# A generative model for the joint registration of multiple point sets 

## I.Abstract

We propose a generative model and its associated algorithm for the joint registration of multiple point clouds. All the point sets are assumed to be rigidly transformed realizations of a Gaussian Mixture Model (GMM), whose means play the role of a central scene model. A formally derived Expectation Conditional Maximization (ECM) algorithm estimates both the GMM parameters and the rigid transformations (one per set), so that the point sets can be jointly registered without favoring any particular set.


## 2. The model

If $v_{j i}$ denotes the $i$-th 3 d point of the $j$-th point set, its realization is modeled by a mixture of $K$ Gaussian and one uniform components

$$
P\left(v_{j i}\right)=\sum_{k=1}^{K} p_{k} \mathcal{N}\left(\varphi_{j}\left(v_{j i}\right) ; \mathbf{x}_{k}, \Sigma_{k}\right)+p_{K+1} \mathcal{U}(h)
$$

with $\varphi_{j}\left(v_{j i}\right)=R_{j} v_{j i}+t_{j}$ where
$R_{j}, t_{j}: 3 \mathrm{D}$ rotation/translation of $j$-th set,
$x_{k}, \Sigma_{k}$ : means/variances of the Gaussian components
$p_{k}:$ prior terms of the mixture
$h:$ the volume that encompasses all the data
This leads to an augmented parameter set of mixture parameters and transformations that has to be estimated:

$$
\Theta=\left(\left\{p_{k}, x_{k}, \Sigma_{k}\right\}_{k=1}^{K}, p_{K+1},\left\{R_{j}, t_{j}\right\}_{j=1}^{M}\right)
$$

## 4.I Results on synthetic data-sets

Registration of four partially-viewed point sets of a 3D model

- 10-degree rotation between successive views
- different down-sampling per point set
- point noise and outliers were added
- Root MSE of rotation error is used for evaluation


Lucy


Armadillo

## References

[4] Besl \& McKay: A method for registration of 3D shapes. PAMI (1992)
[15] Horaud et al.: Rigid and articulated point registration with expectation conditional maximization. PAMI (2011)
[23] Myronenko \& Song: Point-set registration: Coherent point drift. PAMI (2010)
[30] Williams \& Bennamoun: Simultaneous registration of multiple corresponding point sets. CVIU (2001)
[18] Jian \& Vemuri: Robust point set registration using Gaussian mixture models. PAMI (2011)

## 3.The algorithm

We introduce a hidden variable $z_{j i}$, such that $z_{j i}=k$ assigns a transformed observation $\varphi_{j}\left(v_{j i}\right)$ to the $k$-th mixture component, and we aim to maximize the expected complete-data log-likelihood. This reduces into the constrained optimization problem:

$$
\begin{gathered}
\max _{\Theta} f(\Theta) \\
\text { s.t. } R_{j}^{T} R_{j}=I,\left|R_{j}\right|=1, \forall j \\
f(\Theta)=-\frac{1}{2} \sum_{j i k} a_{j i k}\left(\left\|\phi_{j}\left(v_{j i}\right)-x_{k}\right\|_{\Sigma_{k}}^{2}+\log \left|\Sigma_{k}\right|-2 \log p_{k}\right)+\log p_{K+1} \sum_{j i} a_{j i(K+1)}
\end{gathered}
$$

and $\quad a_{j i k}=P\left(z_{j i}=k \mid v_{j i} ; \Theta\right) \quad$ is the posterior probability of an assignment. An ECM scheme is adapted to this problem, thus yielding a novel algorithm that is called JR-MPC (Joint Registration of Multiple Point Clouds) and follows the outline:

Initialize $\Theta$
Repeat until convergence
E-Step: estimate posteriors
$C M$-Step-1: use current mixture parameters to update transformation
CM-Step-2: use current transformations to update mixture parameters

### 4.2 Results on real data-sets

Ten point clouds captured by moving a TOF camera around a scene (color is used for visualization)


## 5. Conclusions

- A probabilistic model to jointly register multiple point sets is proposed.
- The proposed method treats all the point sets on an equal footing
- An ECM-based algorithm estimates an augmented parameter set that consists of GMM parameters and rigid transformations.
- As a by-product, an outlier-free scene model is reconstructed (see our videos).

