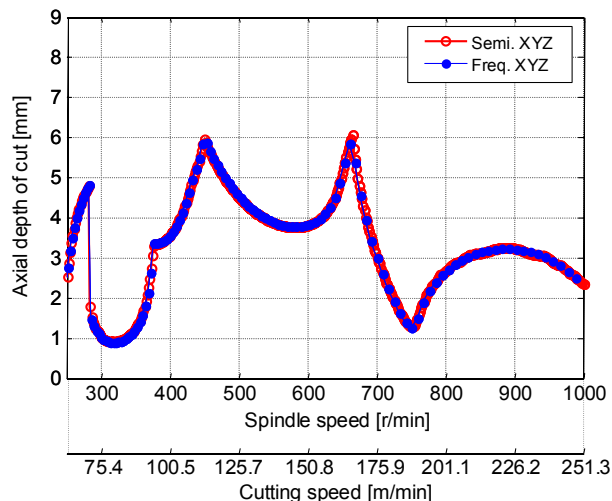


Figure 6: Comparison between Semidiscretisation and Multi-frequency approaches.

By analysing the diagram, it can be seen that the recommended speed region for this tool (400 to 800 r/min) is quite convenient: stability limit above 2 mm is easy to be obtained.

## 8 COMPARISON BETWEEN SKEW MODES AND MODES IN THE CARTESIAN AXES

Figure 7 shows the lobe diagrams for the original data, and for a pair of modes with the same parameters except for the direction of them, which are pure X for the 32.5 Hz mode, and pure Y for the 42.5 Hz mode.

From the lobe diagrams, it can be concluded that approximating the actual direction of the modes by their projection to the Cartesian axes is very far from giving an accurate lobe diagram.

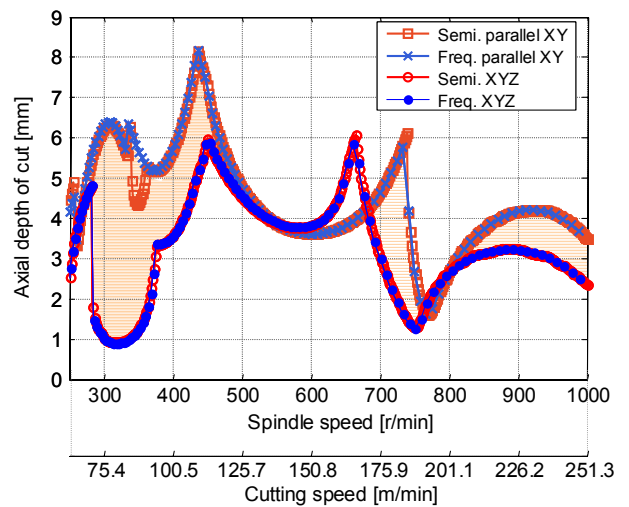


Figure 7: Stability lobes diagram using skew modes and modes in Cartesian axes.

## 9 INFLUENCE OF FEED DIRECTION AND SENSE

For this factor to be analysed, lobe diagrams for feed in directions +X, -X, +Y and -Y were obtained.

Figure 8 shows the stability diagrams for feeds in directions +X (0°) and -X (180°). It can be observed that the system is much more stable when feed is oriented towards +X than when oriented towards -X.

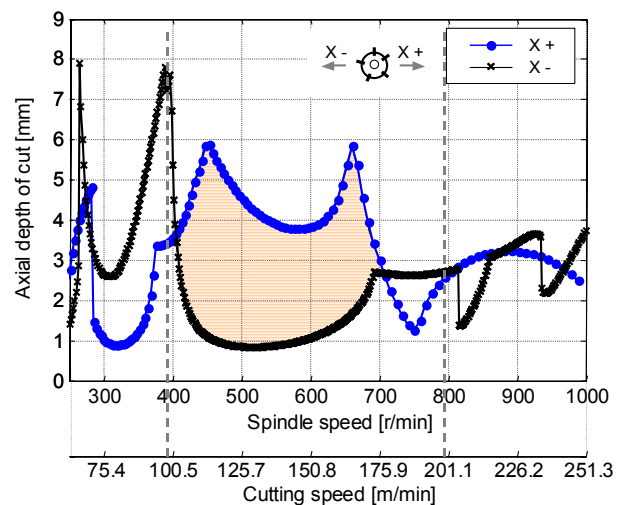


Figure 8: Stability diagram for feed directions X+, X-.

Figure 9 shows the stability diagrams for feeds in directions +Y (90°) and -Y (270°). As for the previous cases, the system is more stable in one sense (+Y) than in the other (-Y).

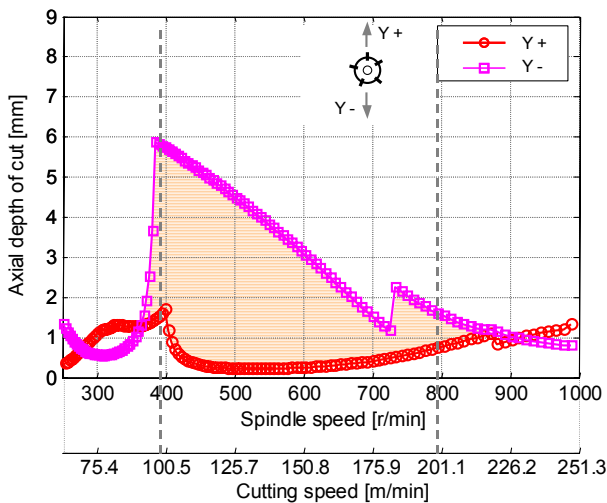


Figure 9: Stability diagram for feed directions Y+,Y-.

It can be concluded that the combination of a lead angle lower than 90°, and vertical components for the modes give rise to system sensitivity to feed sense: the system behaves differently when the feed is inverted.

In many circumstances, the different cutting capacity obtained when inverting the feed is attributed to the non-linear behaviour of the slideways. This analysis shows that the change in the directional factor when reversing the feed has a large influence, probably much larger than any possible non-linearity at the guiding pads.

## 10 SUMMARY

In this paper, some particularities of the process of rough milling of moulds and dies are presented and analysed. The need for mathematical models including the three component of the modes of the machine is assessed from it.

The application of the developed models shows that neglecting the three directional components of the modes can produce very large differences in the stability predicted for the case of roughing mills for moulds and dies.

Also, it is shown that the influence of the feed direction (sense) on the stability is very large, and as a consequence it can be very convenient to take this effect into consideration at the time of producing the CAM program for the rough milling.

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